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Particle accelerator measurements of ^{10}Be in marine accumulation rates Intercomparison with beta counting method

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Abstract. Cosmogenic beryllium-10 activities have been measured in marine accumulation rates of up to ~ 6 m.y age by conventional beta counting technique and by accelerator mass spectrometry. The two sets of data at ^{10}Be levels of 10^9 – 10^{10} atoms/g agree within the errors of the two methods. The detection limit for ^{10}Be by the accelerator mass spectrometry is about five orders of magnitude lower than that with the beta counting method.

Keywords. Particle accelerator; beryllium-10; marine sediments; manganese nodules; accumulation rates.

1. Introduction

With the advent of the accelerator mass spectrometric (AMS) technique, it has become possible to measure long-lived radionuclides at sensitivity levels of orders of magnitude better than what is possible with the conventional decay counting technique (Mullar 1977; Gove 1978; Raisbeck *et al* 1978). Beryllium-10 is now measurable in litres of water (Raisbeck *et al* 1979a, 1980, 1981) and in sub-gram quantities of marine sediments and ferromanganese nodules using the AMS technique (Raisbeck *et al* 1981; Turekian *et al* 1979; Krishnaswami *et al* 1982). These studies are valuable for understanding the related geophysical/geochemical and astrophysical phenomena (Raisbeck *et al* 1981; Somayajulu 1977). Comparison between the two techniques using natural samples in the case of cosmogenic ^{14}C and with standards in the case of ^{10}Be resulted in good agreements (Krishnaswami *et al* 1982; Brown *et al* 1982).

convenient for analysis by AMS.

From a set of 8 samples (table 1) seven were measured at Zurich out of which were meant for intercomparison; the sixth sample was known to contain activity was too low to be precisely measured by the beta counting method. Two samples were 'blanks' prepared to check on the contaminating activity (other than ^{10}Be) arising from chemicals used in the extraction and purification of ^{10}Be . Sample 1 is marine sediment (red clay) from a depth of 32–48 cm of a gravity core from the central equatorial Pacific. Samples 2–6 are sliced sections from ferromanganese nodules out of which samples 2–4 belong to the sample nodule, ANTP 58D and 5 and 6 belong to nodule ARIES 39D; both nodules are from the Pacific Ocean.

^{10}Be measurements by the beta counting method were followed by chemical separation, repurification, recounting and half-thickness measurements of the beta activity. Aliquots of the pure BeO extracts were sent to Zurich for ^{10}Be measurement by the Tandem-van-deGraaf accelerator mass spectrometer (Wolfli *et al* 1982). Each sample was run independently three times and each blank twice. There is a systematic error of 2–3% and the statistical error in the case of all samples during all runs is 1%; for blanks it is 20–30%. The AMS ^{10}Be data presented in table 1 are the means of the three runs and the errors quoted are one sigma standard deviation. The ^{10}Be concentrations obtained by the decay counting method are also presented in table 1. Errors quoted for these values are quadratic sums of counting statistics (1.4–8.7%), errors associated with chemical separation (1.2–2.5%), counting efficiencies (2.0–3.7%) and one sigma counting statistics on the blank activity including background of the detector (4–6%). Two of the PRL blanks (7 and 8) were also analysed by AMS. It must be noted here that the chemical separation introduces a beta blank of about 2 counts/hr (cph) as determined from a large number of blank runs on ^{10}Be free samples (Sharma 1982). The blank activity is not as shown by the accelerator results (table 1); it may be due to ^{40}K and its daughter nuclides of the U–Th series which may follow the chemical separation procedure used for beryllium. The ^{10}Be data are blank corrected. It is seen from

Table 1. Intercomparison of ^{10}Be measurements in marine deposits by different techniques.

Depth

$^{10}\text{Be}/^9\text{Be}$

AMS

De

in those of the samples.

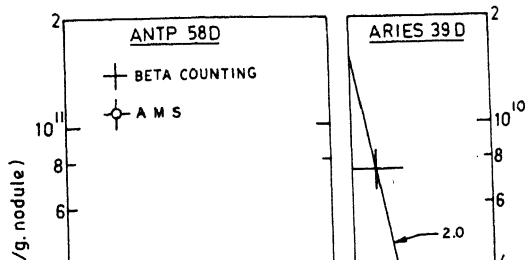
Results and discussion

The agreement between the results from the decay and AMS techniques for sample 1 (table 1) is surprisingly good, if one takes into account the fact that the uncertainty in the decay constant of ^{10}Be ($\lambda = 4.62 \times 10^{-7} \text{ yr}^{-1}$) is of the order of 20% (adopted from Yiu and Raisbeck 1972), and that the $^{10}\text{Be}/^9\text{Be}$ ratio measured relative to that of a standard (produced by neutron activation of ^9Be) is known only within about 15%.

In sample 6, the bottom section of ARIES 39D, where there was no measurable signal above the blank level by beta assay, atom counting yields a ^{10}Be concentration of $(1.79 \pm 0.18) \times 10^9$ atoms/g nodule. Based on the decrease in the ^{10}Be concentration between the two sections of ARIES 39D, its accumulation rate is deduced to be $0.2 \text{ m}/10^6 \text{ yrs}$ (table 1, figure 1).

The beta counting data of ANTP 58D do not lead to a monotonic ^{10}Be decay with depth. There is a change in the slope of the curve with depth, indicating a change in the growth rate of this nodule. Results using the AMS technique confirm this behaviour. This case is similar to that of a small nodule from the Indian Ocean (Krishnaswami *et al* 1982), but departs significantly from the exponential decay with depth observed in the case of about a dozen nodules from the world ocean (Sharma and Somayajulu 1982).

Growth rates based on the first two points of ANTP 58D are 3.8 and 5.3 mm/ 10^6 yrs by the decay and AMS techniques, respectively. The growth rates of the two nodules reported here are in the range observed for other nodules from the world ocean.



activity of the chemical blank.

The present detection limit for AMS method on the other hand is about 1 mg Be (corresponding to a $^{10}\text{Be}/^{9}\text{Be}$ atom ratio of 10^{-13} which is usually below the level of boron contamination of the samples (Brown *et al* 1982; Wolfli *et al* 1982) and of chemical blanks (table 1). Therefore, with typical accumulation rates, the AMS method is applicable in ~ 10 mg of open ocean manganese nodules and about 100 mg of deep-sea sediments with the AMS technique. The studies of depositional histories of manganese coatings on biological debris and of micronodules now appear feasible.

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References

- Bennett C L, Beukens R P, Clover M R, Elmore D, Gove H E, Kilius L, Litherland A E and Pearson G A 1982 *Science* **201** 345
- Brown L, Klein J, Middleton R, Selwyn Sacks I and Tera F 1982 *Proceedings 11th Int. Conf. on Radiocarbon*, June 21-25 Radiocarbon in press
- Gove H E (ed) 1978 *Proc. 1st Conf. Radioactive Dating with Accelerators*, University of Arizona Press
- Krishnaswami S, Mangini A, Thomas J H, Sharma P, Cochran J K, Turekian K K and Ravel J C 1982 *Earth Planet. Sci. Lett.* **59** 217
- Ku T L and Broecker W S 1969 *Deep Sea Res.* **16** 625
- Mullar R A 1977 *Science* **196** 489
- Mullar R A, Stephenson E J and Mast T S 1978 *Science* **201** 347
- Raisbeck G M, Yiou F, Fruneau M and Loiseaux J M 1978 *Science* **202** 215
- Raisbeck G M, Yiou F, Fruneau M, Loiseaux J M, Lieuvain M and Ravel J C 1979a *Nature* **278** 354
- Raisbeck G M, Yiou F, Fruneau M, Loiseaux J M, Ravel J C and Hays J D 1979b *Geophys. Res. Lett.* **6** 103
- Raisbeck G M, Yiou F, Fruneau M, Loiseaux J M, Lieuvain M, Ravel J C, Reyss J L and Sarda S 1982 *Earth Planet. Sci. Lett.* **51** 275
- Raisbeck G M, Yiou F, Fruneau M, Loiseaux J M, Lieuvain M, Ravel J C and Loric V 1982 *Earth Planet. Sci. Lett.* **59** 825
- (London) **292** 825
- Sharma P 1982 *Implications of beryllium-10 measurements in marine sediments and manganese nodules* Ph.D. Thesis Gujarat University p. 196
- Sharma P and Somayajulu B L K 1982 *Earth Planet. Sci. Lett.* **59** 235

Worldwide features of the strength of recurrent geomagnetic ac

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Abstract. Variation of the strength of recurrent geomagnetic activity, which occurs before a sunspot minimum, with local time is studied for a network of observatories in different latitude and longitude zones. For this purpose, hourly averages of horizontal magnetic field (H) for each UT hour for 173 days, which are totally free of disturbances due to geomagnetic transients, have been subjected to spectral analysis. Well-defined spectral peaks associated with periodicities of 28, 14 and 9 days were present in almost all the spectra. The pattern of variation of the strength of the 27-day signal changes from a diurnal one at low latitudes to a semi-diurnal one near the Sq focus and in this region, the 14-day signal appears to have an independent origin irrespective of the longitude zone. A study of 27-day oscillations in the daily H field also indicates that apart from ring current modulation, both Sq and E fields also undergo 27-day oscillations during the declining phase of a solar cycle through the ionospheric wind system.

Keywords. Worldwide features; geomagnetic activity; sunspot minimum; ring current; ionospheric winds.

Introduction

The strength of recurrent geomagnetic activity, as measured by the amplitude of the 27-day oscillation in the mean daily values of horizontal magnetic field has been studied in the equatorial region for different phases of solar activity (Bhargava and Rao 1970). Recurrent activity before sunspot minimum is now known to have features quite distinct from activity during other phases of the solar cycle (Legras and Damon 1981). Bhargava (1973) and Bhargava and Rangarajan (1975) have studied the evening forenoon asymmetry in the amplitude of the 27-day oscillations of horizontal magnetic field at Alibag (dipole lat. 9.5°N) both during the declining phase of the solar cycle just before minimum and during high solar activity. The asymmetry is

In this paper we carry out a systematic search for the local dependance of the 27-day geomagnetic signal due mainly to recurrent solar activity using horizontal intensity observations for the period 20 December 1973 to 10 June 1974 (Bartels rotation No. 1920 to 1926) of a carefully selected network of stations. The coordinates of the stations chosen are given in table 1. In this paper the stations cover the region between the equatorial electrojet belt and the northern Sq focus. Inclusion of Nurmijarvi-dipole conjugate of the main focus in analysis was necessitated to establish the anomalous nature of the 27-day signal at Kerguelen, an island in the southern hemisphere.

2. Data analysis

Hourly averages of horizontal intensity (H) for each UT hour from 20 December 1973 to 10 June 1974 (173 days) were used in spectral analysis. A version of FFT described in detail in Rangarajan and Bhargava (1974) was used as bandwidth for each spectral estimate. Some sample power spectra for Trivandrum and Alibag—are shown in figure 1. For each station two spectra are depicted, one for that particular hour when the strength of the 27-day signal is at a minimum and another for the hour when the power spectral density of the 27-day signal is maximum. In almost all the spectra there are distinct spectral peaks, above the continuum, corresponding to the periodicities of 27 days. Power densities associated with these periodicities, representing the 27-day rotation periodicity and its first harmonic, for each hour are depicted in figures 2 to 6.

Table 1. Geographic and dipole coordinates of observatories used

Station Name	Code	Geographic	
		Latitude	Longitude
Trivandrum	TRD	8° 29' N	76° 57' E
Alibag	ABG	18° 38' N	72° 52' E
Sabhawala	SAB	30° 20' N	77° 48' E
Tashkent	TKT	41° 20' N	69° 37' E
Luning	LNP	25° 00' N	121° 10' E

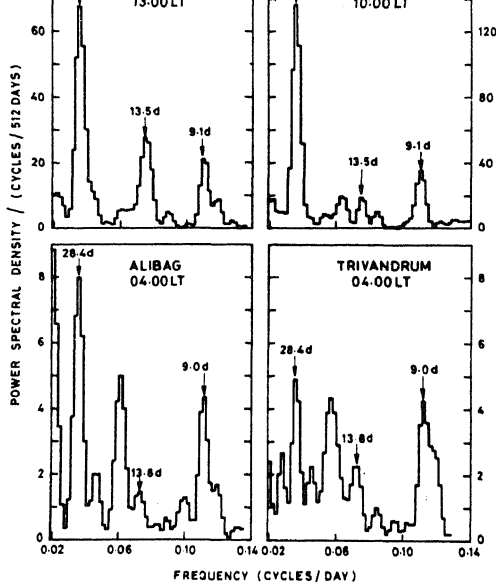


Figure 1. Power spectra of series of hourly averages for two particular hours at Trivandrum and Alibag.

Results and discussion

27-day and 14-day signal in Indian zone

The stations in the Indian zone are within 6° of 75° E longitude so that the time may be taken to be 5 hours ahead of UT. Power densities at the two frequencies as a function of local time for four stations are shown in figure 2. The graphs at once show a strong dependence of the solar rotation signal on time of day with peak magnitude being a function of latitude. Close to the dip equator the power density varies between about 10 nT^2 (unit bandwidth) at predawn hour to a peak value of about 135 nT^2 at local noon. At low latitude (Alibag) the power density varies between about 70 nT^2 and the time of peak amplitude shifts to post-noon hours, similar to

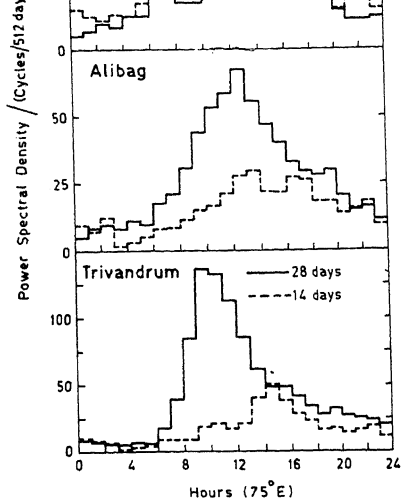


Figure 2. Power spectral densities for 28-day and 14-day signals as for stations in the Indian zone.

and Trivandrum. The present results, on the other hand, clearly show by a factor of 2 in the peak power close to noon time. These differences are attributed to the fact that when the data covering a large time-span of values are utilized, the significant features and fine structure could be the opposite relationship of solar transients and stable coronal activity (Hansen *et al* (1976). The difference in time of peak power between the latitude station outside the belt is consistent with the suggestion (1967) that the electrojet field leads the Sq field by more than an hour. The harmonics of fundamental periodicities appear in the spectrum of the sinusoidal nature of the variations. If the 14-day spectral line were considered the basic 28-day periodicity the local time behaviour of the amplitude will be expected to be similar with reduced magnitude at all hours. The LT pattern for the 14-day line clearly indicates that while this expectation is fulfilled at Alibag, close to the Sq focus (and on either side of it) the results show that the significant oscillation of the H field has a basic periodicity of 14 days.

Recently Briggs (1979) showed that a 27-day oscillation could

ed. The corresponding variations are depicted in figure 3. Rush and Richmond (1973) and Kane (1978) have shown that such differencing procedure for stations in the same longitude zone but separated in latitude give the time variations of only the ionospheric (electrojet or Sq) part of the field as the magnetospheric contribution is effectively removed.

Significant power at the solar rotation frequency confined to just a few hours before local noon for the (TRD-ABG) data clearly indicates a 27-day oscillation of the electrojet strength. The signal derived from (ABG-SAB) difference, indicative of the ionospheric origin, has nearly the same magnitude for peak power. It may therefore, be inferred that apart from ring current modulation, both Sq and electrojet also undergo 27-day oscillations during the declining phase. As the magnitudes of the two are comparable, it is quite likely that the ionospheric wind system rather than the electrical conductivity shows similar oscillations as inferred by Briggs (1969). The electrical conductivity is more likely to be affected during periods of high solar activity.

Comparison of 27-day signal at a pair of stations in day and night hemisphere

Two magnetic stations, Alibag and Teoloyucan, are located nearly in the same geographic latitude and differ by nearly 180° in longitude. The pair is thus suitable to establish clearly whether the strength of the signal depends on universal (UT) or local (LT) time (LT). Power density change of the 27-day signal as a function of local time for the two stations are shown in figure 4. It is immediately seen that at low latitude the variation is governed by local time (LT) with peak power close to local noon. There is a noticeable difference in the power between the two stations during the afternoon hours. This may be because of the difference in their dipole latitudes, with Teoloyucan being higher by about 20° , leading to a reduction in the magnitude of the contribution of the asymmetric ring current envisaged as the responsible mechanism for the noon/evening asymmetry at Alibag by Bhargava (1973). However a similar difference in strength of asymmetry is not reflected in the chain of Indian zone (figure 2), as has been pointed out earlier in this paper, at Sabhawala and Tashkent the variation pattern for the strength of the 27-day signal assumes a semi-diurnal form.



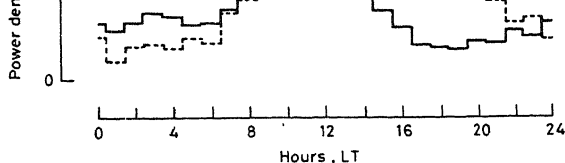


Figure 4. Power spectral density of the 28-day signal as a function of stations Alibag and Teoloyucan.

whereas at Teoloyucan it is diurnal. This goes to show that the position becomes important for the quiet time currents which prevail during a sunspot activity. Dip latitude will play a major role in disturbance field context the effect appears to be a combination of geographic and dependences.

In recent times it has been suggested (Crooker and Siscoe 1981) dusk-centred partial ring current hypothesis for low latitude asymmetry. Disturbance field suffers from ionospheric closure and orientation problem and is abandoned in favour of a system of distributed Birkeland currents. Kamide (1973) estimated the contribution of Birkeland currents to the asymmetry at various geomagnetic latitudes from 0° to 40° using a partial ring current system which would consist of a west-ward flowing partial ring current in the equatorial plane, the field-aligned Birkeland current and an east-ward electrojet in the auroral zone ionosphere. They found that the contribution of Birkeland currents to the asymmetry is larger than that of the partial ring current. However, the contribution of the partial ring current decreases with increasing geomagnetic latitude, that of the Birkeland currents increases with increasing geomagnetic latitude. The total asymmetry due to the whole current system (including the partial ring current and the auroral zone) was found by these authors, to decrease with increasing geomagnetic latitude. In their model calculation we point out that if the asymmetry is due to Birkeland currents alone it may be difficult to explain a decrease in asymmetry with increasing geomagnetic latitude as we find above in the case of Teoloyucan. A more thorough study of the asymmetry at different latitudes is required in order to arrive at a current system responsible for the low-latitude

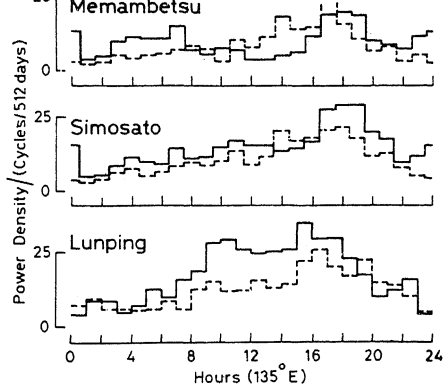


Figure 5. Power densities for 28-day and 14-day signals as functions of 135° E time stations in the Japanese sector.

tral power again indicates it to be an independent periodicity close to the Sq focus. It may be inferred that for a fairly wide longitudinal region in northern hemisphere, the local-time and latitudinal modulation of the solar signal are similar and consistent.

27-day oscillation of the field in southern hemisphere

The local-time change in power densities of the 27- and 14-day spectral lines for selected southern hemisphere stations are shown in figure 6. The noteworthy aspect of the plot is the significant reduction in the peak power density at low latitude stations like Steenbeesthoek, when compared to the stations in the Indian zone. Similar north-south asymmetry has been shown to exist in irregular geomagnetic activity by Mathan (1970) and Rangarajan (1979). At Hermanus, close to southern Sq focus, the amplitude variations are again semidiurnal, while at lower latitude it is diurnal. At Cape Town, located further south, amplitude of the solar rotation signal appears hemispherical. The peak power of 150 nT^2 at dawn is comparable to the noon value in the Tropic of Cancer region (Trivandrum). With increasing daylight, the power density rapidly diminishes reducing almost to zero indicative of a completely different generation mechanism for the field oscillation at this location in contrast to that at other latitudes.

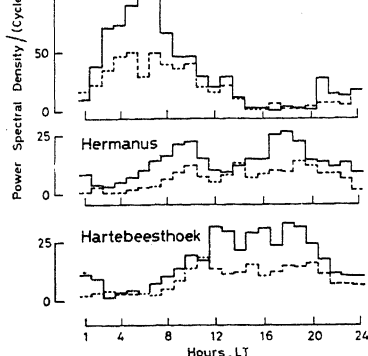


Figure 6. Power spectral densities for 28-day and 14-day signals as functions of local time for three stations in the southern hemisphere are shown in the three lower diagrams. The top diagram shows the power density for the 28-day signal as a function of local time for Nurmijarvi, a station in the northern hemisphere geomagnetically conjugate to Hermanus.

other periodicities and for other island stations. Such work is in progress and will be reported later.

3.5 27-day oscillation in mean daily H field

Mean daily field of H at low latitudes, to a considerable extent, represents the ring current (Bhargava and Rao 1970). Spectra derived from the mean daily field once again indicate a dominant 27-day oscillation. Power density as a function of geographic as well as dipole latitude is shown in figure 7. The ring current field will be symmetric with respect to the equator, and only positive values (positive values) only are considered. It is evident that there is a systematic variation in the strength of the solar rotation signal with latitude. Kerguelen shows an anomalous local time dependance again deviates from the linear trend. At low latitudes the strength of the signal appears slightly better with geographic as compared to dipole latitude.

If the 27-day oscillation of mean daily field arose on account of the ring current alone, one would expect the power density for the 27-day oscillation to have an approximately $\cos^2 \theta$ dependance on the dipole latitude in the tropic latitudes. However, we find that the calculated power densities show a

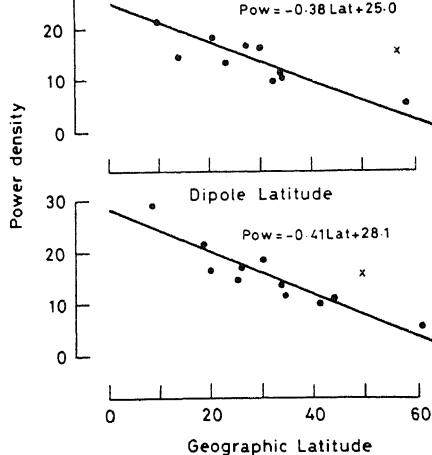


Figure 7. Dependence of power density for the 28-day signal in mean daily values on dipole latitude (top) and on geographic latitude (bottom). Straight lines are least squares with adjustment made for the anomalous behaviour at Kerguelen (marked by X).

During a solar cycle, the solar daily variation undergoes 27-day oscillations passing through the ionospheric wind system. This wind system is controlled by the magnetic equator and this calls for a more organised dependence of the 27-day variation of the power density of the mean daily field on the geographic latitude. In the presence of transient disturbances such as solar flares it would be difficult to detect this 27-day oscillation in the ionospheric wind system. Hence the period that we have chosen for our analysis, being totally free of such transient solar activity, enabled us to pick up this

References

- Rangarajan B N 1973 *J. Atmos. Terr. Phys.* **35** 567
- Rangarajan B N and Rangarajan G K 1975 *J. Geomag. Geoelectr.* **27** 257
- Rangarajan B N and Rao D R K 1970 *Planet. Space Sci.* **18** 287
- Aggarwal B H 1979 XVII IUGG Gen. Assembly Abstract E 2.09 p. 375
- Booker N U and Siscoe G L 1981 *J. Geophys. Res.* **86** 11201
- Kushima N and Kamide Y 1973 *Radio Sci.* **8** 1013
- Booker N U and Siscoe G L 1976 *Planet. Space Sci.* **24** 201

Anal representation of spatial and temporal variations of the netic field in the Indian region

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Abstract. The magnetic measurements of declination (D), horizontal (H) and vertical (Z) components of earth's magnetic field, collected from ground surveys between 1962 and 1966, are used to develop an analytical model of geomagnetic field variations over Indian region for the epoch 1965. In order to reflect spatial features with wavelengths of approximately 1000 km, sixth degree polynomial as a function of differential latitude and longitude is calculated by the method of least squares. The root mean square fit of the model to the input data is better than that accounted by the International Geomagnetic Reference Field for 1965.0. Isomagnetic charts drawn for D , H , Z and total force (F) reflect more details than that shown on world magnetic charts. Further, the values of the field at common repeat stations recorded between 1962 and 1974, after eliminating the field values for the epoch 1965.0, are used to get the secular variation as well as its spatial dependence again by means of polynomial which now includes coefficients which are functions of time and of geographical locations. The accuracy of coefficients is tested against the behaviour of secular variation at permanent magnetic observatories. The merits and limitations of the model are discussed.

Keywords. Geomagnetic reference field; isomagnetic charts; secular variation; polynomial fit; analytical representation; spatial variation; temporal variation.

Introduction

Application of geomagnetic field measurements in delineating crustal magnetic features, which are of longer wavelengths than those investigated by exploration geophysics, is being increasingly recognised (for example see Hinze 1979; Blakely and Hinze 1979). This has resulted from significant developments in the modes of data collection as well as due to the development of new analytical interpretation tech-

regional charts were contoured by hand directly from the observations fitting the data to a selected mathematical function. The process is subject to human bias but also provides data in a form less adaptable to computer treatment. In the past decade, some attempts have been made to fit magnetic charts analytically, wherein the field values are represented by power series in longitude and latitude (Nagata 1971). Occasionally the term corresponding to the secular variation of the field has been included to model the secular variation of the field (Dawson and Nevanlinna 1979).

The Survey of India (soi) has been periodically conducting field magnetic surveys for graphical preparation of regional magnetic charts over Indian region for navigational needs. In their endeavour to participate in the World Magnetic Survey (wms) during 1964–65, soi carried out an extensive survey of the geomagnetic measurements at field stations in addition to a regular survey of repeat stations. This survey was extended over the years 1962–66. The two successive field stations ranged from 30 to 80 km. This is in contrast to repeat stations which have interstation spacing of the order of 200–300 km. At both field and repeat stations, isomagnetic charts were graphical for the epoch of 1965.0. Subsequently, data from two other surveys corresponding to epoch 1970.0 and 1975.0 have also become available. During these surveys, repeat stations only were acquired. The present study is aimed at an extensive data base in developing an analytical model representing the temporal variation of geomagnetic field in the Indian region.

2. Data acquisition and reduction procedure

The magnetic measurements undertaken by Survey of India consist of observations and computations of horizontal force (H), vertical force (Z) and declination at repeat and field stations using QHM, BMZ and T2 theodolite respectively. The instruments used in the survey were calibrated against the permanent magnetic observatory standards at Alibag. From the data, corrected for instrumental errors, the quiet-day daily variation and magnetic disturbance effects were removed. The aid of data from permanent magnetic observatory close to the field stations. During the magnetic survey extending over years 1962–1966, a total of 1000 repeat stations were covered. Of these, only 732 stations, where all three elements namely H , Z and D were observed, were selected for the subsequent surveys, spreading over the years 1966–1970 and 1972–1976.

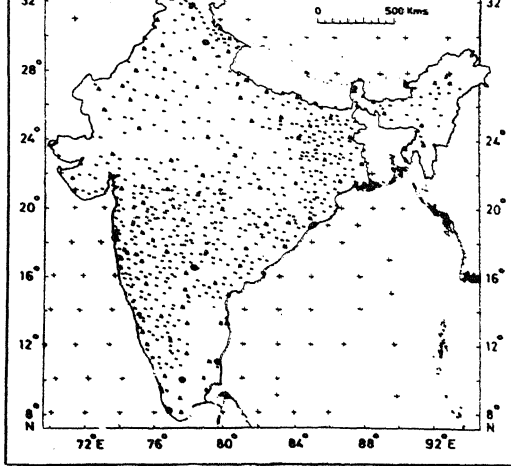


Figure 1. Distribution of magnetic field stations for the epoch of 1965.0 and repeat stations where three observations are available during 1962-75. ● represent permanent magnetic observations and + mark indicates locations at seas where control values evaluated from IGRF are provided.

the spatial characteristics of the field to be revealed are controlled by the area and the degree of polynomial adopted. In order to produce magnetic Indian region, reflecting wavelengths of about 1000 km, it is necessary to use a polynomial fit of degree six which according to Bullard's (1967) equivalent degree would reflect magnetic features of required wavelengths. This follows from the fact that the Indian sub-continent bounded by geographic latitudes 8° to 36° N and 70° to 96° E represents a mapping area of about $8.8 \times 10^6 \text{ km}^2$ and a sixth degree polynomial fit with 28 coefficients to this size is nearly equivalent to a spherical harmonic fit of degree and order 40 for the world, and therefore, should reflect wavelengths close to 1000 km. Although the knowledge of spatial characteristics of the magnetic field is of primary importance for geological applications, but as the magnetic field undergoes slow temporal variations, called secular variation, it is desirable that a complete description of magnetic field by analytical function should also incorporate time coefficients in addition to spatial coefficients. The great advantage of including time coefficient is that the analysis can be performed

time domain data were available only for the years 1962–1966, therefore, the coefficients could at best be estimated by a parabola. Using all the data base for the period 1962–1966, the set of coefficients a 's in the above equation were calculated by the method of least squares separately for H , Z and D . The geographic latitude 22°N was taken as the central point and the time deviations are reckoned from the epoch to 1965.0. The validity of time and spatial coefficients has been checked by computing the temporal variation at a few magnetic observatories and by constructing the isomagnetic charts for the whole region. The temporal variation of the expansion of polynomial at different observatories fail to reveal any anomaly noticed on the magnetic observatory data, both in regard to form and magnitude. It is hard to ascertain the cause for the lack of compatibility of modelled magnetic field with the observatory data but one probable source of ambiguity could be the non-uniform distribution of data in time domain over different parts of the continent.

The peculiar feature seen on isomagnetic charts was that the spatial variation of the geomagnetic field over the continent was found to converge into sharp lines along the boundaries of the Indian continent. Essentially such an abrupt variation is an artifact of the analysis approach, as no such behaviour is noticed in the isomagnetic charts compiled from IGRF coefficients for 1965.0 (Faber and IGRF, 1969). This may be attributable to the absence of observations on the continent. This preliminary exercise clearly reveals that with presently available data it is hard to analytically represent the magnetic field variation both in space and time by a single polynomial expression of the type described in equation (1). The preliminary attempt of modelling the data of first magnetic survey, which had a total of 732 observations, after reducing them to a common epoch of 1965.0, and comparing with the observatory data, was rewarding. After eliminating the geomagnetic field variation from the epoch of 1965.0 from observations at common repeat stations of the continent, the residual values are used to get time variation as well as its spatial variation. The following sections describe the results of the above two approaches.

3.2 *Analytical representation of spatial variations*

The magnetic measurements, collected as part of WMS from 732 stations obtained over a time span of 1962–1966 were reduced to the common epoch by applying a correction corresponding to the change in field values between the period of observation and epoch. Correction factors were calculated by comparing the permanent magnetic observatory with an underlying assumption that the time coefficients of the

ns that the vertical component of $\text{Curl } F=0$, relates the surface variation of X and Y (East) elements as

$$\frac{\delta X}{\delta \lambda} = \frac{\delta (Y \cos \theta)}{\delta \theta} = 0. \quad (3)$$

ver series expression for X and $Y \cos \theta$ can be written in its expanded form as

$$X = a_0 + a_1 \Delta \theta + a_2 \Delta \lambda + a_3 \Delta \theta^2 + a_4 \Delta \theta \cdot \Delta \lambda + a_5 \Delta \lambda^2 + \dots + a_{27} \Delta \lambda^6 \quad (4)$$

$$Y \cos \theta = b_0 + b_1 \Delta \theta + b_2 \Delta \lambda + b_3 \Delta \theta^2 + b_4 \Delta \theta \cdot \Delta \lambda + b_5 \Delta \lambda^2 + \dots + b_{27} \Delta \lambda^6 \quad (5)$$

3) as a necessary condition for consistency, a relationship between the s of (4) and (5) is obtained. These are listed in table 1. These relations were equation (4) and both equations (4) and (5) were expanded to include all the s as follows:

$$X = a_0 (1) + b_0 (0) + a_1 \Delta \theta + b_1 \Delta \lambda + b_2 (0) + \dots \quad (6)$$

$$Y \cos \theta = a_0 (0) + b_0 (1) + a_1 (0) + b_1 \Delta \theta + b_2 \Delta \lambda + \dots \quad (7)$$

mbined set of coefficients in (6) and (7) for the X and $Y \cos \theta$ system is usly determined by the least square solution. The coefficients were finally be compatible with (4) and (5). These are given in table 2. The coefficients rmined separately, are also given in table 2. A preliminary exercise carried has revealed that due to the lack of data in the neighbourhood of Indian modelled variation showed a sharp convergence off the continental bound-

$a_5 = b_4/2$	$a_{14} = b_{13}/4$	$a_{23} = 5b_{22}/2$
$a_7 = 3b_6$	$a_{16} = 5b_{15}$	$a_{24} = 4b_{23}/3$
$a_8 = b_7$	$a_{17} = 2b_{16}$	$a_{25} = 3b_{24}/4$
$a_9 = b_8/3$	$a_{18} = b_{17}$	$a_{26} = 2b_{25}/5$
$a_{11} = 4b_{10}$	$a_{19} = b_{18}/2$	$a_{27} = b_{26}/6$

Table 2. Polynomial coefficients describing spatial variation of geomagnetic expansion point (θ, λ_0) is taken at 22° N and 80° E. In variable column DLON = $\Delta\lambda$; DMLT = ΔT . A number associated with these indicates the power (DLAT2 = $\Delta\theta^2$)

Variable	X	Y cos	Z	
1	+ .38629E + 05	— .78887E + 03	.22125E + 05	1
DLAT	— .38622E + 03	.10706E + 03	.16145E + 04	DLAT
DLON	.10706E + 03	— .16999E + 02	— .22551E + 02	DLON
DLAT2	— .16894E + 02	.78698E + 00	— .11910E + 02	DLAT2
DLAT * DLON	.15739E + 01	— .43233E + 01	.66245E + 01	DLAT * DLON
DLON2	— .21617E + 01	.21229E + 01	— .13381E + 01	DLON2
DLAT3	.42979E + 00	— .24074E 01	— .14471E + 00	DLAT3
DLAT2 * DLON	— .72222E 01	.32351E + 00	.11547E + 00	DLAT2 * DLON
DLAT * DLON2	.32351E + 00	— .36370E + 00	.68370E — 01	DLAT * DLON2
DLON3	— .12124E + 00	.16235E + 00	— .15162E — 01	DLON3
DLAT4	.62882E — 02	— .10244E — 01	— .36934E — 02	DLAT4
DLAT3 * DLON	— .40976E — 01	.61520E — 02	.40920E — 01	DLAT3 * DLON
DLAT2 * DLON2	.92280E — 02	— .55860E — 02	.79408E — 01	DLAT2 * DLON2
DLAT * DLON3	— .37240E — 02	.23389E — 01	— .62950E — 01	DLAT * DLON3
DLON4	.58473E — 02	— .62096E — 02	.27867E — 01	DLON4
DLAT5	— .20117E — 02	— .70554E — 03	— .38560E — 03	DLAT5
DLAT4 * DLON	— .35277E — 02	— .26323E — 02	— .12395E — 03	DLAT4 * DLON
DLAT3 * DLON2	— .52646E — 02	.78982E — 03	— .86618E — 03	DLAT3 * DLON2
DLAT2 * DLON3	.78982E — 03	.30752E — 02	.64792E — 03	DLAT2 * DLON3
DLAT * DLON4	.15376E — 02	— .16880E — 02	— .53596E — 03	DLAT * DLON4
DLON5	— .33760E — 03	— .48307E — 03	— .60981E — 03	DLON5
DLAT6	— .75790E — 04	— .21558E — 04	.60118E — 04	DLAT6
DLAT5 * DLON	— .12935E — 03	.12645E — 03	.15450E — 03	DLAT5 * DLON

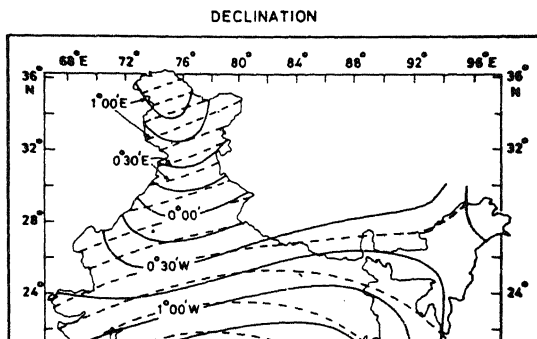
For the purpose of comparison similarly drawn isomagnetic maps using harmonic coefficient corresponding to IGRF model of 1965.0 are superimposed by broken line. The RMS values pertaining to deviation of observed respect to IGRF are also computed and included in table 3. It may be noted for purpose of computing RMS values, the deviations at dummy stations provide oceans are not considered.

3.3 Analytical representation of temporal variation

The behaviour of secular variation at Indian magnetic observatories has been studied by several workers (e.g. Bhargava and Yacob 1969; Yacob and Rangara

Table 3. Root mean square deviation of observed data with respect to polynomial from IGRF

Element	RMS Values (nT)	
	Polynomial	IGRF
X	178	214
Y	172	213
Z	231	275
H	178	214
F	193	235
D	0.25°	0.31°



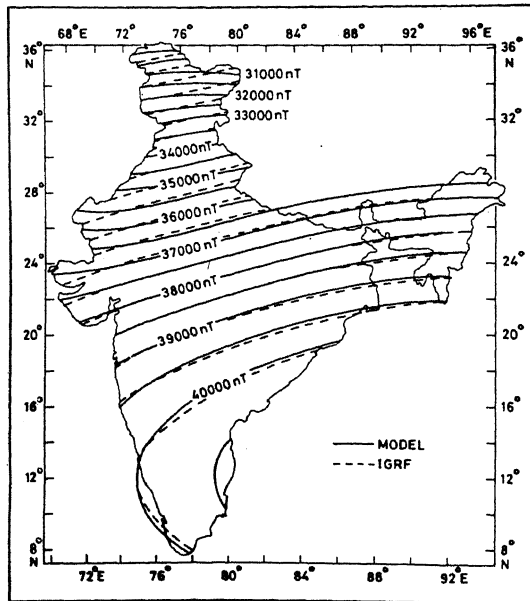
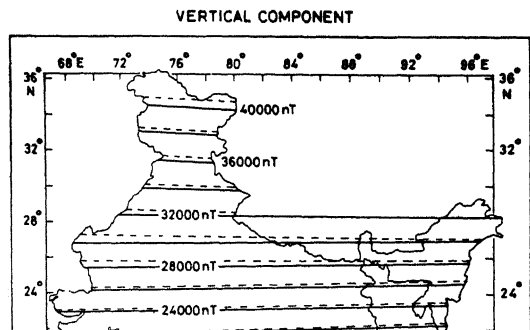


Figure 3. Isomagnetic charts of horizontal component for the epoch 1990 through polynomial model and IGRF.



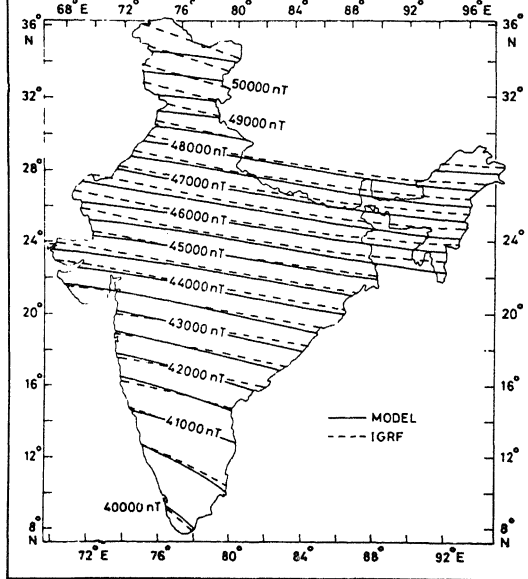


Figure 5. Isomagnetic charts of total force for the epoch 1965.0 obtained through p
ial model and IGRF.

(Vastava and Abbas 1977). Broad agreement in secular trends with some diff
detail is registered at various observatories. In order to bring out the regio
cal characteristics of temporal variations over Indian region, a set of 93 un
tributed repeat stations, where observations in H , D and Z from all three
are available, were selected. From these observations, the field values correspo
the epoch of the 1965.0, *i.e.* input data used in modelling spatial variati
ottracted. It may be emphasized that in the process of subtraction the consta
the core field as well as magnetic field of crustal origin is effectively elim
aving behind the component of the field which has varied in time with res
55.0. With the aid of these residual data, the behaviour of secular variation as
spatial dependence on latitude and longitude were determined for any elen
ing a polynomial of the following form:

observations are not consistent among elements and are scattered in directions which do not fit into the known tectonic framework of the country. The region of Deccan traps is known for its prolonged seismic activity beginning or so. The detailed anomalous nature of geomagnetic anomalies is reported (Arur *et al* 1980). Nevertheless, it should be noted that, in recent times reported from different parts of the world that in certain regions of very extent secular variation at occasions tends to be anomalous. These are attributed to the seismo-magnetic effects arising due to the changes in the position of rocks in the earth's crust under the accumulating mechanical stresses with earthquakes.

In order to minimise the effect of these sporadic observations on the present study, the observations with deviation of more than one standard deviation from the mean values, which in no case exceed 7% of data, were excluded. From the remaining fresh set of coefficients in (8) were re-evaluated. The coefficients in respect of D , H , Z and F are given in table 4. As judged by the low RMS values, the fit is fairly good. The accuracy of the coefficients is tested by comparing the secular variation given by polynomial fit as against the observed variations at magnetic observatories. Figure 6 gives the secular variation in D , H and Z at the separated magnetic observatories, namely Trivandrum (TRD), Alibag (ALB), and Kavaratti (KAV). The spatial dependence of secular variation as characterised by secular change and secular acceleration, *i.e.* linear and quadratic polynomials, is illustrated in figures 7 to 10 for D , H , Z and F respectively.

4. Model fit

The RMS deviations given in table 3 suggest that the field values computed by the polynomials, worked out here, fit the observed data better than those reported by the IGRF. This is in accord with the expected behaviour. The IGRF being evaluated by the spherical harmonic analysis of degree and order eight would account for the magnetic features which have wavelengths greater than 5000 km whereas the polynomial of the field used here include contributions from anomalies with wavelengths down to 1000 km.

The isomagnetic charts given in figures 2-5 show all the normal magnetic field variations for such latitude range. The line of zero variation giving the location of dip equator, lies between Tirunelveli (TIR) and

e 4. Polynomial coefficients in respect of temporal variation.

<i>D</i>	<i>H</i>	<i>Z</i>	<i>F</i>	<i>V</i>
.57694E — 02	— .15273E + 02	— .83670E + 01	— .14658E + 02	DEL
.12155E — 02	— .19447E + 01	— .56762E + 00	— .23254E + 01	DEL
.11219E — 02	— .16686E + 00	+ .67726E + 00	— .11218E + 00	DLA
— .38851E — 03	— .68877E — 01	— .27861E + 00	— .12625E + 00	DLA
— .78305E — 03	— .71285E + 00	+ 12920E + 01	.70971E — 01	DLO
.14402E — 03	+ .11999E + 00	— .25315E — 01	.91324E — 01	DLO
0.112°	47.7	40.9	46.2	

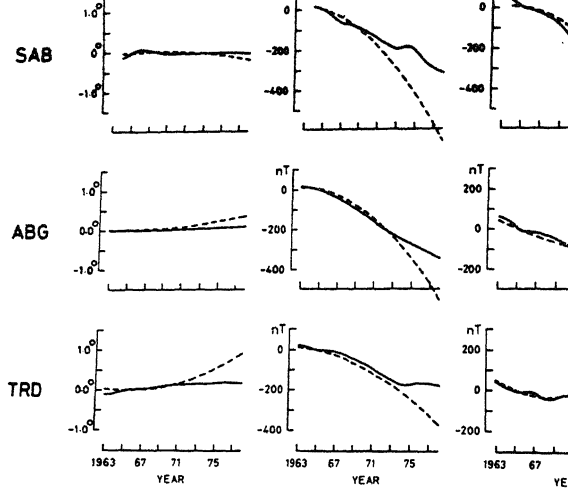
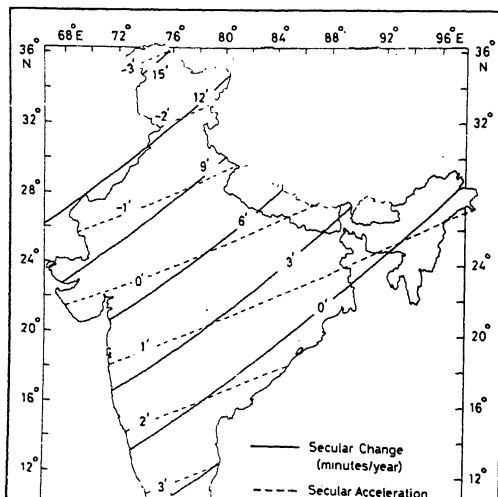


Figure 6. Comparison of observed and calculated (by polynomial fit) D , H and Z at Trivandrum, Alibag and Sabhawala.



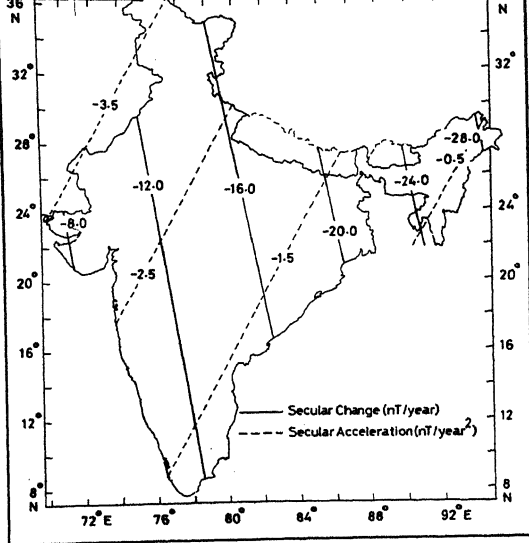
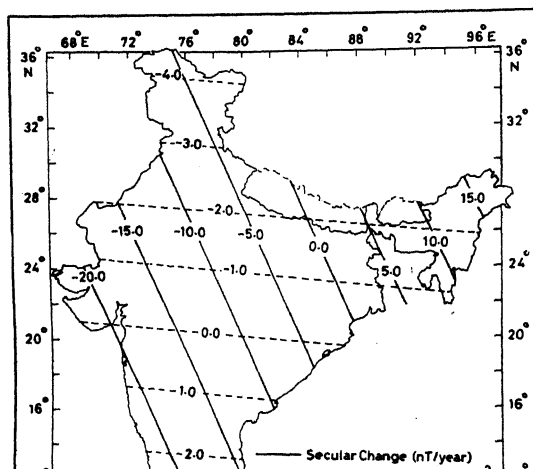


Figure 8. Spatial characteristics of secular variation in the horizontal component.



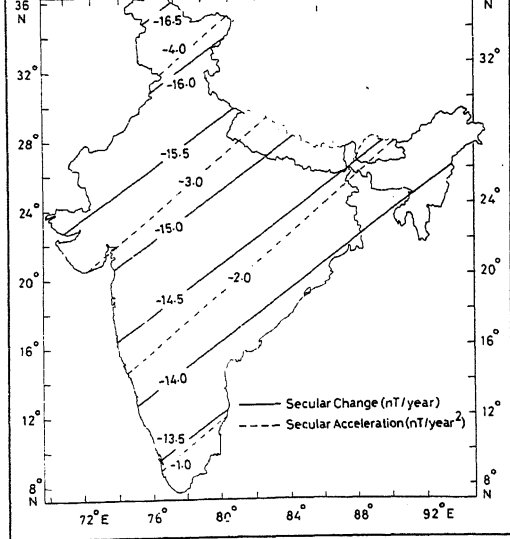


Figure 10. Spatial characteristics of secular variation in the total force

attribute this divergence to the sparse distribution of magnetic measurements in the northern part of the country and on the other hand in view of the close proximity of this region to Himalayas, the differences between the two models could be due to the contribution from anomalies not included in IGRF model but included in the present representation, *i.e.* anomalies with spatial scale lengths in the range of 1000–5000 km. Such anomalies may arise from tectonic features in the upper mantle under the Himalayas. It is interesting to note that magnetic anomaly maps compiled from POGO (Regan *et al* 1975) and MAGSAT (Langel *et al* 1986) clearly indicate a large negative anomaly over the Himalayas. In the present model, the differences noticed between the present model and IGRF for Z and F are also influenced by geological bodies, are also negative. Thus the divergence between the two models may be attributed to the influence of long wave anomalies in the region rather than to the paucity of the observations. Following this argument, the differences between the present model values and those of IGRF over the central and

method in the analytical representation of sv, it should be noted that outside the period of observations the differences between the observed and calculated values increase, suggesting that polynomial coefficients have little predictive value. Further, it might be added that certain short term changes registered in magnetic field around 1973–76, for example seen in H traces of TRD and SAB in figure 6, correspond to secular variation impulse (sudden change in the rate of secular variation) of the type reported by Alldredge 1975; Mizuno 1980; Nevanlinna and Hoff 1981. Obviously such impulses cannot be modelled by low degree polynomials. With polynomials are expected to reveal only the long-term trend in secular variation. The most salient feature of the nature of secular variation to emerge from the inspection of time coefficients as well as charts of sv given in figures 7–10 can be summarised as follows for different elements:

in D : At the southern tip of India, sv is dominantly determined by acceleration (SA) term and the resulting sv has easterly trend with respect to magnetic declination. On the upper half of the country secular change (SC) and SA terms have comparable contributions. Their relative strengths are such that SA contribution would exceed SC contribution around 1970, resulting in the reversal of sv trend.

in H : In the case of H , sv is dominantly linear. Except on the eastern flanks of the country, SC and SA are additive in nature. There is a marked east–west asymmetry in the rate of secular variation per year, the decrease being more pronounced on the eastern part of the country than on the western part.

in Z : Nature of sv in Z tends to be much more complex with a strong regional dependence. In the northern part, SA and SC are negative indicating that magnetic field component is decreasing with increasing rate year after year. In the southern part of the country SC and SV have a tendency to arrest each other. The SC controlling the sv during the years 1963–68, thereafter SA determines the behaviour of SV, the reversal occurring around the year 1968–69.

in F : SC as well as SA contributions have negative trend all over the country, suggesting a continuous decrease in the total field of earth over this part of the world. The isogonic lines run NE–SW, with their magnitude increasing with increasing latitude.

Conclusion

The present study clearly shows that polynomial representation, adopted here,

- Allredge L R 1975 *J. Geophys. Res.* **80** 1971
- Arur M G, Bains P S and Jeevan Lal 1980 *Geophys. Res. Bull.* **18** 67
- Bhargava B N and Yacob A 1969 *J. Geomagn. Geoelectr.* **21** 385
- Blakely R J and Cande S C 1979 *Rev. Geophys. Space Phys.* **17** 204
- Bullard E C 1967 *Earth Planet. Sci. Lett.* **2** 293
- Chapman S 1942 *Terr. Magn. Atmos. Elec.* **47** 1
- Chatterji J 1970 *Proc. Sem Probs. Equat. Electrojet* Aug 1970 Ahmedabad (PRL) Paper 8
- Dawson E and Newitt L R 1977 *Can. J. Earth Sci.* **14** 477
- Fabino E B and Peddie N W 1969 ESSA Tech. C & GS **38**
- Hinze W J 1979 *Rev. Geophys. Space Phys.* **17** 267
- IIG, NGRI, SOI 1972 *Geophys. Res. Bull.* **10** 167
- Langel R A, Phillips J D and Horner R J 1982a *Geophys. Res. Lett.* **9** 269
- Langel R A, Schetzler C C, Phillips J D and Horner R J 1982b *Geophys. Res. Lett.* **9** 273
- Mizuno H 1980 *J. Geomagn. Geoelectr.* **32** 413
- Nagata T 1971 in *World magnetic survey 1957-1969* (ed.) A J Zamuda IAGA Bull. No. 28,
- Nevanlinna H 1979 *J. Geophys.* **46** 201
- Nevanlinna H and Sucksdorff 1981 *J. Geophys.* **50** 68
- Regan R D, Cain J C and Davis W M 1975 *J. Geophys. Res.* **80** 794
- Srivastava B J and Abbas H 1977 *J. Geomagn. Geoelectr.* **29** 51
- Yacob A and Rangarajan G K 1973 *Pure Appl. Geophys.* **111** 2286

Solar cycle and equatorial stratopause temperature

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Abstract. Variations in a solar activity and their effect if any on the temperature of the stratopause are studied. Monthly mean stratopause temperatures during 1969–1979 are compared with the monthly mean value of eight-year period, determined for four equatorial rocket launching stations, viz. Ascension Island, Kwajalein, Fort Sherman and Thumba. The data are statistically compared with sunspot number departures. To study the effects of solar activity variation on stratopause temperature, the data have been divided into winter, summer and equinoctial periods. Seasonwise study indicates that the correlation between the stratopause temperature and sunspot number is positive and just below the significant level for Thumba and positive and highly significant over all the other stations with a better relation during equinoctial periods than in both winter and summer. The estimated regression coefficients are positive and significant.

Keywords. Solar cycle; stratopause temperature; middle atmosphere; correlation coefficient; regression coefficient.

1. Introduction

The stratopause is the level of maximum vertical temperature as a consequence of absorption of solar ultraviolet energy by ozone. This level represents the equilibrium altitude. The heat input, capacity and assorted loss mechanism combine to produce the maximum temperature. The temperature variation in this region has been thought to be caused by atmospheric heating produced by energetic particles and electromagnetic radiation from the sun. Heath (1973) indicated that during a solar cycle, the intensity of the solar spectrum may be enhanced by a factor of 2 near 1800 Å and by 20% near 2000 Å during sunspot maximum. If this is true, according to Fritz and Angell (1973), the stratopause temperature might vary with the sunspot cycle, and they reported a 1% stratopause temperature difference during sunspot maximum and minimum. Using the formula $dT/dt = -0.214T + 49.8^\circ \text{ K/day}$. This could be compared with the rocketsonde data.

temperature of stratopause and sunspot number, the results were tested by student's t test. Accordingly, the correlation is significant only if the calculated value is greater than the tabular value above at 95% level of significance.

Since R is only a measure of the degree of relationship between stratopause temperature and the sunspot number one has to resort to regression analysis to establish the nature of the relationship, delineating the cause and effect. The regression coefficient of stratopause temperature on sunspot number is

$$B_{TZ} = \frac{\sum_{i=1}^N T_i Z_i}{\sum_{i=1}^N Z_i^2}$$

where T_i = departure of temperature from the mean, Z_i = departure of sunspot number from the mean and B_{TZ} is represented by the slope of the regression line.

The correlation coefficient, regression coefficient and the value of t are given in table 1, were calculated for the seasons summer (May to August), winter (December to February) and the two equinoctial periods (March–April and September–October) for the case of the stations located in the northern hemisphere, and for the stations in the southern hemispheric station, where the summer and winter seasons are opposite to those of the northern hemisphere.

3. Results and discussion

The monthly departures of sunspot number from 55.1 (the 8 year, *i.e.* mean value) were plotted against time and is shown in figure 1. 1969 was the year of sunspot maximum and 1976 was the year of sunspot minimum.

The deviations of stratopause temperature from the 8-year mean for the four stations are also graphically presented in figure 1. As the activity of the sun quiet the stratopause temperature deviations change from positive to negative which is a common trend seen over the four stations. Nearly quasi-biennial (26 to 32 months periodicity) are present in the temperature data after 1972. These oscillations are present in the sunspot number data. These oscillations are not in phase at all the stations this may explain in part the correlation coefficient from station to station resulting in poor correlation.

Regression coefficients

Summer	Winter			Equinoctial			All seasons				
	B_{TZ} (°C/sunspot number)	N	R	B_{TZ} (°C/sunspot number)	N	R	B_{TZ} (°C/sunspot number)	N	R	Level of significance (%)	B_{TZ} (°C/sunspot number)
0.051		27	0.698	0.062	26	0.820	0.051	79	0.689	99.9	0.055
0.055		25	0.406	0.035	26	0.609	0.054	71	0.528	99.9	0.047
0.033		18	0.569	0.084	16	0.547	0.037	56	0.511	99.9	0.051
0.043		21	0.350	0.039	20	0.417	0.021	63	0.362	99.5	0.036

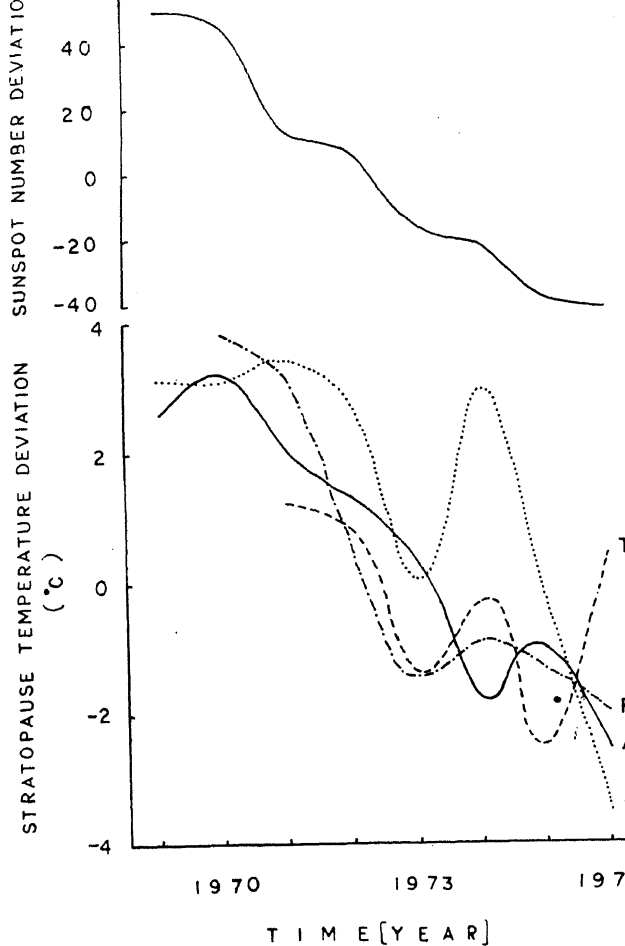


Figure 1. Variation of sunspot number departure and stratopause temperature deviation from the respective 8-year mean, with time. (ASC - Ascension Island, FSH - Fort Sherman, THU - Thumba)

than the western hemispheric stations than in the eastern hemispheric stations. Thumba.

During the period of sunspot maximum, increased amounts of ultraviolet radiation enter the atmosphere. These radiations are absorbed by the ozone present in the stratosphere causing an increase in the temperature of the stratosphere. Thus, a temperature inversion can be observed in the stratopause (Webb 1966). It is well-known that diurnal variations observed in the stratosphere have a greater amplitude in the stratosphere, especially in the stratopause. A diurnal temperature variation of about 10°C is present at stratopause because heating due to ozone absorption occurs only during the day time, whereas cooling due to the emission from carbon dioxide takes place during the night time (Houghton 1977). This might form a basis to explain the observed temperature anomaly at Thumba related with the post-sunset rocket launching.

Fritz and Angell (1976) suggested that the temperature change in the stratosphere seems to have its greatest amplitude at the equator and to have a wide duration and widespread aerial extent. From the analysis of Angell and Kopp (1978) of the high altitude meteorological rocket data over ten stations for almost 20 years or the period 1965–1977, a 6° C lowering for summer in the layer 46–55 km was observed when the nearly active sun becomes quiet. The combined station data involving 10 seasons and a three-year smoothing yield a temperature decrease of about 4° C in the layer from solar maximum to minimum.

Quiroz (1979) investigated temperature at two discrete levels, 35 and 50 km for the period 1965–1977 for seven stations lying in the latitude 8° S–64° N with solar activity. His investigation yielded overall coefficients of correlation in the range + 0.7 between upper stratospheric temperature and sunspot number. He found a decrease in the average temperature from solar maximum to minimum for summer which is about 10 km at 50 km. A theoretical analysis by Callis and Nealy (1978), assuming an approximately 20% increase in the uv radiation during sunspot maximum, results in a temperature increase of about 15° C in the 40–50 km at this time.

A high significant correlation over stations in the western hemisphere and a significant negative correlation over Thumba have been reported earlier (Mohandas and Devanarayanan 1982) and this anomaly was not explained. But the present study attributes the anomaly over Thumba to the presence of waves over the region (figure 1). During the sunspot minimum year 1976, Thumba alone displayed a decrease in temperature. This higher stratopause temperature actually led to the correlation to be insignificant and negative whereas it was found to be significant and positive over other stations.

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References

- Angell J K and Korshower J 1978 *J. Atmos. Sci.* **35** 1758
Callis L B and Nealy J E 1978 *Geophys. Res. Lett.* **5** 249
Fritz S and Angell J K 1976 *J. Geophys. Res.* **81** 1051
Heath D F 1973 *J. Geophys. Res.* **78** 2779
Houghton J T 1977 *Physics of Atmospheres* (London: Cambridge Univ. Press)
Mohanakumar K and Devanarayanan S 1982 *Mausam* **33** 405
Quiroz R S 1979 *J. Geophys. Res.* **84** 1979
Rajaram M, Singh B P and Gupta H V 1979 *Tellus* **31** 515
Webb W L 1966 *Structure of the Stratosphere and mesosphere* (New York: Academic Press)

Intense night-time equatorial radio scintillations by sporadic *E* layer

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Abstract. Almost saturated scintillations of radio beacons from geostationary satellites received at an equatorial station during night-time have been shown to occur even in the complete absence of spread *F* on the vertical incidence ionograms at the same location. Scintillation events were observed when the ionograms showed blanketing type of sporadic *E* layers simultaneously at different heights. It is suggested that strong equatorial radio scintillations during night-time are caused by multiple scattering between different layers of large plasma density gradients in the *F* or sometimes in the *E* regions of the ionosphere.

Keywords. Radio scintillations; sporadic *E* layers.

Introduction

Equatorial radio scintillations have been considered to be basically a night-time phenomenon associated with spread *F* irregularities (Wright *et al* 1956; Koster and Tyagaraj 1970; Bhargava 1964). A detailed comparison of equatorial radio scintillations with the corresponding ionograms at a close-by station was undertaken by Rastogi (1982a) using the amplitude scintillation records of 41, 10 and 1.6 MHz radio beacons from the geostationary satellite at 94° W longitude received at the station during the period October 1974 to May 1975. It was shown that it was the type of range spread where different layers of strong reflections were recorded on the ionograms which produced scintillations exceeding the amplitude fluctuation of 3 dB or the scintillation index of 100%. The range spread showing uniform frequency spreading of echoes caused only moderate scintillations on the VHF band with fluctuations of less than 5 dB. The frequency spread with no range spread causes no scintillations or some times produce scintillations of only 1-2 dB. The blanketing type of sporadic *E* layer during the night-time was found to cause scintillations of about 10 dB.

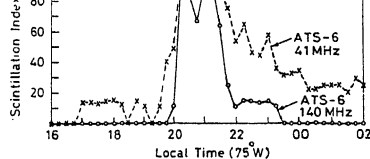
Early results of ATS-6 beacons recordings at Ootacamund indicated scintillations occurring in about 25% of time (Rastogi *et al* 1977a) suggested the day-time equatorial scintillations to be due to type identified by the radar as well as Esq echoes on ionograms. Rastogi (1980) showed that intense day-time scintillations were due to non- q type flat type of E_s which blankets the higher reflections. The scintillation index was studied by Rastogi (1980) for day-time of different epochs of solar cycle.

Chandra *et al* (1979) found that some of the events of scintillation of ATS-6 beacons received at Ootacamund were associated with night-time E layer irregularities. They found that during phase I of ATS-6 project (1974–75), when the satellite was in the western hemisphere, the blanketing type of sporadic E layer which was observed at night-time caused scintillations of about 1–2 dB on VHF and of about 1–2 dB on HF bands. During day-time, of course, VHF scintillations of index greater than 1 were detected associated with blanketing sporadic E at Huancayo (Rastogi *et al* 1981). This paper describes two events of intense night-time VHF scintillations causing the amplitude of 18 dB or 100% index during the complete absence of E layer irregularities (spread F). The necessary conditions and mechanism of such intense scintillations are discussed.

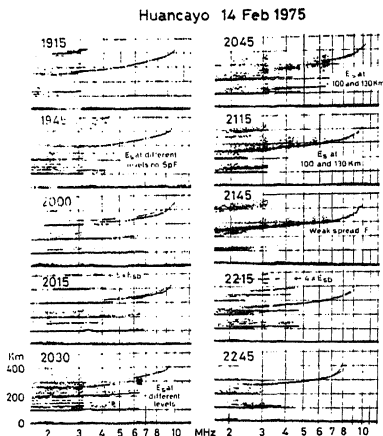
2. Observations

The first set of observations is based on recordings of the amplitude of ATS-6 beacons on 41, 140 and 360 MHz from ATS-6 satellite received at Huancayo (latitude 10° S, azimuth 58° West of North) on 14 February 1975, a sample of which is shown in figure 1. The E region irregularities causing any scintillations would be at a distance of 100 km from Huancayo and thus a comparison of scintillation index with the ionogram records at Huancayo is quite justified. The amplitude of the 41 MHz beacon was fluctuating at a very slow period until 2016 LT which then became very deep in intensity. Similarly the fluctuations of 140 and 360 MHz beacons were feeble till 2016 LT and then became very clear and strong. These events occurred in all the three frequencies almost simultaneously.

In figure 2(a) are shown the temporal variations of the scintillation index of 140 MHz beacons from ATS-6 on 14 February 1975. Complete



(a)



(b)

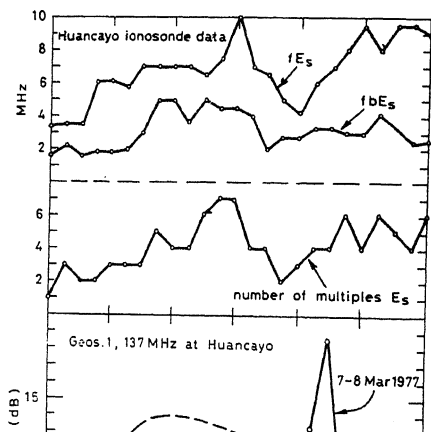
Figure 2. (a) The temporal variation of the scintillation index of 41 and 140 M beacons from ATS-6 to Huancayo (b) Some of the ionograms at Huancayo on 14 1975 during the scintillations of the ATS-6 beacons.

plitude of 360 MHz were not available due to the malfunctioning of the equipment. Ionograms at Huancayo on the night of 14 February 1975 are also reproduced for comparison in figure 2(b). One can clearly notice that the scintillation of the beacons (41 and 140 MHz) started increasing at about 2000 LT and reached saturation (scintillation index 100%) at about 2015 to 2115 LT after which it decreased slowly to normal conditions by 2330 LT. This is a very rapid and strong but fairly short duration scintillation event not characteristic of the scintillations caused by the spread F. It is noteworthy that there was no spread *F* activity observed on the ionograms, the ordinary frequency was roughly identifiable on all the ionograms. There were some scattered radio waves at 2045-2215 LT but such characteristics have been shown to

activity.

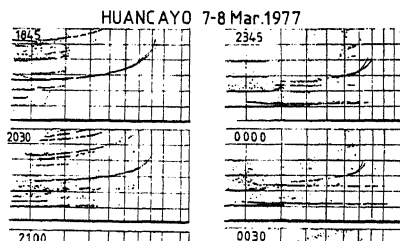
Another set of observations is based on the amplitude of VHF signal from the satellite Geos-1 (at 74.9° W longitude) and the ionosphere at Huancayo (75.3° W longitude) on the night of 7–8 March 1977. The geostationary satellite was situated in the most favourable position at the highest possible elevation (76°) of the satellite from the ground ionosonde station at Huancayo. Therefore, the ionospheric sounder had been free from irregularities from almost the same region of the ionosphere causing the scintillations of satellite radio beacons during this event.

In figure 3(a), the temporal variation of the scintillation depth of the signal received at Huancayo on 7–8 March 1977 is compared with that of the whole month. The temporal variation of the monthly mean scintillation depth showed a rather rapid increase from 1.3 dB at 1900 LT to 14 dB at 0015 LT, which conforms to the general character of VHF scintillations at Huancayo during a high sunspot year (Rastogi 1982b). On the night of 7–8 March 1977, scintillations up to 2000 LT, but later scintillations with a maximum of 4.5 dB at 2130 LT were recorded. After 2215 LT, the scintillation started increasing, reaching 18.5 dB at 2330 LT. It decreased to 7 dB at 0015 LT after which it increased to 7 dB at 0045 LT. No observations were recorded after 0045 LT. The peak value of scintillation at 2330 LT was very



Huancayo was examined to determine the cause of such a large and sharp increase in scintillation during the night.

The ionograms at Huancayo on the night of 7–8 March 1977 reproduced in Figure 1 showed that there was absolutely no spread F activity at Huancayo at any time during the night of 7–8 March 1977. In the evening of 7 March 1977 there was a weak E_s layer at 100 km and a E_2 layer at 140 km with no multiple reflections. At 1845 LT the E_s and E_2 layers recorded but a large number of multiple layers below the second order trace indicating $(2F - E_s)$ type of M reflections. This suggested a weaker E layer but stronger off-vertical sporadic E layer ionization patches around the station. The ionogram at 2030 LT showed strong multitude of layers of ionization below first order F reflection trace. A trace parallel and only 30 km below the second order F trace indicates the presence of scattering between these patches of ionization or between sporadic ionization and the main F layer. The presence of sporadic E activity caused an increase of scintillations at 2015 LT to 2. The ionogram at 2100 LT shows four multiple reflections from the sporadic E layer blanketing of F reflections up to about 6 MHz. The scintillations increased to 3 at 2130 LT. At 2200 LT the ionograms recorded clearly 6 multiple reflections from the E layer at 100 km level. At 2215 LT another sporadic E layer at 120 km appeared and became stronger in terms of both maximum frequency reflected from it as well as the number of multiple reflections. The ionograms at 2345 LT shows E_s at 100 km with $fE_s = 4.2$ MHz simultaneously with E_s at 120 km with $fE_s = 8$ MHz. At 2345 LT simultaneously five multiples from E_s at 100 km and three multiples from E_s at 120 km were recorded. The fE_s for the layer at 120 km exceeded 10 MHz which is much greater than $f2F2$. The important point to be noted is that the first order F reflection at 0030 LT was completely blanketed by 120 km E_s , but the second order F layer reflection was blanketed only up to 3 MHz. This suggests an extremely



vertical variation in the electron concentration. At 0000 LT shows f min F to be much greater for the first than for the second order trace suggesting a much stronger and uniform sheet of ionization overhead the station than at off-vertical areas. Very strong activity of E_s layer is seen in the ionogram at 0000 LT.

The temporal variations of the number of multiple reflections from the sporadic E layer, the maximum frequency reflected from the E_s layer (fE_s) and the maximum frequency blanketed by E_s layer ($f_b E_s$) are shown in figure 3(b) for comparison with the variation of the scintillation index, SI at Huancayo on the night of 7-8 March 1977. The sporadic E layer activity was abnormally high on that night whether one considers the maximum ionization in the E layer in terms of fE_s or in terms of the number of multiple reflections from the E_s cloud by its reflectivity or transmissivity. It may be noted that for a type of E_s layer, fE_s is not the index of ionization in E_s layer but in a blanketed sporadic E layer $f_b E_s$ is close to the maximum ionization density in the E_s layer (Reddy and Rao 1968). Thus the increased scintillation activity in general is in conformity with the increased E_s activity, but when examined in detail the pattern of scintillation does not coincide with the highest frequency reflected from the E_s or the number of multiple reflections from the E_s . The examination of ionograms do indicate some speciality during the period of maximum scintillation. This is the simultaneous occurrence of strong E_s layer at different altitudes; and it seems that the multiple scattering between these different E_s layers which produces strong scintillations rather than the increase of fE_s or $f_b E_s$.

Rastogi and Mullen (1981) have reported two events of intense scintillations at Huancayo during day-time associated with the occurrence of strong E_s activity. From a closer examination of the ionograms reproduced by them it is seen that the event was marked by multiple layers of sporadic ionization below F region of the ionosphere. Thus it appears that a single sporadic E layer is not effective in producing intense scintillations. It is the multiple scattering between the layers that produces strong scintillations. Earlier, Rastogi (1982a) has shown that even in the case of F region irregularities, the strong scintillations are the result of multiple scattering between the layers of large plasma density gradient produced at fully developed stage of irregularity. Even a thick F region irregularity indicated by uniform range spread is not very efficient in generating scintillations of satellite radio signals.

Acknowledgements

- Geophys. 55 145
- Chivers H J A and Greenhow J S 1959 *J. Atmos. Terr. Phys.* **17** 1
- Haung C M 1970 *J. Geophys. Res.* **75** 4833
- Koster J R 1972 *Planet. Space Sci.* **20** 1999
- Mullen J P and Hawkins G S 1975 *Equatorial scintillations at 136 MHz observed over a sunspot cycle* (ed) J Goodman, Naval Research Laboratory, Washington (DC)
- Rastogi R G 1980 *Indian J. Rad. Space Phys.* **9** 219
- Rastogi R G 1982a *Indian J. Rad. Space Phys.* **11** 1
- Rastogi R G 1982b *Indian J. Rad. Space Phys.* (in press)
- Rastogi R G 1982c *Indian J. Rad. Space Phys.* (in press)
- Rastogi R G and Iyer K N 1976 *Curr. Sci.* **45** 685
- Rastogi R G, Deshpande M R, Vats H O, Davies K, Grubb R N and Jones J E 1977a *Planet. Space Sci.* **25** 113
- Rastogi R G 1977b *Geophys. Res. Lett.* **4** 113
- Rastogi R G, Deshpande M R, Murthy B S and Davies K 1977b *Geophys. Res. Lett.* **4** 113
- Rastogi R G and Mullen J P 1981 *J. Geophys. Res.* **86** 195
- Reddy C A and Rao M M 1968 *J. Geophys. Res.* **73** 215
- Wild J P and Roberts J A 1956 *Nature (London)* **178** 377
- Wright R W, Koster J R and Skinner N J 1956 *J. Atmos. Terr. Phys.* **8** 240

Indices of equatorial electrojet and counter-electrojet in the Indian region: Evolution of the indices and their authenticity

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Abstract. Indices of the equatorial electrojet and counter-electrojet in the Indian region have been evolved based on certain reasonable assumptions, by a criterion depending on the difference in the horizontal field strength between a station under the electrojet axis (Trivandrum) and a station outside the jet influence (Alibag). The indices enable one to characterize each day of the year by an appropriate index. Distribution, in the months, seasons and years, of the electrojet and counter-electrojet frequencies at Trivandrum during a 20-year period, 1959–78, is examined. The salient features of the distributions are: (i) Strong electrojet (range ~ 50 nT) and the afternoon counter-electrojets occur, on an average, on about 100 days in the year; (ii) For the counter-electrojet, the monthly frequencies show a seasonal variation with a summer maximum, and for the strong electrojet, a semi-annual variation with maxima in the equinoxes; (iii) A secondary maximum is noted in January in the occurrence frequency of both counter-electrojet and strong electrojet events; (iv) Intense electrojet events with greater frequency in September.

Keywords. Electrojet; counter-electrojet; indices; frequency distribution; solar cycle variation; lunar modulation.

Introduction

The equatorial electrojet, so named by Chapman (1951), is an intense electric current flowing eastward in the ionospheric E-layer in a narrow belt of latitudes centred on the equator. A depression in the daily variation of the horizontal component of the geomagnetic field below the night level, often observed at an observatory near the equator, has led to the concept of a current similar to the electrojet but with the reverse direction. Gouin and Mayaud (1967) have consequently called it the equatorial counter-electrojet. Some of the early studies on the counter-electrojet in the Indian region (the afternoon phenomenon) were based on the enumeration

around noon were considered for the calculation of the afternoon counter-electrojet index (CEJ index). The criterion of both the electrojet and counter-electrojet days is given in table 1.

The assumptions underlying the above procedure are as follows. (i) Extra-ionospheric currents can be considered the same at Trivandrum and Alibag because of the proximity of the stations compared to the distance between the stations.

During the disturbed days, small-period fluctuations (periods less than 1 hr) superposed on the long period variations may not have phase synchronism at the two stations and also the effects of the fluctuating electric field of ionospheric currents may not be the same at Alibag and Trivandrum. These fluctuations are smoothed out to a large extent in the hourly mean values used in the present study. (ii) The residual field is assumed to be not considerable. (iii) The field of the world-wide Sq currents at Alibag fairly represents the Sq field of the world. The index defined by Kane (1973) is based on such an assumption. Y

Table 1. Definition of electrojet (EJ) and counter-electrojet (CEJ) days.

Classification (Index)	
<i>Electrojet Index</i>	
(Based on hourly differences, $\Delta H_T - \Delta H_A$, for 10-14 hr LT)	
(0)	All hourly values are negative
(1)	Maximum value of $\Delta H_T - \Delta H_A$ is not equal to 0
(2)	Maximum value of $\Delta H_T - \Delta H_A \geq 25$ nT but < 50 nT
(3)	Maximum value of $\Delta H_T - \Delta H_A \geq 50$ nT but < 75 nT
(4)	Maximum value of $\Delta H_T - \Delta H_A \geq 75$ nT but < 100 nT
(5)	Maximum value of $\Delta H_T - \Delta H_A \geq 100$ nT

Counter-electrojet index

(Based on hourly differences, $\Delta H_T - \Delta H_A$, for 12-18 hr LT)

- | | |
|-----|--|
| (0) | $\Delta H_T - \Delta H_A$ positive without a minimum in the afternoon |
| (1) | $\Delta H_T - \Delta H_A$ positive but with a minimum in the afternoon |

would be sufficient to subtract, on the average, ΔH_A or at the most $1.1 \Delta H_T$ for the corresponding hour. (iii) The excess field at Trivandrum is attributed to the electrojet effect. The range of 50 nT of the field, which is nearly the diurnal range at Alibag, is treated as the normal electrojet effect because quiet-day range at Trivandrum is found, on an average, to be twice the H range at Alibag (Jayakar 1970). Chapman (1951) states that the electrojet may be about 50 nT of normal current intensity over its narrow band.

3. Analysis

The 20-year period of the analysis, 1959–78, includes a complete solar cycle (cycle 20 (1965–76)). Each day during the period is characterized by an EJ index as a CEJ index by the criteria given in table 1. The frequency distribution of the indices is then calculated for each month. By an appropriate combination of the monthly data, the annual, seasonal and monthly frequencies for the whole period are obtained for each of the indices. Seasonal groupings are made into the three seasons: d – December solstitial season (November, December, January), e – equinoctial season (March, April, September, October) and j – June solstitial season (May, June, July, August). Days with EJ indices 3, 4 and 5 which have a maximum hourly value of $(\Delta H_T - \Delta H_A)$ greater than 50 nT at noon represent, in that order, the days of strong electrojet. Days with CEJ indices 1 and 2 only are counted as counter-electrojet days because these are the days which have at least one negative hourly value of $(\Delta H_T - \Delta H_A)$ in the afternoon hours.

4. Results

4.1 General features

The frequency distribution of different CEJ and EJ indices is given in table 2 for the entire 20-year period as well as in the three seasons. We note that the distribution of different CEJ and EJ indices is not uniform. CEJ indices 1 and 2 and EJ indices 3 and 4 occur more frequently than the other respective indices. The afternoon electrojet events, represented by CEJ indices 2, 3 and 4, occur on an average of 50% of the days. This increases to a little over 70% during the northern summer months. This is consistent with the earlier observation by Bhargava *et al* (1978).

Table 2. Frequency distribution of different EJ and CEJ indices in the seasons

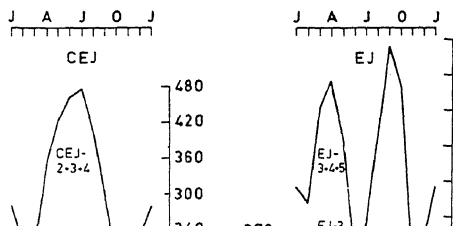
afternoon counter-electrojet variations need not be regarded as constituting one of the patterns of the normal daily variation. The occurrence in summer (j-season) and the minimum in winter (d-season) by Sastri and Jayakar (1972), is maintained in the total frequency indices 2, 3 and 4. Strong CEJ events (CEJ indices 3 and 4), however, occur at the equinoxes. Maximum number of very strong counter-electrojet events occur in either of the solstitial seasons.

Strong electrojet (EJ indices 3, 4 and 5) occurs on about 55% of the period of the analysis. Their frequency exhibits a strong bias towards summer which is even more pronounced in the case of EJ indices 4 and 5. Very strong electrojets occur on nearly 80% of the days reducing to about 50% in any summer day strong EJ is as likely as a weak EJ.

4.2 Frequency distribution in the months of the year

The frequency of occurrence of CEJ and strong EJ over the months of the year for the period 1959-1978 is presented in figure 1.

4.2a CEJ events : A broad dominant maximum around June-July and a secondary maximum in January in the occurrence frequency of CEJ contrast with the features of the annual progression of CEJ occurrence frequency (Sastri, 1979). Such an annual variation is seen individually in the frequency distribution of CEJ indices 2 and 3. Although the two maxima are fairly evident, they are not well marked in the frequency distribution of the strong electrojet events. A feature noted in the CEJ frequency distribution over the months of the year is that in September the highest frequency is registered for the CEJ index 2. Of the months, October to March, index 1 has the maximum frequency.



nts (Rastogi 1974). Thus, in the summer months and a month on either side of the equinoxes, events which could be only partial events tend to register as weak counter-electrojet events.

Strong EJ events: The frequency distribution of the occurrence of strong electrojet (EJ indices 3, 4 and 5 combined) is primarily semi-annual with the maxima occurring near the equinoxes (figure 1). Surprisingly, a secondary peak in January, noticed in the occurrence frequencies, is a conspicuous feature in EJ occurrence also. Some features in the annual progression of the frequency of strong electrojet occurrences are as follows: (i) Strong electrojets are as few in the two summer months, June and July, as in the two winter months, November and December. (ii) There is an approximate symmetry in the occurrence of strong electrojets between the two equinoxes. (iii) Figure 1, which illustrates the annual progression of frequency of EJ indices 3, 4 and 5 combined, individually, further shows equinoctial asymmetry is markedly large in the frequency of intense electrojets (EJ indices 4 and 5) with a peak in their occurrence in the month of September. This peak is accompanied by a pronounced 'dip' in the occurrence frequency of moderately strong electrojets (EJ index 3) in that month, compared to the frequency in the adjacent months, August and October.

Lunar modulation in the occurrence frequency of CEJ and EJ

The frequency distributions of the occurrence frequency of the counter-electrojet events (CEJ indices 2, 3 and 4 combined) and strong electrojet events (EJ indices 3, 4 and 5 combined) over the lunar phase are presented in figure 2a for the three seasons of the year. Frequency distribution of each of the indices characterizing CEJ and strong electrojet, based on the data for the entire period, are shown separately in figure 2b. The lunar modulation is clearly seen in the frequency of both CEJ and EJ indices. A clear semi-monthly wave in the occurrence frequency of CEJ with a maximum occurring near lunar phases 1 and 13 is consistent with the results of Sastri and Jayakar (1973) and Rastogi (1973). Curiously the lunar modulation shows a strong measure of dependence on season, being strong in winter and the equinoxes but very weak or absent in summer. Lunar influence on the frequency of strong electrojet occurrences and its seasonal dependence are, in general, similar to what is observed in the counter-electrojet frequency. There is a perceptible phase shift in the variation of occurrence frequency over lunar phase in the d-season as compared to that in the e-season. Lunar modulation is equally effective in the individual frequency distributions

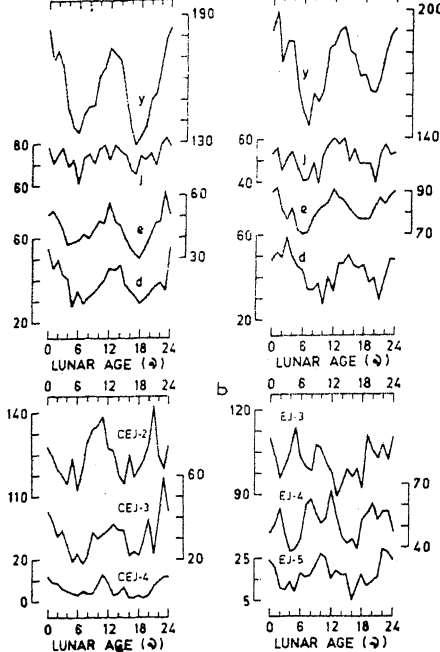


Figure 2. Distribution of the occurrence frequency, based on the data for 1959–78, of the afternoon counter-electrojet (CEJ indices 2, 3 and 4) and indices 3, 4 and 5) events over the lunar phase (a) in the different seasons (y, j, e, d) for the three indices combined, and (b) separately for each of the indices (CEJ-2, CEJ-3, CEJ-4) for strong EJ.

in 1968–69 is close to the broad maximum in R_z . Examining the association between the occurrence of counter-electrojet events, and the annual mean A_p , Ganga and Sastri (1979) found a precise anti-phase relationship between A_p and A_p . The anti-phase relationship observed in the present results is also quite close, particularly in the later part of the period studied. Though the intensity of geomagnetic activity generally varies with the sunspot cycle, there are differences from one cycle to another. It has been suggested (Goswami, 1979) that solar cycle 20 is unusual in that a very large peak has occurred in the late part of the cycle, which has led to a number of studies of the relationship between the sunspot cycle and the geomagnetic activity.

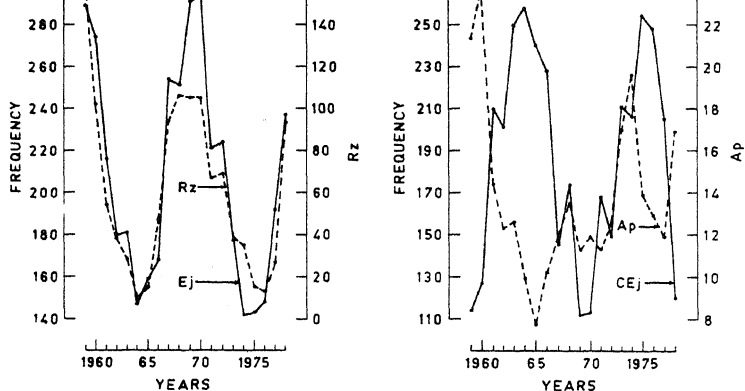


Figure 3. Frequency of occurrence of the afternoon counter-electrojet events (CEj indices 2, 3 and 4 combined) and of strong electrojet days (EJ indices 3, 4 and 5 combined) from 1959 to 1978. The variation in the annual mean relative sunspot number (R_z) and the annual mean magnetic activity index A_p are also included.

frequency is computed separately for days with $A_p \leq 7$ and days with $A_p > 7$, representing respectively the quiet and disturbed conditions. The frequencies obtained for the entire period as well as for the three seasons separately, are given in Table 3 expressed as percentages of the total number of days in the whole period and for the respective seasons. In the 20-year period, days with $A_p \leq 7$ are about 35% of the total. Among such quiet days, CEJ occurs on 52% of the days. This percentage remains the same for the disturbed days ($A_p > 7$) as well as for all the days. Percentages similarly calculated, separately for each season also remain more or less the same, except for a small rise in the j-season percentage for the quiet days. It thus appears that the nature of the day, quiet or disturbed, does not make a difference in CEJ occurrence. The near-antiphase relationship between the annual mean A_p and the annual CEJ occurrence frequency, seen in figure 3, may be more apparent than it is. It could simply be a consequence of the strong association between the solar and magnetic activities on a long-term scale.

Discussion

CEJ occurrence among days		Seasons			Year
		d	e	j	
With $A_p \leq 7$		36	42	86	52
With $A_p > 7$		40	44	70	51
All		38	43	72	51
Days of A_p	7 in all days	37	31	36	35

modulation on the occurrence frequency of CEJ as well as EJ is in a seasonal variation in the strength of L , being maximum in winter and minimum in summer. It thus appears that the nature of the lunar modulation and its dependence is consistent with that resulting from the distortion of the daily variation by the regular lunar daily variation. This view gains strength from the fact that a shift of two lunar phases in the time of maximum fortnightly tide at noon during the equinoxes, relative to the winter solstice, resulted in a similar shift in the occurrence maximum of strong EJ phases.

The strong lunar modulation seen in the CEJ occurrence has led to the conclusion that, on the average, the counter-electrojet event results from the superposition of the lunar tidal wave on the solar daily variation of H appropriate for the longitude as well as the epoch of the year and of the solar cycle (Sastri and Jayakar 1972; Forbes and Lindzen 1976). However, we note that even in the winter months, when the lunar perturbation is strongest, CEJ events that occur irrespective of the lunar phase are half as frequent as events that occur in favourable lunar phases and the frequency of summer events is similar to that of the total frequency. Further, if the events were solely due to the lunar perturbation one would expect the maximum frequency of events in the winter months and a minimum in the summer months. But a contrary feature is observed in the occurrence of CEJ events over the months where summer months show large frequencies. It has been noted (Sastri and Jayakar 1972; Forbes and Lindzen 1976) that when CEJ events occur on a series of consecutive days, the minimum usually occurs at the same solar hour. Thus the results favour the earlier conclusion that the Moon's effect is not to cause but only to modulate the occurrence of CEJ events.

180) is due to the vertical winds or gravity wave origin, which with a proper latitudinal extent and magnitude could account for the different intensities and widths of the CEJ observed on different days. Rastogi and Patel (1975) and Rastogi *et al* (1981) show that a number of counter-electrojet events are associated with changes of the latitude (θ) of the interplanetary magnetic field (IMF) from southward to northward direction, in the presence of large and rather steady value of the IMF (B) which produces pure electric fields that would be anti-parallel to the IMF at the magnetic equator. Such events have their signatures as abrupt and short duration variations in H at the equatorial stations over an extended longitude zone and at non-equatorial stations and occur at the same universal time. Rastogi (1981) suggests a lunar tidal source as well as a magnetospheric source for the CEJ. However, the feature, that the afternoon minimum is found at about the same solar hour for CEJ occurs on a succession of relatively quiet days, remains unexplained.

The strength of the equatorial electrojet is known to undergo a strong semi-annual variation with maxima in the equinoxes, which is ascribed (Tarpley 1977) to the equatorward movement of the northern and southern current foci in September and March respectively. The equinoctial maxima are well reflected in the pattern of the frequency distribution of strong EJ. The approach of the focus towards a low latitude station outside the influence of the electrojet would, however, cause a decrease in the H amplitude. Such a decrease in the amplitude of the daily variation of the H field in September was clearly noticed by Sastri *et al* (1978) in the annual profile of the monthly amplitudes at Alibag. The strength of the electrojet is evaluated as the difference in the H field at a station under the electrojet (Trivandrum) and outside its influence (Alibag). As a consequence electrojets of weak or moderate strength around September would tend to be enhanced in their strength to be considered as strong electrojets. This would result in a higher frequency for the strong electrojets and a reduced number for the weak electrojets in September, a feature brought out in the frequency distributions of EJ indices 3, 4 and 5.

A close parallelism between the occurrence of strong electrojets and the number of sunspots is consistent with the hypothesis that intensity of the electrojet increases with increasing solar activity, a behaviour well observed and reported.

In view of the poor understanding of the possible sources of counter-electrojet, at present, it is difficult to explain the inverse relationship between the counter-electrojet occurrence and R_z . However, the possibility of CEJ-related current distortion due to the electrojet current to the extent of its reversal is large when normal electrojet is weak, as during the sunspot minimum.

by an objective criterion, broadly conform to the known characteristics of occurrence in the Indian region. The primary features of the seasonal control of strong electrojets (EJ) is also consistent with the known regularity of the electrojet. Thus, the criteria based on which the CEJ and EJ events are categorized by the respective indices appear valid indicating that the assumptions in deriving the indices are fairly reasonable. Prominent features of frequency of occurrence of the afternoon counter-electrojet events and strong electrojets at Trivandrum during the 20-year period 1959-78 can be summarized as follows:

- (i) Strong electrojets (range $> 50\text{nT}$), like the afternoon counter-electrojet, occur, on an average, on about 50% of the days in the year.
- (ii) While summer (j-season) has the highest frequency of CEJ events (on about 80% of the days) strong EJ occurs most in the equinoxes (on about 80% of the days).
- (iii) The variation in the occurrence frequency of CEJ is primarily semi-annual and strong EJ is clearly semi-annual with equinoctial maxima.
- (iv) A secondary maximum in January, noted earlier in the CEJ frequency, is also seen in the occurrence frequency of strong electrojets although EJ frequency is at minima in the solstitial months.
- (v) Very intense electrojets (EJ indices 4 and 5) occur more frequently in the winter which month consequently shows a highly reduced percentage of weak electrojets (indices 1 and 2).

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References

- Arora B R 1972 *Indian J. Meteorol. Geophys.* **23** 195
Bhargava B N and Sastri N S 1979 *Geomagn. Geoelectr.* **31** 97
Bhargava B N, Sastri N S, Arora B R and Rajaram R 1980 *Ann. Geophys.* **36** 231
Brown G M 1975 *J. Atmos. Terr. Phys.* **37** 107
Brown G M and Williams W R 1969 *Planet. Space Sci.* **17** 455
Chapman S 1951 *Arc. Meteorol. Geophys. Bioklim Series A Band IV* 368
Chapman S and Garriot O C 1959 *Electromagnetic Phenomena in the Ionosphere* (Cambridge University Press) 100 pp

- etri N S and Jayakar R W 1970 *Indian J. Meteorol. Geophys.* **21** 279
- etri N S and Jayakar R W 1972 *Ann. Geophys.* **28** 589
- etri N S, Arora B R and Rao D R K 1978 *Ann. Geophys.* **34** 325
- alapp D M and Malin S R C 1979 *Geophys. J. R. Astron. Soc.* **59** 161
- ning R J 1977 *J. Atmos. Terr. Phys.* **39** 1071
- pley J D 1973 *J. Atmos. Terr. Phys.* **35** 1063
- cob A 1977 *J. Atmos. Terr. Phys.* **39** 601

Power spectrum of brightness temperature fluctuations derived from solar eclipse observations at 2.8 GHz

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Abstract. In this paper we report on the MEM power spectrum analysis of brightness temperature fluctuations observed at 2.8 GHz during the total solar eclipse of 16 February 1980. The observed periodicities range from 3.5 min to 64 min. These periodicities are due to spatial and/or temporal variations in the solar radio emission. The observed periodicities imply presence of scale sizes ranging from 70,000 to 600,000 km assuming that the brightness fluctuations arise because of spatial variation only. On the other hand, if the fluctuations are due to temporal variation, the observed periodicities correspond to the predicted modes of solar global oscillations.

Keywords. Solar microwave flux; solar oscillations; solar eclipse measurements; power spectrum; brightness temperature fluctuations; maximum entropy method.

1. Introduction

Radio emissions from the sun were recorded at the Japal-Rangapur observatory (17° 43.7'E; 17° 05.9'N) of the Osmania University, Hyderabad which was situated on the path of totality during the total solar eclipse of 16 February 1980. Table 1 summarizes the eclipse parameters. A total power radiometer of the Dicke-type operating at 2.8 GHz had a continuously tracking equatorially mounted parabolic dish antenna with a half-power beamwidth of about 5°. The receiver had an integration time-constant of 1 sec and the solar flux was recorded on a fast moving strip-chart. Before and after the duration of the eclipse the radiometer was calibrated by noting the deflection on the strip-chart due to radio emissions from the earth, sky background and the sun. In order to check the stability of the receiver was checked using a noise generator. The stability of the radiometer was very satisfactory throughout the duration of the eclipse.

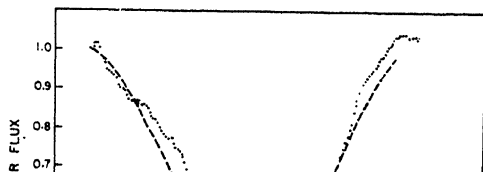
$$G(\theta, \phi) \int d\Omega + T_a \beta \sec z \} (1 -$$

where θ, ϕ = angular coordinates from the centre of the sun, G = normalised power pattern of antenna, α = fractional loss of input cable, β = atmospheric attenuation of the atmosphere toward zenith, z = zenith angle and T_a = effective brightness temperature of the sun, moon and atmosphere.

After substituting the various parameters in (1), the values of T_A were which were then subjected to low pass-filtering by the 3-point averaging performed iteratively to remove high frequency fluctuations which were the system noise. The T_A values vs time give the eclipse curve. For comparison the observed eclipse curve compared with artificial eclipse curve for uniform disk at 2.8 GHz. The changes in the brightness temperature across the eclipse are proportional to the departure of the slope of the eclipse curve from that of a uniformly bright circular solar disk. This is used to derive the distribution of brightness temperature across the solar disk using the method of Swanson (1975). In figure 2 is shown this distribution at 2.8 GHz.

Table 1. Elements of eclipse on 16 February 1980 at Japal-Rangarajapuram

Times of contact (UT)				
First	Second	Third	Fourth	Magnitude
08 58 32.4	10 16 02.4	10 18 11.4	11 25 59.9	1.00
Distance of the observatory to the central line of the path of totality (km)				



defined line indicates the level of the brightness temperature equal to uniformly bright solar disk. The increase in the brightness temperature is the radio bright regions scanned by the moon's edge. The enhancement seen at about solar radius is probably due to the well-known limb-brightening effect at centimetre wavelengths.

In order to see which periodicities were present in the fluctuations of brightness temperature as shown in figure 2, these were first expressed as a time series. Using the fact that the moon's edge takes about 75 min to traverse across the solar disk and their spectrum was computed using the maximum entropy method (MEM) (1967). The length of data used for the spectral analysis, corresponded to the time taken by the moon's edge from the first contact to the centre of the sun and by the moon's edge from the centre to the fourth contact. This was chosen to simplify the analysis of brightness temperature. The applicability of MEM was tested using control data of known spectrum, like the 11-year solar activity cycle, and it was found that MEM gives stable spectra without frequency-splitting are obtained when the data contains at least one cycle. The resulting power spectrum at 2.8 GHz is shown in figure 3, which was computed with a spectral resolution of 0.001 MHz. The figure presents relative values of power normalised by the maximum value and expressed on a ten times their logarithm. The spectrum as a whole shows a slight decrease towards higher frequencies. This is due to the fact that solar power spectrum has a background variation as f^{-2} . Superimposed on this are many oscillatory features whose periodicities in minutes are indicated on top of the peaks. The error bars for each peak indicate 99% confidence intervals. Thus the significant periodicities are 14.1, 14.6, 9.9, 6.9, 5.2 and 3.5 min. The periodicities adjusted within ± 2 min for the longest period and less than ± 0.5 min on the shorter periodicities when the resolution was decreased from 0.001 to 0.01 MHz.

In order to ascertain genuineness of the oscillatory features shown in figure 3, spectra of control data, obtained before and after the solar eclipse on 16 Feb 1980, were computed using the method described above. The results are shown in figures 4(a) and 4(b). The error bars indicate 99% confidence intervals. The error bar at about 4 MHz in figure 4(a) may be due to the fact that radioactive

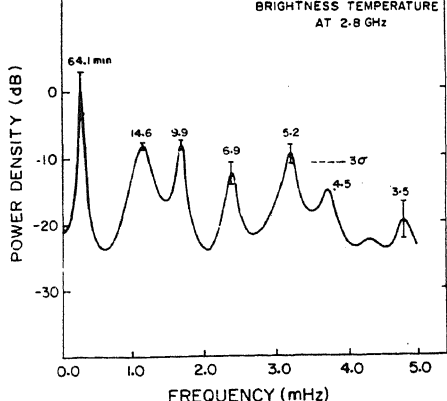
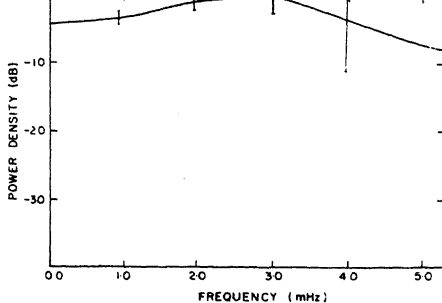


Figure 3. MEM spectrum of solar brightness temperature fluctuations with spectral resolution of 0.001 MHz. The number on each peak gives the frequency and the error bars show 99% confidence intervals.

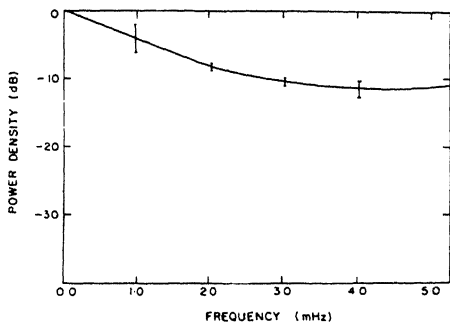
were detected a little before the commencement of the first contour. The West limb of the sun. In general, these two spectra are devoid of oscillatory features. This is expected if there are almost no radio features on the solar disk which would otherwise have been scanned by the method. The features seen in the spectrum of figure 3 are, therefore, a result of the method of resolution scanning of solar radio features by the edge of the sun.

3. Discussion

The observed radio brightness temperature variations used in this study could arise due to spatial and/or temporal variations across the solar disk. It is difficult to separate the contributions due to these causes from the observations reported in this paper. To detect the presence of features, if any, in the radio brightness temperature over the solar disk, radio features should be tracked for a sufficiently long period of time using narrowband observations. A comparative study of such observations may indicate the presence of features on the sun. A large number of annuli are contained in the



(a)



(b)

Figure 4a-b. **a.** Spectrum of solar brightness temperature at 2.8 GHz before the 1980 contact on 16 February 1980. **b.** Spectrum of solar brightness temperature at 2.8 GHz after the 1980 contact on 16 February 1980.

m about 70,000 to 600,000 km. The lower end of this range compares with the size of supergranules.

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Quantitative estimation of groundwater potentiality and rainfall infiltration in a typical crystalline environment

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Abstract. A sophisticated method of measuring water levels is adopted to determine groundwater potentialities in a typical weathered and fractured environment by Thiessen polygon method. The frequency of the rainfall has shown a direct bearing on recharging the water system compared to the intensity of the rainfall. The Thiessen polygon method is used to determine the weighted average of the rainfall over the basin area to estimate the groundwater infiltration rates. The long term average infiltration to the medium is established as 10% of total rainfall.

Keywords. Crystalline environment; fluctuation; specific yield; recharge; correlation coefficient; Thiessen polygon; infiltration rate.

Introduction

The replenishable character of groundwater systems always warrants systematic assessment for proper planning and development. Techniques of assessment of this resource are broadly classified into direct and indirect methods (Niyogi 1971). In direct methods, recharge is calculated by determining the specific yield of the formation from the water level fluctuation. Indirect methods involve comprehensive study of the environment by quantitatively assessing various inputs to and outputs from the system. The application of these techniques to crystalline environment is more difficult because of differential weathering, fracturing and absence of primary porosity. Several investigators (Radhakrishna 1971; Singhal 1971; Venkatanarayana 1972) attempted to estimate recharge in these terrains by the above methods. Their findings show wide variations because of the experimental nature of the investigations spread over a long number of years and the inherent complexity of the environment. Hence long

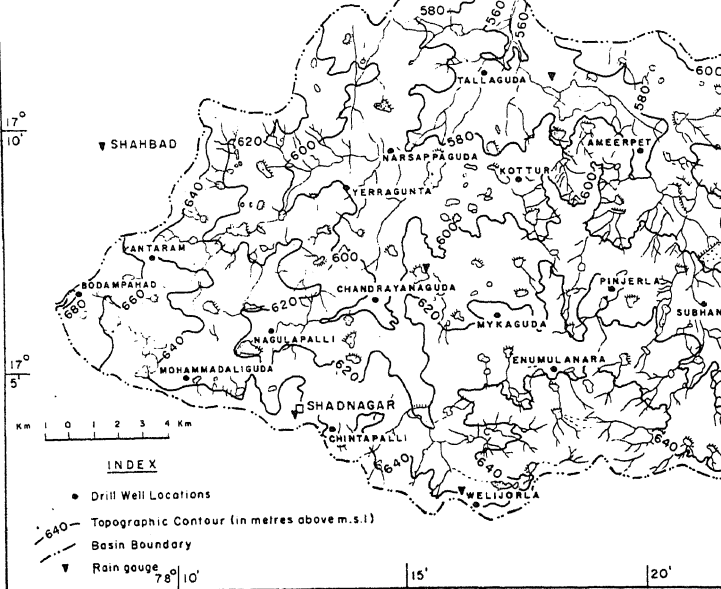


Figure 1. Drainage and topography of the basin along with well

Table 1. Rainfall data at different stations in basin (1973–1978)

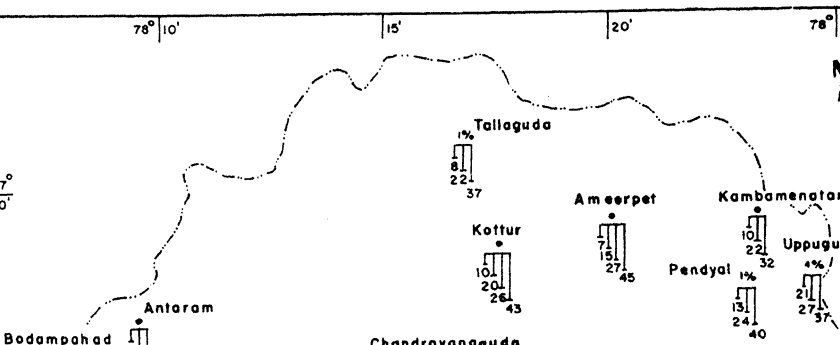
Location of rain gauge station	Rainfall (cm)				
	1973	1974	1975	1976	1977
Chippalapalli	72.9	61.8	157.2	119.9	—
Maisaram	85.5	76.0	120.1	83.3	150.8
Nandigaon	80.9	87.3	182.4	85.3	79.2
Palmakul	93.9	62.2	111.9	67.4	49.4
Parandla	—	72.3	155.6	72.7	49.4
Shahbad	100.7	88.0	113.4	77.9	73.7
Shadnagar	66.8	80.4	91.0	75.0	55.6
Welijorla	71.8	77.6	84.9	63.9	52.4

Water level measurements

The locations of the 14 piezometric nests and the depths at which they are installed are shown in figure 2. All the nests are continuously monitored at monthly intervals from January 1974 and the data up to December 1978 are used in the present studies. During these measurements water levels in all the piezometric tubes installed in any piezometric nest are nearly the same (Briz-Kishore and Bhimasankaram 1981) and consistent with each other during pre-monsoon and post-monsoon periods. This behaviour has continued over the entire period of study of five years and reveals that water levels in shallow weathered mantle and deep-seated fractures are the same and the aquifer is continuous. Hence the average water levels in all the piezometric tubes for each piezometric nest are considered. The pre-monsoon (max bgl) and post-monsoon (min bgl) water level data are presented in table 2 along with the fluctuation. The reference level of the well site for the years 1974 to 1978.

1 Correlation study between elevation and fluctuation

Table 2 shows that the annual fluctuations in water levels in different wells range from 1.5 to 8.27 m bgl throughout the basin area, with an average annual water level fluctuation ranging from 2.23 to 5.34 m bgl. These fluctuations also seem to be accentuated whenever the drought periods are succeeded by high rainfall periods in 1975 and 1978.



water level fluctuations (1974–78).

[illegible]

ing areas and higher fluctuations are observed over higher topographic areas. Statistical correlation coefficient is calculated between elevations and fluctuations for the years 1974 to 1978. High values of correlation coefficients ranging from 0.8 to 0.9 are noticed between these two variables for different years of observation. The exact correlation can be more clearly visualised from figure 3 where the fluctuations are presented graphically with the varying elevations of different wells for 1974 to 1978.

Studies on intensity and frequency of rainfall

To study the effect of rainfall on water level fluctuation the average rainfall intensity is compared with average fluctuation in the basin and presented in table 3(a). It is observed that the average fluctuation in the basin does not directly correspond with the intensity of rainfall. To find the probable explanation for this behaviour, the events of rainfall (number of rainy days) recorded yearwise in three representative locations of the basin, viz. Welijorla, Nandigaon and Palmakul are presented in table 3(b). The effect of rainfall over the groundwater levels is seen to depend on frequency apart from intensity.

In order to present more vividly the influence of the intensity of rainfall and frequency, the data in tables 3(a) and 3(b) are classified into two groups having different average fluctuations. In group I, although greater intensity of rainfall is recorded in 1976, the average level of fluctuation has become lesser since the effect is completely countermanded by much smaller frequency of rainfall in this year. A similar observation could be made from group II comparing the data for the years 1975 and 1978.

Average rainfall in the basin (cm)	75.95	80.68	127.06	103.04
Average fluctuation in the basin (M)	3.10	2.23	4.95	5.34

Table 3(b). Comparison of events of rainfall (number of rainy days) in different years

	Group I		Group II	
	1974	1976	1975	1978
Welijorla (upland area)	56	32	37	49
Nandigaon (mid slope area)	72	49	42	55
Palmakul (low lying area)	67	39	49	63

3. Point and net recharges of the basin area

3.1 Estimation of point recharges

Specific yield values determined (CGWB 1975) at five representative places over the basin area are used to estimate point recharges. The location of these wells along with specific yield values is given in figure 2. The point recharge at all the well locations is calculated for the years 1974 to 1978 as the product of mean annual fluctuations and the specific yield factor as inferred from the nearest well site. As an example, the recharge contour map is prepared for the normal rainfall year 1977 and is presented in figure 4. The point recharge ranges from 0.01 to 0.2 m/year over the entire area of the basin depending on the location and rainfall during the year.

3.2 Estimation of net recharges

To estimate the net recharge of groundwater system in different years the specific yield factors obtained at different well sites in the basin area are analyzed. It is observed that the estimated value of specific yield is around 4% at most of the wells in the basin area but a lesser value at two well sites, Pendyal and Tallaguda. In fact the Pendyal well site whose specific yield is observed to be 1% is very close to Uppugudatanda having specific yield of 4%. Even Tallaguda well site whose specific yield also is 1% is situated at a low topographic elevation with potential aquifer condition. In addition, at both the well sites good yields of around 5 lps are noticed with a thick-weathered zone of 15 m. Hence, these two specific yield values may be due to purely local conditions and may not be considered as the representative of the entire basin area.

To verify the already arrived specific yield values, a digital model with a computer program (Briz-Kishore and Avadhanulu 1981) is utilized. These values are obtained through a process of adjustment while simulating short duration pumping tests over a period of 100 minutes and by obtaining a close match between the observed and

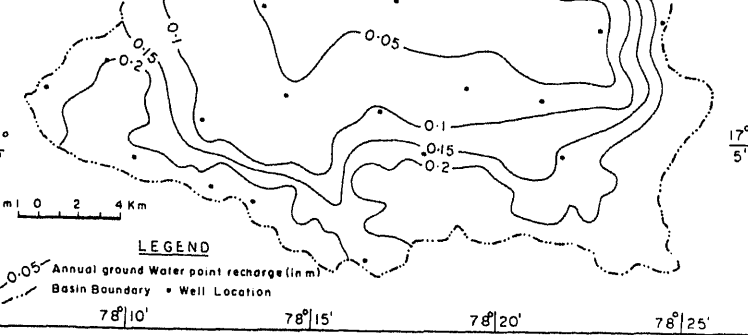


Figure 4. Recharge contour map of normal rainfall year (1977)

Table 4. Computed specific yield values at representative well sites

Village	Computed value of specific yield
Ameerpet	0.04
Chandrayanaguda	0.044
Enmulanara	0.05
Kottur	0.034
Mohammadaliguda	0.04
Uppuguda Tanda	0.04
Yeerakunta	0.04

drawdowns. The specific yield values thus computed at seven representative sites presented in table 4 show that the values are mostly restricted to the range of 0.04 to 0.05, indicating that the overall specific yield value is around 4%. This estimated value is close to the value reported by Ground Water Overexploitation Committee, Agricultural Refinance Development Corporation (ARDC 1979) after systematic investigations over different parts of the granitic terrains. Hence considering the specific yield, mean annual fluctuation and area of the basin, the net recharges are computed for the years 1974 to 1978 and are presented in table 5. It is seen that the annual recharge of the basin ranges from 38.99 to 54.20 MCM during normal rainfall

Annual rainfall infiltration

Weighted averages of rainfall

To determine the percentage of the rainfall recharging into the system, the total rainfall and the total recharge are considered. The total rainfall over the entire basin

seen from the table that the volume of rainfall at different stations varied from 528.7 to 8624 sq km cm. Similarly, the total volume of rainfall for subsequent years was also estimated and presented in table 7

4.2 Groundwater infiltration rates

Using the estimated groundwater recharge in table 5, the percentages of the rainfall infiltration into the basin are calculated and presented in table 7, which shows that the annual infiltration rates vary from 11.4 to 19.8% over a period of five years (1974-78) and the average infiltration coefficient of the basin comes to about 15%.

Table 5. Estimates of groundwater recharge in different years

Year	Average water level fluctuations in the basin (M)	Point recharge in water column when specific yield is 4%	Net recharge (MCM)
1974	3.10	0.1240	54.20
1975	4.95	0.1980	86.55
1976	2.23	0.0892	38.99
1977	2.53	0.1012	44.23
1978	5.33	0.2132	93.19

Average Recharge (5 years) = 63.43 MCM

Area of the basin = 437.1 sq km

Table 6. Volume of rainfall by Thiessen method at different stations

Rain gauge station	Thiessen polygon (sq km)	Observed rainfall (cm)	Rainfall volume (sq km cm)
Chippalapalli	52.5	61.8	3244.5
Maisaram	37.5	78.0	2925.0
Nandigaon	61.0	87.3	5325.3
Palmakul	8.5	62.2	528.7
Parandla	43.8	72.3	3163.1
Shahbad	98.0	88.0	8624.0
Shadnagar	39.5	80.4	3175.8
Welijorla	97.0	77.6	7527.2

Total volume of rainfall over the entire basin area = 345.1362 MCM

1974	345.136	54.20	15.704
1975	545.848	86.55	15.856
1976	351.703	38.99	11.086
1977	310.865	44.23	14.228
1978	461.290	93.19	20.202

Average infiltration rate = 15.415%

Conclusions

A direct relation is observed between elevation and fluctuation with high positive correlation coefficient. The frequency of rainfall is shown to have a direct relation on the amount of recharge compared to the intensity of rainfall itself. Representative specific yield values for the entire basin area are computed using a numerical model. Weighted averages of rainfall at different stations are also computed by Thiessen polygon method which is designed to account for the irregular distribution of the rain gauge stations. The average rainfall infiltration using this method is estimated at 15% of the total rainfall over the entire basin area.

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References

1. Central Groundwater Over Exploitation Committee Report, Bombay
2. Bhimanikant B H and Avadhanulu R V S S 1981 *Groundwater* 19 400
3. Bhimanikant B H and Bhimasankaram V L S 1981 *Groundwater* 19 476
4. Canadian-assisted groundwater project, Hydrogeology Technical Report, December 1971 *Groundwater potential in hard rock areas*, Seminar Volume, GSI, Bangalore 45
5. Bhimanikant B P 1971 *Groundwater potential in hard rock areas*, Seminar Volume, GSI, Bangalore 27
6. Bhimanikant B S 1971 *Groundwater potential in hard rock areas*, Seminar Volume, GSI, Bangalore 42
7. Jayaramana B 1972 *Problems in Groundwater development*, Procs. of the Seminar, Directorate of Groundwater, Madras 36

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Abstract. The igneous alkaline rocks at Elchuru start from a parent ijolite-melteigite association to basic malignite, melalusitanite and shonkinite followed by nepheline syenites and then biotite lamprophyres (as dykes) at the waning phase of the evolutionary course of the complex. The distinct alkalinity of the rocks is manifested by the development of modal nepheline and calcic amphibole (kaersutite). For both the basic rocks, *i.e.* alkali gabbro and biotite lamprophyre, the percentages of normative nepheline are always higher than modal nepheline, indicating silica deficiency and alkali enrichment of the mafics. It is evident from detailed petrological and geochemical studies that the two basic members are very much akin to each other and there is no major deviation in their bulk chemistry.

Keywords. Alkaline complex; ijolite-melteigite; malignite; melalusitanite; biotite lamprophyre.

Introduction

Elchuru alkaline intrusion ($16^{\circ}05'N$; $79^{\circ}55'E$) is an arcuate ring complex outcropping in the Eastern Ghats high grade metamorphic series (Bose and Nag *et al* 1982; Nag and Chakravorty 1983) with abundant development of gabbro, hypersthene bearing granulites and gneisses, anorthosites, granite-calc-granulites, quartzites and migmatites (Perraju *et al* 1979). These rocks have been variously dated by different workers as ranging from 3000–1000 Ma (Nag *et al* 1982), indicating a complex history for the Eastern Ghats, largely in the Archeans. The petrological and temporal relationships along with limited radiometric age data on the alkaline plutons of the Eastern Ghats belt, suggest a probable Proterozoic magmatism (Sarkar 1968; Deans and Powell 1968; Crawford 1969; Subba Rao 1971; Bose 1972; Ray and Bose 1975; Moralev *et al* 1975), emplaced along regional fractures across the high grade host rocks, particularly at Tamil Nadu (Srinivasan 1973, 1977; Subramaniam *et al* 1977). The Elchuru alkaline complex, being a part of Eastern Ghats alkaline episode, is composed of a series of rock types ranging from ijolite-melteigite to alkali gabbro (starting with malignite, melalusitanite and shonkinite) and then to hypersolvus and nepheline syenites which are penetrated by late stage biotite lamprophyre. In this paper, the petrography, mineralogy, geochemistry and modal content of these rocks are studied to develop a model of the magmatic evolution of the complex. It is to be mentioned here that the ijolite-melteigite association at Elchuru is a very rare one (through detailed mapping on $16'' = 1$ mile scale) apart from alkaline rocks as reported earlier (Madhavan and Leelanandam 1979; Nag 1979; Bose and

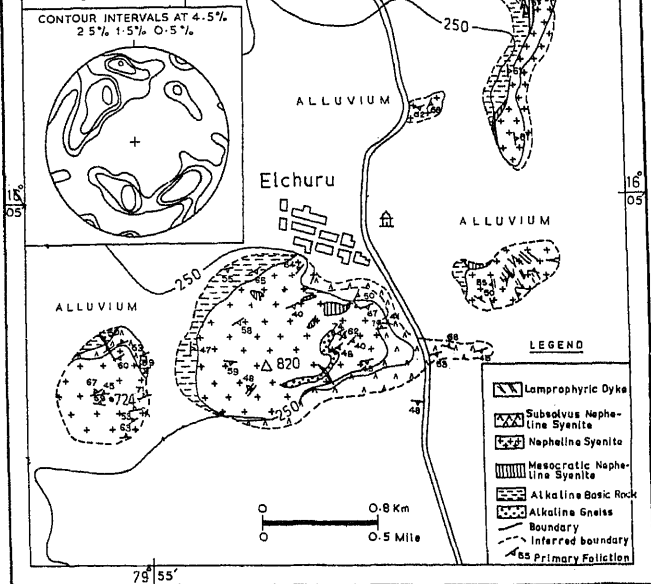


Figure 1. Geological map of Elchuru alkaline complex, Prakasam District, Andhra Pradesh. For inset map: 1. Koraput, Orissa; 2. Kunavaram, Andhra Pradesh; 3. Elchuru, Andhra Pradesh; 4. Sivamalai, Tamil Nadu. Cd—Cuddapah basin.

Nag 1980). An extensive geochemical and petrological account on ijolite-melteigites at Elchuru is discussed elsewhere (Nag 1983).

The purpose of the present paper is to test, using Student's *t*-statistic, whether there is statistically significant difference in modal contents or major and trace element contents and elemental ratios of the early phase alkali gabbros and the biotite lamprophyres. In all instances the null hypothesis was satisfied at the 1% confidence suggesting that both rock groups were derived from the same population.

2. Method of study

Six samples from alkali gabbro and four from biotite lamprophyre were selected for chemical analyses and their modal contents are presented in table 1. Major elements were analysed by rapid method (Shapiro and Brannock 1962) using USGS standards GSP-1 and BCR-1 for countercheck of precision and accuracy of results. Total alkalis were determined by flame photometric technique. The samples were analysed at the Geological Laboratory, Presidency College, Calcutta. Trace elements of both the rock-members were analysed using a Philips PW 1410 Universal vacuum x-ray spectrometer at the University of Windsor, Ontario, Canada. Compton scatter peaks were measured for each sample, for USGS standard rock powders W-1, AGV-1, GSP-1

hibole	28.13	—	46.37	5.72	1.24	28.12	1.23	—	—	—
te	9.76	46.86	14.04	38.80	50.73	19.01	51.70	41.17	47.90	51.00
li feldspar	31.81	17.44	24.76	17.43	31.25	43.80	18.50	36.52	8.50	8.00
cline	14.16	2.90	8.36	6.00	—	3.60	1.17	5.95	4.40	—
ite	2.91	0.42	0.48	1.25	—	1.07	0.22	1.05	—	—
te	0.67	0.56	0.69	0.71	3.18	0.29	0.41	0.69	0.99	—
ne	—	—	—	3.57	—	—	—	—	—	—
que	0.19	—	—	0.40	—	0.19	0.63	0.26	—	—
ur index	53.97	79.95	66.88	76.53	68.75	52.60	68.55	70.41	87.19	92.00

-2, and for pure quartz on both Cr and Mo target tubes. From this data mass absorption values were determined for each sample (Turek *et al* 1977). The mass absorption values derived from the measurements using Mo tube were used to correct analyses of Rb and Sr. The mass absorption values derived from the Cr tube measurement were used to correct the analysis of Ba.

discussion

basic malignite, melalusitanite and shonkinite are generally made up of coarse-grained subhedral minerals, grading into each other according to variation in mineral proportions (Streckeisen 1973). The biotite lamprophyres are fine-grained microporphyritic rocks containing clinopyroxene phenocrysts. The pyroxenes in alkaline gabbros are ferrosalite ($\text{Wo}_{54}\text{Fs}_{26}\text{En}_{20}$) and those in the biotite lamprophyres are salite ($\text{Wo}_{45}\text{Fs}_{16}\text{En}_{39}$). Both the basic rocks contain calcic amphibole (magnetite; Leake 1978) and Fe-Ti biotites. Microperthitic and cryptoperthitic orthoclase occur in the early basic rocks whereas the feldspar is mesoperthite orthoclase in the lamprophyre.

Analyses of $(\text{Na}_2\text{O} + \text{K}_2\text{O})$ -CaO-MgO-(FeO + Fe_2O_3) of alkali gabbros and biotite lamprophyres show no significant chemical differences (figure 2) between the two rock types which also have similar Fe/Mg ratio. The ratio $\text{TiO}_2 \times 100 / (\text{FeO} + \text{Fe}_2\text{O}_3)$ when plotted against TiO_2 (wt%) shows that the combined effects of Fe-Ti do not widely vary between early and late stage magmatic events (figure 3). The ratio $(\text{Na}_2\text{O} + \text{K}_2\text{O}) / (\text{CaO} + \text{MgO})$ being always less than 1 ranges from 0.83 to 0.93 for basic gabbros and 0.63 to 1.00 for biotite lamprophyres (figure 2).

Geochemical studies reveal that the mean K₂O, Rb, Ba contents and K/Rb ratios of early basic rocks (5.30%, 116 ppm, 2416 ppm and 395.65 respectively) and biotite lamprophyres (5.13%, 135 ppm, 2322 ppm and 336.13 respectively) are very similar. The average CaO contents show depletion in lamprophyres (8.23%) compared to the gabbros (9.52%). Strontium shows no significant difference being 1258 ppm in gabbros and 1204 ppm in lamprophyres. The average ratios Rb/Sr and Ba/Sr are in accordance with their similar magmatic character showing highly significant results at 1% confi-

Table 2. Chemical analyses and cnpw Norms of alkaline basic rocks of Elchuru, Andhra Pradesh, India.

Serial Numbers	Alkali gabbro					Biotite lamprophyre			
	1	2	3	4	5	6	7	8	9
SiO ₂	45.92	45.71	47.81				45.20	45.54	45.83
TiO ₂	3.02	2.22	2.55				1.46	2.72	2.55
Al ₂ O ₃	17.02	12.02	13.54				17.52	14.80	12.08
Fe ₂ O ₃	0.62	4.12	4.29				1.37	4.59	2.07
FeO	7.75	7.18	7.75				6.89	6.90	7.03
MnO	0.26	1.08	1.13				0.26	1.07	0.70
MgO	4.55	7.70	4.20				8.19	5.01	8.51
CaO	7.68	9.38	7.22	9.54	10.82	12.48	8.54	6.30	9.23
Na ₂ O	5.24	3.79	3.62	2.88	4.55	4.28	4.25	3.15	4.35
K ₂ O	5.31	4.56	5.62	5.61	5.58	5.13	3.82	6.91	5.04
H ₂ O	1.72	1.90	1.60				2.08	1.98	2.18
P ₂ O ₅	0.99	0.59	0.59				0.56	0.79	0.39
Total	100.26	100.30	99.92				100.14	99.76	100.46
CaO/MgO	1.69	1.22	1.72				1.04	1.26	1.08
Na ₂ O + K ₂ O/Al ₂ O ₃ (mol. prop.)	0.83	0.93	0.89				0.63	0.86	1.00
TiO ₂ × 100	36.08	19.65	21.18				17.67	23.67	28.02
FeO + Fe ₂ O ₃									
Solidification Index	19.38	28.15	16.48				33.40	18.86	31.51
Mafic Index	64.78	59.47	74.14				74.14	51.68	50.21

Table 2 (Contd.)

CIPW Norms

Or	26.00	22.72	33.36	11	22.54	34.27	11
Ab	—	—	6.55	—	—	—	—
An	7.78	2.36	4.17	—	17.51	5.75	—
Lc	4.03	3.33	—	—	—	5.17	14
Ne	23.86	17.32	13.03	19.31	19.31	14.43	19
Di	20.08	32.77	23.37	17.27	17.27	16.90	34
Ol	7.82	8.25	6.41	16.66	16.66	7.55	9
Mt	0.70	6.03	6.26	1.86	1.86	6.66	3
Il	5.63	4.25	4.86	2.75	2.75	5.17	4
Ap	2.35	1.34	1.34	1.34	1.34	1.85	0
Rb	105	120	88	161	81	220	14
Ba	1480	1870	1590	4833	1547	2490	226
Sr	810	1070	2510	970	827	1850	125
K/Rb	420.00	315.45	530.15	289.44	391.49	260.74	29
K/Ba	29.78	20.24	29.34	9.64	20.50	23.04	1
Ca/Sr	103.21	62.65	20.56	87.32	93.71	24.34	5
Ba/Sr	1.83	1.75	0.63	4.98	1.87	1.35	—
Rb/Sr	0.13	0.11	0.04	0.17	0.10	0.12	—

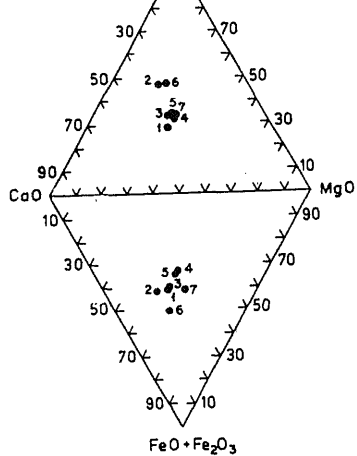


Figure 2. (Na₂O+K₂O)-CaO-MgO-(FeO+Fe₂O₃) diagram (wt%) for alkaline basic members of the investigated complex. 1-3. Alkaline gabbro. 4-7. Biotite lamprophyre.

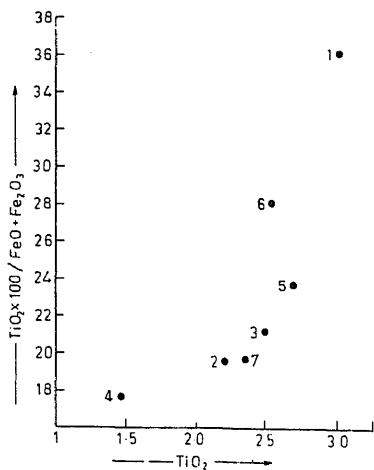


Figure 3. Plots of $\text{TiO}_2 \times 100 / (\text{FeO} + \text{Fe}_2\text{O}_3)$ (wt%) as a function of TiO_2 (wt%) of the alkaline basic members of Elchuru. Explanation same as in Figure 2.

	χ_1	χ_2	σ_1	σ_2	df	t^*	Significance
	27.748	9.202	17.88	11.548	8	1.340	H ₀
	15.563	10.329	30.225	10.511	8	1.953	H ₀
	29.87	16.206	47.943	4.163	8	1.952	H ₀
	66.447	10.303	79.538	10.221	8	1.766	H ₀
	46.48	9.444	45.228	5.558	5	1.851	H ₀
	14.193	2.093	14.21	2.178	5	0.0086	H ₀
	7.233	1.886	0.57	0.143	5	1.034	H ₀
	5.483	1.574	7.86	1.743	5	1.572	H ₀
	9.52	1.786	8.228	1.169	8	1.097	H ₀
	4.05	0.746	3.64	0.672	8	0.811	H ₀
	5.302	0.377	5.128	1.123	8	0.314	H ₀
	0.883	0.041	0.818	0.134	8	0.695	H ₀
	1.543	0.229	1.078	0.119	5	2.950	H ₀
	25.637	7.411	22.265	3.962	5	0.645	H ₀
dex	21.337	4.961	29.318	6.089	5	1.568	H ₀
	66.13	6.065	57.75	9.618	5	1.119	H ₀
	116.333	25.642	135.25	53.364	8	0.669	H ₀
	2416.833	1133.97	2321.75	519.354	8	0.140	H ₀
	1257.833	568.95	1204.25	406.25	8	0.146	H ₀
	395.648	86.55	336.13	57.93	8	1.079	H ₀
	21.103	6.90	18.8	3.636	8	0.548	H ₀
	71.093	26.09	60.85	25.572	8	0.548	H ₀
	2.252	1.324	2.148	0.840	8	0.125	H ₀
	0.108	0.041	0.11	0.007	8	0.073	H ₀

level of significance where t 3.365, 2.896 (one sided test) and 4.032, 3.355 (two sided test) at degrees of freedom (df) = 5 and 8.
 hypothesis at 1% level "that the samples are coming from the same population in 99 cases out of 100 is accepted.

Table 3) and hence, all the geochemical characters suggest a close coherence between the two basic members of Elchuru alkaline complex.

on

the two basic members of Elchuru are very similar in chemistry and it is suggested that the biotite lamprophyres were formed from a basic magma which appeared late in evolutionary sequence of the complex as a result of a hydrous environment by earlier solidification of magmas, metamorphism (Sengupta 1964). Abundance of modal biotite (always > 40%) and phenocrysts

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References

Bailey D K 1964 *J. Geophys. Res.* **69** 1103
Borodin L S, Gopal V, Moralev V M and Subramanian V 1971 *J. Geol. Soc. India* **12** 101
Bose M K 1972 *24th Int. Geol. Congr.* Section **14** 18
Bose M K and Nag S 1980 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **89** 383
Crawford A R 1969 *J. Geol. Soc. India* **10** 117
Deans T and Powell J L 1968 *Nature (London)* **218** 750
Grady J C 1971 *J. Geol. Soc. India* **12** 56
Leake B E 1978 *Can. Miner.* **16** 501
Madhavan V and Leelanandam C 1979 *N. Jb. Miner. Abh.* **136** 276
Moralev V M, Voronovski S N and Borodin L S 1975 *Doklady Akad. Nauk. SSSR* **222** 46
Nag S 1979 *Sci. Cult.* **45** 207
Nag S, Chakravorty P S and Smith T E 1982 *Proc. 69th Indian Sci. Congr.*, pt. III 16 (Abs.)
Nag S and Chakravorty P S 1983 *Proc. 70th Indian Sci. Congr.* Pt. III 6 (Abs.)
Nag S 1983 *N. Jb. Miner. Abh.* **147** (in press)
Perraju P, Kovach A and Svingor E 1979 *J. Geol. Soc. India* **20** 290
Ray S and Bose M K 1975 *Chayanica Geol.* **1** 1
Sarkar S N 1968 *Precambrian stratigraphy and geochronology of Peninsular India* (Dhanbad: Dhanbad Publishers) p. 33
Shapiro L and Brannock W W 1962 *U.S. Geol. Surv. Bull.* **1144-A** pp. 56
Srinivasan V 1973 *J. Geol. Soc. India* **14** 313
Srinivasan V 1977 *J. Geol. Soc. India* **18** 598
Streckeisen A 1973 *N. Jb. Miner. Abh. H-4* 149
Subba Rao K V 1971 *J. Petrol.* **12** 621
Subramaniam V, Viladkar S G and Upendran R 1978 *J. Geol. Soc. India* **19** 206
Turek A, Riddle C and Smith T E 1977 *Can. J. Spectrosc.* **22** 20

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Preliminary pedogeochemical and biogeochemical studies on the Hinota kimberlite, Panna District, MP

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Abstract. Distribution of a number of trace elements in the Hinota kimberlite pipe, its overlying soil and selected vegetation is discussed in relation to the distribution of the same elements in the soils and vegetation overlying the quartzite country rock through which it is emplaced. The values of most elements in Hinota are similar to those of kimberlites from some other parts of the world, but Ni and Cu are higher. These elements in a "floating reef" of the Dhandraul Quartzite in the pipe closely follow the averages for other sandstones. It is observed that most elements in the soil over the kimberlite are significantly higher than in the soil over the surrounding country rock, particularly Ni, V, Cu and Cr. These could serve as reasonable guides to prospecting for kimberlites. Trace elements in the ash of selected trees growing over the diatreme and the country rock do not show any significant differences.

Keywords. Hinota kimberlite; trace elements; pedogeochemistry; biogeochemistry.

1. Introduction

Pedogeochemical and biogeochemical methods of prospecting have already proved useful in several cases of mineral exploration. Such studies have also been effectively used in India, particularly for tracing the extension of some known base metal mineralised belts, potential mineralised zones and around areas of non-favourable geological regions (Bose and Banerjee 1976). However, these studies for prospecting of hidden kimberlite pipes have not been carried out so far in our country, and only at a few places elsewhere. Accounts of geochemical prospecting for kimberlites in general are also not common. Applied geochemical studies in the Katanga area (Van de Steen 1961) and kimberlite pipe near Kenieba, West Africa (Aicard 1959), however, have yielded encouraging results. Theoretically, emplacement of kimberlite diatremes (which have rather unusual chemistry with specifically high concentrations of Cr, Ni, Co, Cu) in the country rocks of the cratonic areas should be in geochemical contrast in respect of these trace elements. Buried kimberlite diatremes could therefore possibly be located by detailed pedogeochemical and biogeochemical surveys provided such studies yield positive correlation in the already known areas of kimberlite occurrences. The aim of this paper is to explore such a possibility. Trace element distributions in kimberlite, the soil cover and three of the common occurring plants over it are reported for the Hinota diatreme, Panna District, MP (figure 1). The choice of the Hinota pipe was obvious as it is more or less in its natural state and the surface soil has not been generally disturbed. The results therefore are expected to be unbiased unlike they would be in the case of the Majhgawan diatreme, situated about 3 km southeast, which has been extensively churned over by old and recent workings.

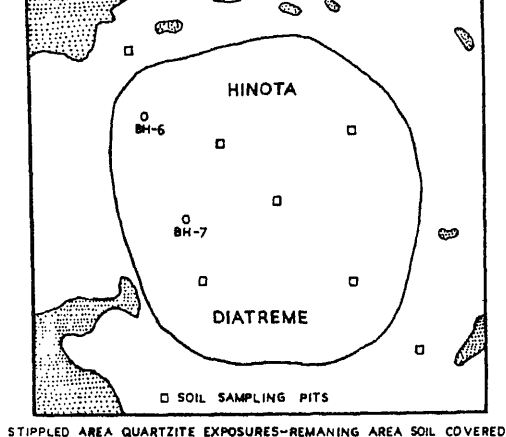


Figure 1. Simplified plan of Hinota diatreme.

Table 1. Selected trace elements in the Hinota and some other kimberlites (all values in ppm).

	Hinota			Basutoland (Dawson 1962)	Yakutia (Litinski 1961)	South-West Africa (Jones 1964)
	Borehole No. 6	Borehole No. 7	Surface			
Ni	285-750	65-500	< 500	1140	450	1200
Co	50-90	25-90	10	77	40	• 50
Cu	150-200	50-200	5	100	60	—
Pb	25	10-25	10	9	—	30
Ga	10-50	10-25	5	9	—	10
V	100-150	50-200	10	120	170	160
Cr	—	—	< 500	1440	1500	1000
Ba	—	—	10	740	—	1000

Analyses of the Hinota samples by the Geological Survey of India.

The Hinota kimberlite diatreme is not economic for diamond production unlike the Majhgawan diatreme which sustains a regular production of about 10,000 carats per annum. Extensive petrological, mineralogical, geochemical and geochronological studies have been carried out on the Hinota diatreme (Kresten and Paul 1976; Paul 1979; Paul *et al* 1975a, b, 1976, 1977). This paper embodies the results of some selected trace element data on the kimberlite by one of the authors together with trace element analyses on the soils and selected vegetation in and around the diatreme acquired by the late Dr D R Grantham (who passed on his papers to one of the authors before his death). The results are shown in tables 1 to 4. For comparison trace element data of some other kimberlites are also given in table 1.

	Dhandraul Quartzite floating reef	Vindhyan (Rewah) quartzite near Sagar	Average for sandstone (Hawkes and Webb 1962)
Ni	10	15	2-10
Co	25	5	1-10
Cu	50	29	10-40
Ga	10	—	—
V	50	—	10-60
Cr	—	50	10-100

Table 3. Trace elements in soil covering the Hinota diatreme and surrounding country rock (all values in ppm).

	Soil over diatreme			Soil over Dhandraul Quartzite
	From 30 and 90 cm depth (6 samples)	From 1.8 m depth	From 2.4 m depth	
	Range	Average		
Pb	15-37	34	10	10
Sn	5-7	6	3	3
Ga	15-18	16	10	5
V	100-150	127	110	30
Cu	45-60	49	30	10
Ti	3500-4750	4000	4000	5000
Ni	45-65	56	130	120
Co	13-30	17	25	30
Mn	450-950	641	450	650
Cr	100-150	120	130	180
Zr	200		200	200

Analyst: Dr. I. Nichol, Imperial College, London.

2. Experimental

Samples were analysed semiquantitatively by Hilger emission spectrograph using visual comparison with synthetic standards. The samples were burned in a 7-amp DC arc. A 60-second exposure of the spectrum was recorded between 2500 and 3675 Å. The limit of error for most of the elements reported was within 20-30%, which is reasonable for such determinations (Ahrens 1961).

3. General

As in all ultramafic rocks, Ni, Cr, Co and Cu are fairly high in kimberlites along with abnormally high Ba, La, Li, Nb, Pb, Rb, Sr, Th, U and Zr. However, only the first set of

	Mahua		Teak		Palas	
	On pipe	Outside pipe	On pipe	Outside pipe	On pipe	Outside pipe
Pb	5	6	2	4	5	10
Ga	n.d.	n.d.	n.d.	n.d.	tr	4
Mo	n.d.	n.d.	15	tr	n.d.	n.d.
Cu	115	100	80	60	85	80
Ti	200	150	200	150	325	500
Ni	30	40	12	3	40	30
Co	n.d.	n.d.	n.d.	n.d.	n.d.	tr
Mn	225	400	125	300	750	2500
Cr	7	5	5	5	15	15
Mg %	6	6	4	6	10	7

Analyst: Dr. I. Nichol, Imperial College, London.
n.d. — not determined.

elements could be considered suitable for prospecting kimberlites. This is also found true in the present case. It is observed that vanadium also exhibits considerably anomalous values in the soil over the kimberlite pipe in contrast to the soil over the Dhandraul Quartzite country rock.

4. Kimberlite borehole samples

Among the trace elements that have been determined in the kimberlite (table 1) Ni is fairly low, ranging from 285 to 750 ppm and with an average of 400 ppm in borehole No. 6 and much lower in borehole No. 7 (65-500 ppm with an average of 270 ppm). In comparison, the Basutoland kimberlites and South-West African kimberlites have higher Ni contents, 1440 and 1200 ppm, respectively; Yakutia kimberlites, however, with 450 ppm Ni match closely with the kimberlite under study (Dawson 1967). Cobalt values are comparable with the world averages for kimberlite but some of the Hinota samples record higher concentration of this element, as high as 90 ppm. Copper concentration in the Yakutia and Basutoland kimberlites on the average is 60 and 100 ppm, respectively, while the Hinota kimberlite has a generally higher Cu concentration of 150-200 ppm in borehole No. 6; the upper Cu range in borehole No. 7 is also 200 ppm but values as low as 50 ppm are also found. Vanadium in the present case ranges from 50-200 ppm and gallium from 10-50 ppm. Both these elements seem to be comparatively enriched in the Hinota kimberlite in comparison to the Basutoland (V = 120 ppm and Ga = 9 ppm) or South-West African kimberlites (V = 160 ppm and Ga = 10 ppm).

5. Quartz arenite and other sandstones

Although there are no trace element data available for the Dhandraul Quartzite (which is in fact a quartz arenite) of the country rock into which the Panna kimberlites are

have given the same values showing the highly consistent concentration of these elements in the quartz arenite. For comparison, trace element data of a Vindhyan sandstone (possibly Rewah) at Sagar of similar petrology and chemistry and the averages of a large number of sandstones (Hawkes and Webb 1962) are also recorded. This "floating reef" of quartzite, though only about 10 m thick and not as huge as the Waterberg Quartzite occurring in the Premier pipe (South Africa), presents interesting implications. It is observed that most of the elements (Ni, Co, Cu and V) in the Hinota "reef" are within the usual range of these elements found in other normal sandstones. Cobalt, however, has a slightly higher value. How far is this difference real and meaningful can only be assessed when its concentration in the surrounding Dhandraul Quartzite is known. On the whole it implies that the kimberlitic magma seemingly had no effect on the caught-up quartz arenite in modifying the trace element chemistry of the refractory quartzite so far as these elements are concerned. A comparison of the kimberlite and sandstone analyses in tables 1 and 2 shows that most of the elements are enriched several times in the kimberlite as compared to the sandstone of the country rock.

6. Pedogeochemistry

Distribution of 11 trace elements in the samples collected over the diatreme and over the adjacent country rock of Dhandraul Quartzite is shown in table 3. Soil samples were collected from three different horizons to determine the most anomalous horizon with respect to the elements of interest. The six samples at depths of 30–90 cm were collected along two lines laid at right angles, crossing at a point approximately at the centre of the pipe area. The soil samples from 1.8 m depth and 2.4 m depth were collected from the side of a pit. The last sample was obtained from the soil covering the Dhandraul Quartzite about 1.5 km north of Panna (figure 2).

Concentrations of four elements of interest considered here namely, Cr, Ni, V and Cu, are highly variable in the soils in general. Cr has a range of 5–1000, Ni 5–500, Co 1–40 and Cu 2–100 ppm (Levinson 1972). The average concentration of these four elements in the top soil covering the Hinota kimberlite is as follows: Cr 120, Ni 56, V 127 and Cu 49 ppm. In contrast, the soil covering the neighbouring quartzite has the following figures for the same elements; Cr 40, Ni 30, V 30 and Cu 10 ppm. This means that Cr, Ni, V and Cu are enriched 3, 2, 4 and 5 times, respectively, in contrast to the soils over quartzitic rocks.

Considering the distribution of these elements in the entire soil profile, it is observed that all the elements do not have a similar pattern. Cr and V have their maximum concentrations in the deeper levels (2.4 m depth) having 180 ppm of Cr and 150 ppm of V. Nickel, on the other hand, has the most anomalous value in the intermediate depths of the soil profile (1.8 m depth), having a concentration of 130 ppm in contrast to 56 ppm in shallower depths and 120 in deeper parts. Copper and cobalt have more or less similar distribution patterns; their enrichment is found both in the shallower as well as deeper horizons. In the appraisal of the Katanga area geochemical anomalies with respect to Cu, Zn, Sn, Cr and Ni in the soils overlying the kimberlite pipe have been found to be useful (Van de Steen 1961). Similarly, Cu and Ni anomalies at depths of 30 and 100 cm in soils overlying a kimberlite pipe near Kenieba, West Africa, have proved to be useful indications for prospecting hidden kimberlite pipes. Chromium, in

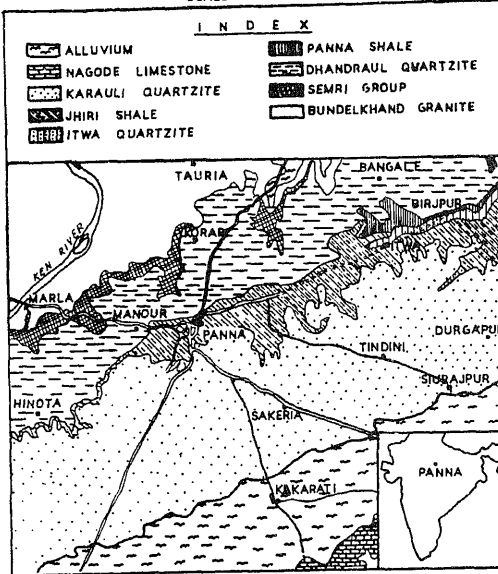


Figure 2. Part of the Panna diamond field.

particular, at a depth of 100 cm, has a more pronounced difference than Ni, and has been recommended specifically in aid of kimberlite prospecting (Aicard 1959) This work also proves that anomalous distributions of Cr and V at 2.4 m depth in the quartzitic area can be useful geochemical guides for prospecting of kimberlites. Similarly, Ni at 1.8 m level seems to be a good indicator for the occurrence of ultramafic rocks like kimberlite in an otherwise nickel-poor, soil-covered region derived from sandy rocks. Cu does seem to be as useful as Cr and Ni in the present case. On the other hand, V has values which seem to be nearly four to five times enriched in soil over the kimberlite pipe in contrast to the general background, and could also be a useful geochemical guide in prospecting.

The soil on the kimberlite pipe area in general is of residual type but a certain amount of contribution from transported material cannot be ruled out, particularly in the depressions formed by the diatreme. Efforts should be made to test the validity of the suggested geochemical guides by a thorough study of the soil profile itself and the extent of residual and transported nature of soils to give a more realistic and meaningful interpretation.

7. Biogeochemistry

Biogeochemical prospecting has had very limited application so far in our country mainly because of its various variables and complications involved. However, because of greater depth of penetrations, this technique has great potential provided systematic and detailed orientation surveys are undertaken. The present findings are most preliminary in this direction and are aimed only at pointing towards more serious attempts in future for using trace element contents of plant ash as potential indicators

for rocks of ultramafic composition. Table 4 gives results of 9 trace elements determined spectrographically in the plant ash obtained from incinerating leaves and twigs of three selected common trees growing over the diatrema as well as the surrounding quartzite. These include Mahua (*Madhuca indica*), Teak (*Tectona grandis*) and Palas (*Butea monosperma*).

Unlike the pedogeochemical findings, the results on the plant ash analysis are not encouraging. Nickel shows practically no difference in *Madhuca indica* and *Butea monosperma* as values on the trees growing on the pipe and growing outside it are nearly the same. Only *Tectona grandis* shows that 12 ppm of Ni found in the ash in the plant growing over the pipe may have some relation with the source rocks as outside the pipe this value drops down to 3 ppm only. However, this difference cannot be called notable and has to be taken with caution. Concentration of Cu is also similarly not significant, and that of Cr in all the three species is more or less the same, whether it grows on the kimberlite pipe or outside it. Among the elements recorded in table 4 only Mn shows appreciable amounts, but then the Mn content of average plant ash is normally very high, as much as 4800 ppm (Hawkes and Webb 1962). From the present work it appears that all the three plants are capable of extracting more manganese from the sandstone bedrock than from where the bedrock is kimberlite. This is particularly noticeable in *Butea monosperma* where the Mn content in the country rock is 2500 ppm against 750 ppm in the ash of trees growing over the pipe. Thus these elements seem to have no applied geochemical significance in the present case.

Unfortunately, there are about 20 different variables which can affect the accumulation of elements by vegetation (Brooks 1972). Some of the more important ones include the type of plant studied, the plant organ sampled, the age of plant and/or plant organ, pH of the soil, depth of the root system, drainage conditions, etc. In this preliminary work there was no scope for investigating the effect of the above variables and it is just likely that the plants chosen for this work were not the right type for accumulating the elements of interest. Cannon (1960) has shown that both Cr and Ni are enriched more in grasses, herbs and shrubs than in bigger trees (deciduous, conifers, etc.). For example, Cr content of grasses, herbs and shrubs is 19, 10 and 14 ppm, respectively, as against 5 and 8 ppm in deciduous trees and conifers. Similarly, Ni is highest in shrubs (91 ppm) as against 57 ppm in conifers. It is believed that a detailed biogeochemical study of different species growing over and surrounding the kimberlite diatremes will be worthwhile in testing the potential of this technique for prospecting kimberlites.

8. Conclusions

Pedogeochemical studies on the Hinota kimberlite diatrema and the surrounding quartzitic terrain suggests that Ni, V, Cu and Cr anomalies in the soils overlying the pipe rock could serve as geochemical guides in prospecting for kimberlites. Other anomalies are less significant but could be useful. Biogeochemical study attempted on a limited scale does not bring out a positive relationship. Systematic and detailed work on these lines could be rewarding in search for hidden kimberlite pipes.

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- Ahrens L H 1961 *Spectrochemical analysis* (London: Addison-Wesley)
- Aicard P 1959 *Ann. Minas* 103
- Bose S K and Banerjee S 1976 *Indian Miner.* **30** 48
- Brooks R R 1972 *Geobotany and biogeochemistry in mineral exploration* (New York: Harper and Row)
- Cannon H L 1960 *Science* **132** 591
- Dawson J B 1962 *Bull. Geol. Soc. Am.* **73** 545
- Dawson J B 1967 *Ultramafic and related rocks* (ed.) P J Wyllie (New York, London, Sydney: John Wiley) p. 241
- Hawkes H E and Webb J S 1962 *Geochemistry in mineral exploration* (New York: Harper and Row)
- Jones A J A 1964 *Kimberlites and related rocks of the Nama Plateau, South-West Africa* Ph.D. Thesis (unpublished) University of Leeds, U.K
- Kresten P and Paul D K 1976 *Can. Mineral.* **14** 487
- Levinson A A 1972 *Introduction to exploration geochemistry* (Calgary: Applied Publishing Ltd.)
- Litinski V A 1961 *Geochemistry* 813
- Paul D K 1979 *Geochim. Cosmochim. Acta* **43** 389
- Paul D K, Buckley F and Nixon P H 1976 *Chem. Geol.* **17** 125
- Paul D K, Gale N H and Harris P G 1977 *Geochim. Cosmochim. Acta* **41** 335
- Paul D K, Potts P J, Gibson I L and Harris P G 1975a *Earth Planet. Sci. Lett.* **25** 151
- Paul D K, Rex D C and Harris P G 1975b *Bull. Geol. Soc. Am.* **86** 364
- Van de Steen J 1961 *Symposium on Geochemical Prospecting in the Western Region*, 3rd Meeting Kaduna 1960 p. 42

The Kalpatta granite, Wynad District, Kerala (India) and its petrochemistry

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Abstract. The elliptical grey biotite-granite stock of Kalpatta, Wynad district, Kerala is emplaced within the Precambrian crystallines comprising gneisses and charnockites and is probably one of the intrusive granites of the Kerala region which were emplaced during the late Precambrian-early Palaeozoic tectonomagmatic episode. Chemically, the rock is peraluminous and granodioritic in composition with element distributions and presence of mafic enclaves suggesting supracrustal participation. Derivation of the granite by alkali feldspar and quartz fractionation from a partial melt generated at deeper crustal levels is envisaged.

Keywords. Stock; granite; late Precambrian; geochemistry; fractionation; enclave.

1. Introduction

The Kerala region which forms part of the western continental margin of India is understood to have suffered a prominent tectogenetic episode during the late precambrian-early Palaeozoic periods as shown by the recent radiometric data (Nair and Vidyadharan 1982; Soman *et al* 1982) with associated rifting and granitic activity (Santosh and Nair 1983). Intrusive granites are known to occur in the Precambrian crystallines at different localities, distributed along regional lineaments.

The granite of the present study occurs as an elliptical stock covering about 50 sq km around Kalpatta, Wynad district, Kerala (figure 1). It is a medium-to-coarse grained grey biotite granite emplaced within the Precambrian crystallines comprising biotite-hornblende gneisses and charnockites. The granite shows rather sharp contacts with the country rocks as observed in some localities where the leucocratic granite sharply abuts against the melanocratic biotite-hornblende gneiss, with no discernible transition zone. Near the granite contact, the trend of foliation of the surrounding rocks shows gentle flexures deviating from its regional trends. The general feeble foliation imparted by biotite in the granite becomes pronounced near the contact with the country rock. Numerous veins of pegmatites and aplite varying in width from 5 to 15 cm traverse the country rock. In some localities, tongues and apophyses of the granite are seen cutting into the country rock.

Enclaves of hornblende gneisses and amphibolites (0.5 m to a few metres in dimension) with locally-diffused contacts are observed within the granite body, especially towards the peripheries. The foliations in the enclaves and the enclosing granite are discordant. Irregular veins of pegmatite and aplite in the granite cut across the enclaves also.

The gneisses surrounding the granite have been polymetamorphosed with an overprinting of a retrogressive amphibolite facies on the original granulite facies (Nair 1976). The age of the granite is not known. Based on structural relations of the

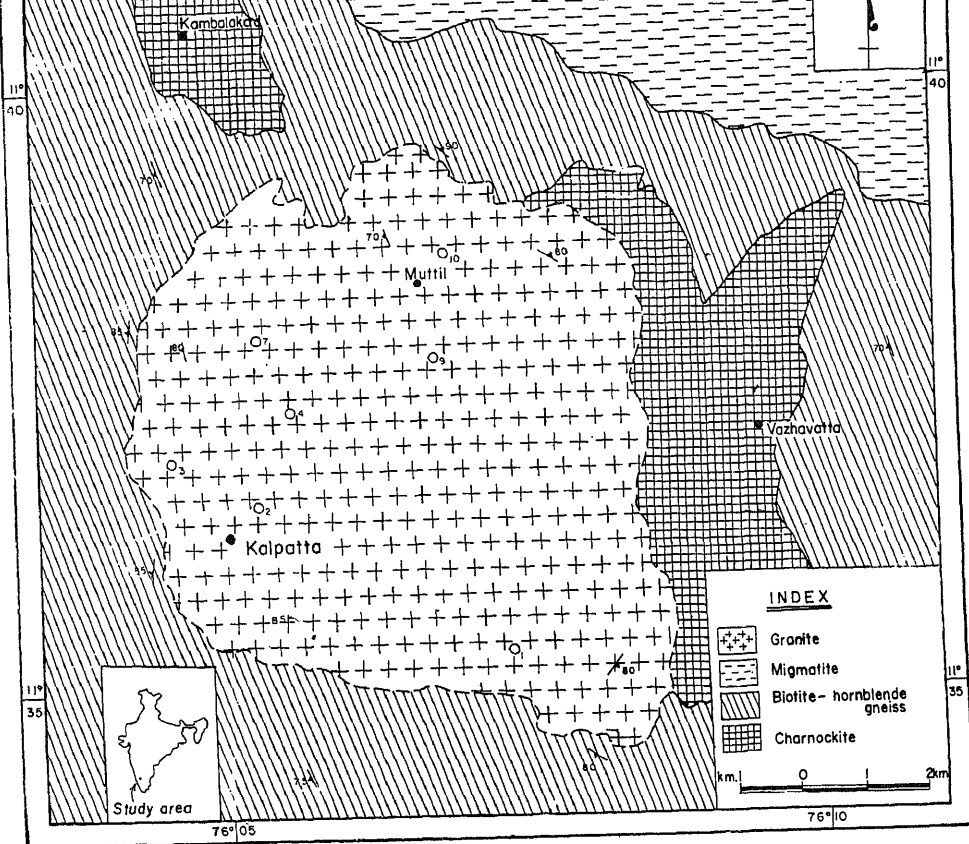


Figure 1. Generalised geological map of Kalpatta area.

south-western contact of the granite pluton with the country rocks, Nair (1977) felt that the Kalpatta granite might have formed during the culmination of the Eastern Ghat orogeny at 1600 m.y. Being a grey biotite granite in contrast to the other pink granites of the Kerala region, the Kalpatta granite probably bears a different genetic history. Nevertheless, we prefer to assume a late Precambrian-early Palaeozoic correlation for the granite.

The present study is aimed as a preliminary work in order to understand the genetic aspects of the granite based on its petrochemistry.

2. Petrography

The principal mineral constituents of the granite are K-feldspar, plagioclase and quartz, set in a hypidiomorphic granular texture. The dominant K-feldspar is microcline (3–6 mm) which is occasionally perthitic. The microcline exhibits varying intensities of cross-hatched twinning which probably corresponds to different degree of triclinicities (Marmo 1971). Subhedral grains of plagioclase (3–5 mm) show multilamellar twinning and range in composition from $Ab_{95}An_5$ to $Ab_{85}An_{15}$. It is fresh in

the major mafic component. Hornblende occurs rarely and is seen at places breaking down to a granular aggregate with associated iron-oxides. Sphene and apatite are ubiquitously present as accessories. Minor amounts of secondary epidote and calcite are also observed.

The modal compositions (table 1) show that plagioclase (av. 30.91) is slightly in excess to K-feldspar (av. 28.85). Moderately high amount of accessories (av. 13.64) is also characteristic.

The mafic enclaves within the granite are inhomogeneous having an abundance of hornblende with subordinate biotite, epidote, sphene, feldspars, little quartz and iron oxide. Appreciable amount of apatite is also noted. Locally, the hornblende is partially altered to biotite, which in turn shows conversion to chlorite and granular sphene.

3. Geochemistry

Major element composition of six representative samples from the granite and one from the enclave are presented in table 2 and the normative compositions are given in table 3. Major elements were analysed by conventional wet methods and trace elements by atomic absorption spectrophotometer (Perkin-Elmer 4000). Na and K were analysed by flame photometer.

Among the major elements, SiO_2 and Al_2O_3 values indicate a granitic to granodioritic nature. Na_2O generally exceeds K_2O which is characteristic of this granite in contrast to the higher K_2O and consistently higher $\text{K}_2\text{O}/\text{Na}_2\text{O}$ ratios for the other magmatic granites of Kerala (*cf.* Santosh and Nair 1983). Based on K_2O versus Na_2O values (Harpum 1963), the rock type is chemically classified as granodiorite (figure 2).

Al_2O_3 levels of the granite are generally high, ranging from 15.3 to 16.83. Since the molecular proportion, $\text{Al}_2\text{O}_3/(\text{CaO} + \text{Na}_2\text{O} + \text{K}_2\text{O})$ is greater than unity, the granite could be considered as peraluminous (Clarke 1981) but not essentially an S-type (White and Chappell 1977) since characteristic aluminous minerals like garnet or aluminium silicates are lacking. FeO/MgO , $\text{Na}_2\text{O}/\text{CaO}$ and $\text{K}_2\text{O}/\text{CaO}$ ratios are generally high, typical of magmatic granites. $\text{Fe}_2\text{O}_3/\text{CaO}$ values are also high and might indicate considerable water vapour pressure during crystallization.

Harker variations of the major elements are shown in figure 3. The sharp depletion of Al_2O_3 and K_2O are consistent with crystallization of K-feldspar which, as seen from petrography, was a principal fractionating phase. Na_2O shows rather scattered plots.

Table 1. Modal composition of Kalpatta granite.

	KA-1	KA-2	KA-3	KA-4	KA-7	KA-9
Quartz	28.68	25.34	23.09	27.77	26.05	27.71
K-feldspar	28.28	29.52	31.16	29.66	28.47	26.03
Plagioclase	30.22	32.98	30.09	29.44	30.28	33.44
Biotite	5.32	5.68	6.97	5.39	6.03	4.61
Hornblende, Sphene, Apatite and others	7.50	6.48	8.69	7.74	9.17	8.21

Wt%	KA-1	KA-2	KA-3	KA-4	KA-7	KA-9	KA-10
SiO ₂	73.94	72.32	69.47	73.79	72.19	74.38	55.52
Al ₂ O ₃	15.30	15.30	16.83	15.81	15.81	15.30	17.34
TiO ₂	0.12	0.14	0.39	0.09	0.21	0.08	0.47
FeO	0.68	0.81	1.35	0.32	1.44	0.45	2.05
Fe ₂ O ₃	0.11	0.70	1.06	0.68	0.80	0.62	6.75
CaO	1.35	1.79	2.02	1.57	2.02	1.79	8.30
MgO	0.48	0.48	0.81	0.16	0.65	0.16	3.23
Na ₂ O	4.02	4.83	4.17	4.07	3.99	4.27	4.12
K ₂ O	2.93	3.49	3.63	3.05	2.88	2.54	0.67
MnO (ppm)	234	160	208	122	227	105	107
P ₂ O ₅ (ppm)	526	458	0.13%	32	549	18	0.26%
L.O.I.	0.18	0.22	0.25	0.11	0.18	0.11	0.49
Moisture	0.05	0.12	0.15	0.07	0.11	0.12	0.08

KA-1 to KA-9 Kalpatta Granite; KA-10—'Enclave' rock.

Table 3. Normative composition of Kalpatta granite

		KA-1	KA-2	KA-3	KA-4	KA-7	KA-9
Q		34.68	24.96	24.66	34.74	31.49	35.23
Or		17.24	20.57	21.68	17.79	17.24	15.01
Ab		35.91	42.44	36.64	35.63	35.11	37.20
An		6.67	8.92	9.17	7.78	10.01	8.90
Hy	En	0.79	1.20	2.00	0.40	1.60	0.40
	Fs	1.20	0.66	0.92	—	1.58	0.13
Mt		0.23	0.93	1.62	0.70	1.16	0.93
He		—	—	—	0.15	—	—
Il		0.32	0.32	0.80	0.16	0.48	0.16
Ap		—	—	0.16	—	—	—
C		2.86	—	2.35	2.75	2.14	2.04

The depletion in CaO, MgO, FeO⁺ and TiO₂ suggest early crystallization of mafic minerals which are ubiquitously present as accessory constituents in the granite.

The chemical composition of the mafic enclave rock shows low silica and high alumina values. CaO, Fe₂O₃ and MgO values are very high. These are in accordance with mineralogy.

Among the trace elements (table 4), the consistently lower levels of Ba and Sr imply that the magma underwent considerable degree of feldspar fractionation. Cr shows high values and are in contrast to those expected in a fractionated granite, even if the melt had equilibrated from a mafic source. Moreover, these values do not correlate with Ni, which shows lower levels. Hence it is assumed that Cr in the granite was contributed from the mafic supracrustals which occur as enclaves within the granite. The values of other trace elements are in general similar to those for the magmatic granites of Kerala.

Variation of trace elements are plotted against the index of differentiation, Mg/Mg+Fe⁺ (figure 4). Li, Ni, Co and Zn show depletion consistent with their

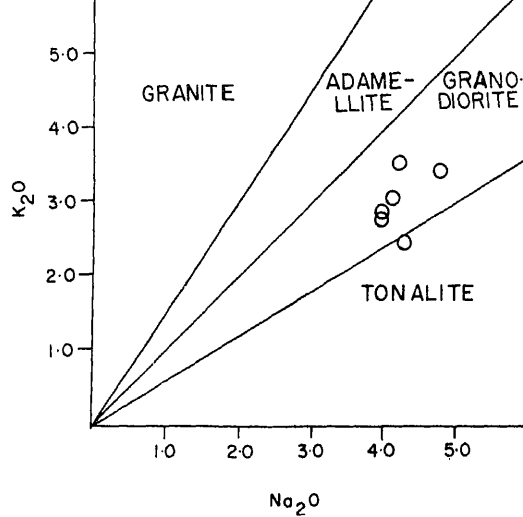


Figure 2. Na_2O vs K_2O plots of Kalpatta granite. Rock classification after Harpum (1963).

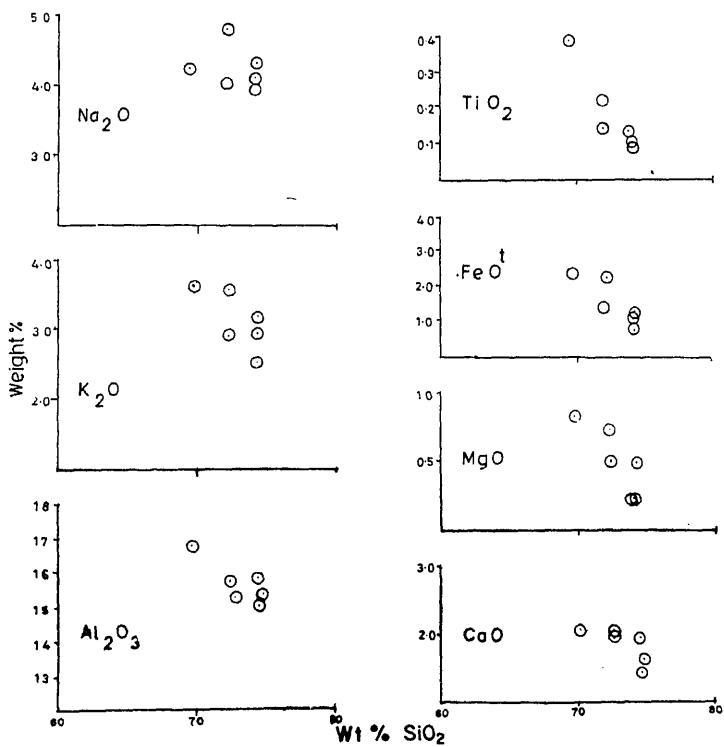


Figure 3. Major element values vs SiO_2 plots of Kalpatta granite.

Element (ppm)	KA-1	KA-2	KA-3	KA-4	KA-7	KA-9	KA-10
Li	7.6	3.5	11.7	1.6	8.6	0.9	3
Bi	n.f.	n.f.	22	n.f.	n.f.	n.f.	n.f.
Pb	11	8	12	5	9	21	9
Zn	33	24	41	15	37	12	12
Cu	3	2	7	n.f.	1	2	37
Ba	39	30	157	31	74	54	128
Sr	n.f.	1	3	n.f.	2	3	10
Sb	8	152	6	59	n.f.	36	8
V	n.f.	n.f.	10	n.f.	n.f.	n.f.	10
Ni	13	66	32	6	32	29	44
Co	n.f.	2.8	10.4	n.f.	8.2	9.7	20.1
Mo	20	7	18	6	27	23	11
Cr	193	146	169	117	171	173	63

KA-1 to KA-9—Kalpatta Granite; KA-10—'Enclave' rock.
n.f. — not found

entrance in early crystallizing mafic phases. Cr shows a bimodal distribution. Sb and Mo yield rather scattered plots. Sr, which substitutes Ca in plagioclase, shows depletion with progressive fractionation. Ba, which is generally partitioned in K-feldspar or biotite, does not exhibit the expected smooth depletion. The distribution of Ba seems to be obliterated due to the interference of K-bearing mafic phases of the enclaves which are carries of Ba also.

4. Discussion

The microcline-albite-quartz assemblage of the Kalpatta granite is comparable to subsolvus granites (Tuttle and Bowen 1958) of the late-kinematic type (Marmo 1971). Evidences of an initial hypersolvus crystallization history are offered by the rare occurrences of perthitic feldspar. However, the principal phase of crystallization which resulted in the present mineralogical constitution was under-depressed P-T conditions as reflected by the subsolvus mineral assemblage.

An overall estimate of the P-T conditions is achieved by the normative plots of the granite in a Q-Ab-Or plane (Tuttle and Bowen 1958). The plots lie close to the ternary minimum (figure 5) with a trend migrating from the orthoclase field towards higher quartz field as the pressure drops from 5 to 3 kb. Perthitic feldspars are observed to form under magmatic conditions and at temperatures above 700° C (Tuttle 1955). The nature of plots in the Q-Ab-Or plane clearly indicates similar initial crystallization temperatures with a subsequent drop which led to the separate formation of the two feldspar phases.

The mode of distribution of major and trace elements are consistent with fractionation involving alkali feldspar. The higher levels of Al_2O_3 and Na_2O and the behaviour of certain trace elements imply that there was significant supracrustal participation. It is reasonable to assume that during emplacement, minor quantities of the supracrustal rocks were incorporated in the granite magma. Partial hybridization is displayed by the enclaves even at the present level of exposure and might be more extensive at

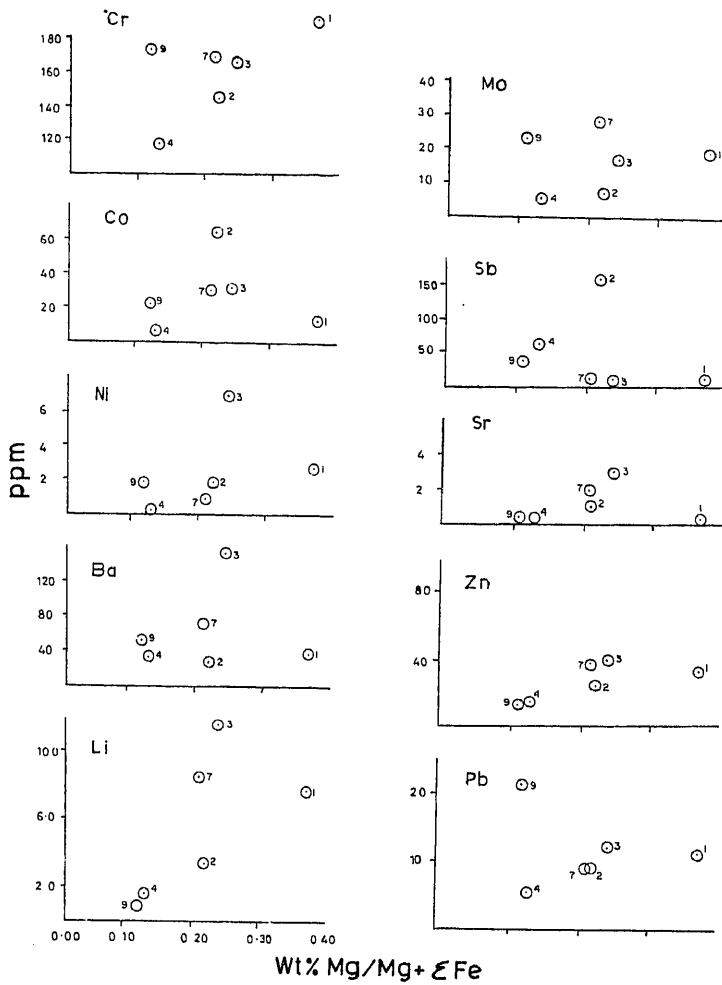


Figure 4. Trace element values vs $\text{wt}\% \text{Mg}/(\text{Mg} + \Sigma \text{Fe})$ plots of Kalpatta granite.

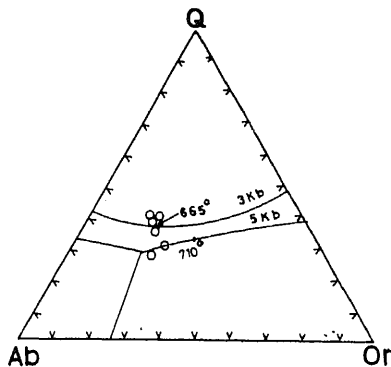


Figure 5. Polybaric diagram (after Tuttle and Bowen 1958) showing Q-Ab-Or plots of Kalpatta granite.

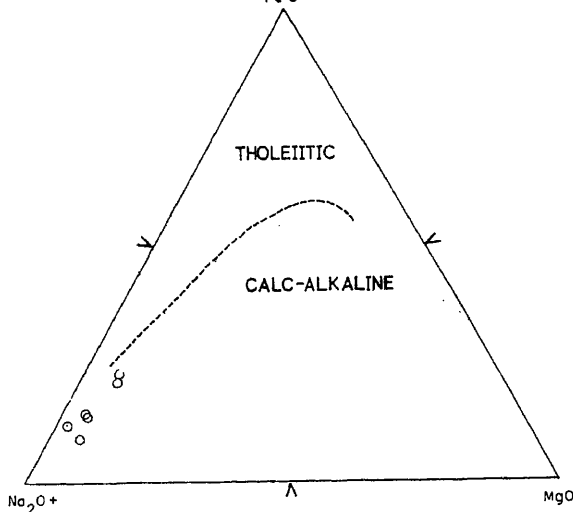


Figure 6. A-F-M plots of Kalpatta granite.

depth. The enclaves are elliptical or rounded in shape and are different from the micaceous 'restites' (Didier 1973) resulting from anatectic process. Higher levels of CaO, MgO, FeO' and Cr in the granite imply that there was significant reciprocal reaction between the granite and the enclaves resulting in the addition of these elements to the former. However, complete equilibrium between the mineral phases present in the enclaves and the granite is lacking. Nevertheless, the regional distribution of accessory minerals like apatite and sphene in considerable amounts in the granite (table 1) support the evidence of contamination. The apatite concentration in the enclaves is in accordance with the view of Nockolds (1935) that volatiles play a significant role in the mutual transfer of elements. Experimental deduction of Luth *et al* (1964) that vapour-phase transfer of alkalis could modify a metaluminous granitic liquid to peraluminous may be applied in the present case. Accordingly, the subsolvus nature is also in support of such mass transfer. Plots in an A-F-M diagram (figure 6) show a calc-alkaline trend, indicating a progressive decrease in the fo_2 regimes during the course of crystallization.

The elliptical shape of the granite body, its intrusive relationship with the country rocks, presence of enclaves and absence of significant replacement textures along with geochemical characters assign a magmatic parentage for the Kalpatta granite. Petrochemistry suggests that the granite was derived by feldspar and quartz fractionation from a parent magma, probably generated by partial melting at deeper crustal levels with subsequent supracrustal contamination.

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- Clarke D B 1981 *Can. Mineral.* **19** 3
- Didier I 1973 *Granites and their enclaves* (Amsterdam: Elsevier), p. 393
- Harpum J R 1963 *Rec. Geol. Surv. Tanganyika* **10** 80
- Luth W C, Jahns R H and Tuttle O F 1964 *J. Geophys. Res.* **69** 759
- Marmo V 1971 *Granite petrology and the granite problem* (Amsterdam: Elsevier), p. 244
- Nair M M and Vidyadharan K T 1982 *J. Geol. Soc. India* **23** 46
- Nair P K R 1976 *Curr. Sci.* **45** 379
- Nair P K R 1977 *Curr. Sci.* **46** 266
- Nockolds S R 1935 *Geol. Mag.* **72** 289
- Santosh M and Nair N G K 1983 *J. Geol. Soc. India* (in press)
- Soman K, Nair N G K, Golubyev V N and Arakelyan M M 1982 *J. Geol. Soc. India* **23** 458
- Tuttle O F 1955 *Colloq. Internat. Centre Natl. Rech. Sci.* (Nancy)
- Tuttle O F and Bowen N L 1958 *Geol. Soc. Am. Mem.* **74** 1
- White A J R and Chappell B W 1977 *Tectonophysics* **43** 7

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Temperature variation in the mesosphere during solar eclipse

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Abstract. Rocket-borne observations of the extinction of solar hydrogen Lyman-alpha radiation made during the solar eclipses of 20 May 1966 at Karistos for normal and 44% visibility and of 7 March 1970 at East Quoddy for normal, 10% and 0.6% visibility have been used to study the variation of temperature in the mesosphere during these events. It is seen that near the mesopause the decrease of temperature at Karistos is by 20° K for 44% visibility and at East Quoddy by 100° K for 0.6% visibility. Possible causes of these temperature variations *vis-a-vis* molecular oxygen and pressure variations have been briefly discussed.

Keywords. Temperature; mesosphere; Lyman-alpha; solar eclipse.

1. Introduction

The temperature of the mesosphere, particularly of the mesopause, is a sensitive parameter which undergoes variations with any change in the geophysical condition of the ionosphere. While in phenomena like winter anomaly and polar cap absorption an increase of temperature has been reported (Thrane *et al* 1979; Faire and Murphy 1972), it is not known with certainty whether during solar eclipse the temperature undergoes any change in this region. The solitary measurement made so far by Horvath and Theon (1972) during the solar eclipse of 7 March 1970 by Pitot tube probes could not detect any change in temperature from that of a normal day. However, to explain the existence of the higher percentage of positive water cluster ions observed by Narcisi *et al* (1972) and Kopp (1979) during the solar eclipses of 12 November 1966 and 26 February 1979, a decrease in temperature of the mesopause region is a must. In the absence of any reliable information it would be worthwhile to examine the available extinction profiles of solar Lyman-alpha radiation measured during various solar eclipses and see what picture of variation of temperature of mesosphere emerges.

2. Theory of temperature derivation

If n_i be the number density and m_i the mass number of the i th constituent in a layer of thickness dz , then the change in the intensity I of solar radiation as it traverses through the layer is given by

$$dI = -I \sum n_i \sigma_i \sec \chi dz$$

where σ_i is the absorption cross-section and χ is the solar zenith angle. Since O₂ molecules are the dominant absorbers of Lyman-alpha,

$$dI = I \cdot n(O_2) \cdot (\sigma + f) \cdot \sec \chi \cdot dz, \quad (1)$$

where f is a small correction to account for other absorbers and its value is of the order of $5 \times 10^{-26} \text{ m}^2$ (Hall 1972). If dp be the pressure difference across the layer then

$$dp = -g \sum_i n_i m_i dz \quad (2)$$

Combining (1) and (2) one obtains

$$dI/I = -(\sigma + f) dp / K \cdot \cos \chi$$

$$\text{where } K = g \sum_i n_i m_i / n(O_2),$$

K is constant for practical purposes at all heights. For monochromatic radiation

$$I = I_\infty \exp \left(- \frac{(\sigma + f) \cdot p}{K \cdot \cos \chi} \right)$$

From (1) one can write

$$n(O_2) = \frac{\cos \chi}{\sigma} \cdot \frac{1}{I} \cdot \frac{dI}{dz} \quad (3)$$

The parameters on the right side are all known and hence one can estimate $n(O_2)$ from (3). From (2) one gets

$$dp/dz = -K \cdot n(O_2). \quad (4)$$

Substituting for $n(O_2)$ from (3) one gets

$$dI/I = -\sigma dp / K \cdot \cos \chi$$

$$\text{or } I = I_\infty \exp \left(- \int \frac{\sigma dp}{K \cos \chi} \right) \quad (5)$$

Hall (1972) has plotted I/I_∞ vs $p/(K \cdot 10^{23} \cdot \cos \chi) \text{ m}^{-2}$. Once p is known, one can calculate the scale height in terms of p and $n(O_2)$ which is given by

$$\frac{1}{H} = -\frac{1}{p} \frac{dp}{dz} \quad (6)$$

Using (4) we get

$$H = p / K \cdot n(O_2) \quad (7)$$

The scale height H is also related to the temperature by the relation

$$H = kT/mg \quad (8)$$

and hence T can be estimated.

3. Data

Rocket observations of the extinction of solar hydrogen Lyman-alpha radiation have been used by several workers to derive the temperature of the mesosphere (Thrane *et al* 1979; Thrane and Johannessen 1975; Hall 1972) under various geophysical conditions. Hall (1972) studied this method in detail and demonstrated that, if the absorption cross-section of Lyman-alpha in molecular oxygen is known, then this technique can give the values of pressure, density and temperature of the mesosphere with good accuracy. While many workers have used a constant value of absorption cross-section, Hall (1972) points out that it is essential to take into account the variation of this parameter with wavelength across the 0.1 nm wide solar line. Thrane and Johannessen (1975) find that the laboratory measurements of the absorption cross-sections of Lyman-alpha in molecular oxygen do not apply to conditions in the upper atmosphere. The scope of this work is not to examine the correctness of this absorption cross-section value but to use the same value of absorption cross-section to the Lyman-alpha extinction profiles measured during and out of eclipse conditions (Hall 1970; Dickinson 1972) in two solar eclipse events and estimate the temperatures for both the conditions. One of the events occurred on 20 May 1966 over Karistos, Greece (38° N, 24° W) where Lyman-alpha extinctions were measured during normal and 44% visibility conditions. The other eclipse event took place in East Quoddy, Canada (45° N, 62° W) where four rockets were launched during normal, 0.15%, 0.6% and 10% visibility condition. The last two rockets measured after the eclipse totality *i.e.* in the regaining phase of the ionisation. There was considerable error in the 0.15% eclipse totality measurements (Dickinson 1972) and hence we have not used this measurement in our present work. Using these profiles, the temperature values of the mesosphere have been derived by the method described by Hall (1972) and which has been described briefly above. It takes into account the variation of absorption cross-section across the Lyman-alpha line. Calculations have been done at an interval of 1 km.

4. Results

In figure 1, the temperature values during normal and different visibility conditions have been plotted for both the solar eclipse events. It shows that at Karistos the decrease is maximum by about 20° K in the region 80–83 km. At East Quoddy the maximum decrease by 100° K is around 80–83 km when the visibility decreased from full sun to 0.6%. An interesting point is the near similarity of the shapes of temperature profiles of flight 4 which is in the regaining phase of ionization and flight 1 which is in the full sun condition. Also, of the three temperature profiles obtained for the East Quoddy eclipse, the temperature profile of flight 3 shows a minimum around 80–82 km while the other two temperature profiles for flights 1 and 4 show a bulge around this height. A temperature bulge around the mesopause level for normal conditions was reported earlier (Chakrabarty and Chakrabarty 1979) for Wallops Island where the temperature was estimated from rocket measurements of $[\text{NO}^+]/[\text{NO}^+\text{H}_2\text{O}]$ ratio.

The accuracy of the temperature estimations depend critically on the reliability with which the Lyman-alpha extinction profiles are measured during these events. The sensitivity of the instruments is a parameter which may not remain constant during and out of eclipse conditions. The principal source of error in the flux determination is contamination of the chamber window. Dickinson (1972), however, puts the overall

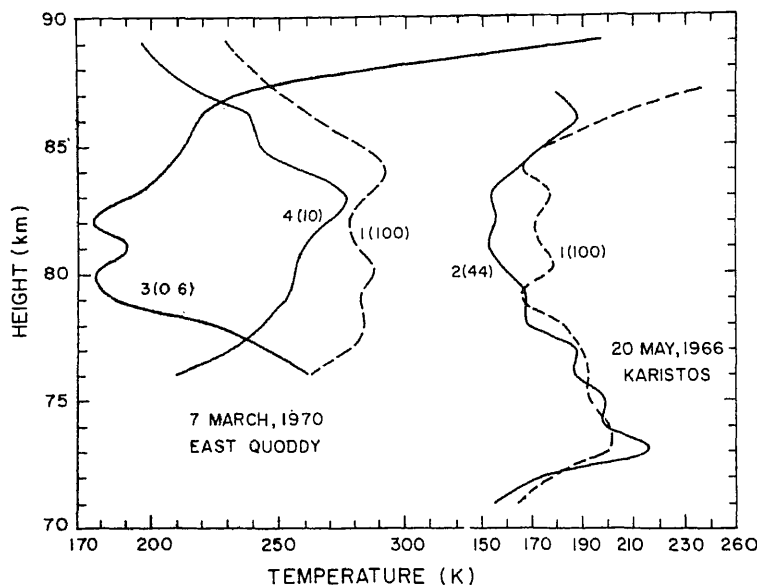


Figure 1. Temperature profiles estimated from Lyman-alpha extinction measurements during the solar eclipses of 20 May 1966 and 7 March 1970 at Kaistos, Greece and East Quoddy, Canada. Numbers inside the parentheses denote visibility in % and the numbers outside denote the sequence of rocket flights.

uncertainty in the absolute calibration of East Quoddy measurements to about $\pm 20\%$. In the present work, since we have considered the ratio dI/I , the overall error in estimated temperature would be less than $\pm 20\%$. It may be mentioned here that the two measurements of Lyman-alpha flux made by OSO-5 satellite at an interval of about 3 hr are available for 7 March 1970 (Vidal-Madjer 1975). These values are $5.7 \text{ ergs cm}^{-2} \text{ sec}^{-1}$ in agreement with the normal time value of $5.3 \text{ ergs cm}^{-2} \text{ sec}^{-1}$ measured by Dickinson (1972) at East Quoddy.

5. Discussion

It is known that in the D -region during solar eclipse the degree of hydration of proton hydrates increases (Kopp 1979; Narcisi *et al* 1972). An explanation of this feature demands a decrease in temperature during solar eclipse. The temperature estimates in figure 1 have been computed from the scale height of molecular oxygen. The scale height is related to the density of molecular oxygen and pressure by equation (7). Hence any variation of scale height could be due to changes in the pressure and molecular oxygen density. The molecular oxygen density has been estimated by Hall (1970) (with constant σ) for the 20 May 1966 eclipse at Karistos. He concluded that the oxygen densities were slightly lower on the eclipse day but the difference is barely significant. We have repeated these calculations of molecular oxygen with σ values from Hall (1972) (in which σ as a function of wavelength has been assumed) for the eclipse events and no significant changes are seen in the molecular oxygen density for the eclipse event at Karistos. For the eclipse event of East Quoddy, however, we see an increase of molecular oxygen density. These are shown in figure 2. The other parameter on which the scale height is dependent is the pressure which also changes during eclipse. The question which arises, therefore, is how does eclipse phenomenon cause a

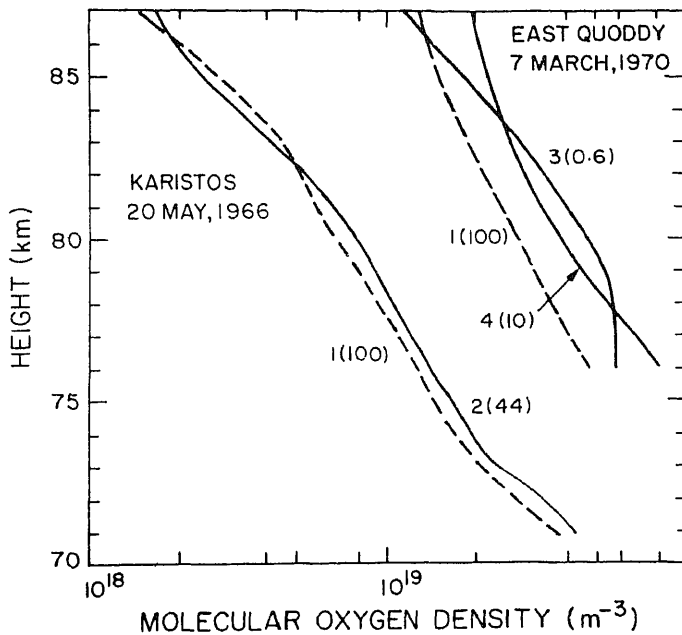


Figure 2. Molecular oxygen density profiles estimated from Lyman-alpha extinction measurements during the solar eclipses of 20 May 1966 and 7 March 1970 at Karistos, Greece and East Quoddy, Canada.

variation in pressure and molecular oxygen density? The two plausible reasons are dynamical and solar radiation effects. It is likely that the supersonic motion of the moon's shadow triggers wave motion causing compression and rarefaction in the whole length of the atmosphere. Gravity waves generated by the shadow of moon as it moves supersonically across the earth's atmosphere had been predicted long ago by Chimonas and Hines (1970). Total electron content observations at four stations in western United States during the solar eclipse of 7 March 1970 detected travelling disturbances which possibly confirmed the suggestion of Chimonas and Hines 1971; Davis and da Rosa 1970. However, TIDs observed in a number of ionospheric experiments in United States during the same period were not conclusively attributed to the eclipse effect (Carlson *et al* 1970; Larford *et al* 1972; Arendt 1972; Sears 1972). From observations of Faraday rotation and group delay measurements at 40 MHz using radio beacons from ATS-6 at Trivandrum, Vaidyanathan *et al* (1978) had reported quasi-periodic fluctuations of about 10 min periodicities during the partial eclipse of 29 April 1976. However, Sethia *et al* (1980) using the ATS-6 satellite data for a number of other locations for the same eclipse reported absence of travelling ionospheric disturbances at all these stations. Thus we see that the presence/absence of eclipse-induced gravity waves is still highly controversial

An alternate reason of the change in the pressure and molecular oxygen density could be the absorption or generation of heat at different regions of the atmosphere when Sun's disc starts getting obscured by the moon. A theoretical calculation has shown that while at stratopause a cooling is produced (by about 10° K), at the tropopause region an increase of temperature is noticed (Davies 1981) during a solar eclipse. A detailed theoretical computation of temperature could show whether the mesopause will also be cooled along with stratopause during solar eclipse.

6. Conclusion

Mesopause temperatures are derived from the rocket-borne observations of Lyman-alpha extinction profiles made during the solar eclipses of 20 May 1966 at Karistos and of 7 March 1970 at East Quoddy. At both places a decreasing trend is noticed. The accuracy of these estimations depends critically on the reliability with which the Lyman-alpha extinction profiles are measured during these events. The temperature variations may be due to either some sort of waves generated by the supersonic motion of the shadow of the moon across the earth's atmosphere or absorption generation of heat at various layers of the earth's atmosphere as the solar disc is rapidly covered by the moon.

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References

- Arendt P R 1972 *J. Atmos. Terr. Phys.* **34** 719
Carlson H C, Herper R, Wickard V, Showen R L, Behnke R, Trost T F, Cogger L R and Nelson C R 1970 *Nature (London)* **226** 1124
Chakrabarty P and Chakrabarty D K 1979 *J. Geophys. Res.* **84** 3403
Chimonas G and Hines C O 1970 *J. Geophys. Res.* **75** 875
Chimonas G and Hines C O 1971 *J. Geophys. Res.* **76** 7003
Davies K 1981 Paper presented at the Int. Solar Eclipse Symp., New Delhi, January
Davis M J and da Rosa A V 1970 *Nature (London)* **226** 1123
Dickinson P H G 1972 *J. Atmos. Terr. Phys.* **34** 621
Faire A C and Murphy E A 1972 Solar particle events of November 1969, Proc. COSPAR Symp., AFCRL-72-0474, Special Report No. 144, p. 445
Hall J E 1970 *J. Atmos. Terr. Phys.* **32** 1849
Hall J E 1972 *J. Atmos. Terr. Phys.* **34** 1337
Horvath J J and Theon J S 1972 *J. Atmos. Terr. Phys.* **34** 593
Kopp E 1979 Private Communication
Larford G M, Jurgens R M, Vesscky J F and Warhbann T W 1972 *J. Atmos. Terr. Phys.* **34** 733
Narcisi R S, Bailey A D, Wlodyka L E and Philbrick C R 1972 *J. Atmos. Terr. Phys.* **34** 647
Sears R D 1972 *J. Atmos. Terr. Phys.* **34** 727
Sethia G, Chandra H and Deshpande M R 1980 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **89** 153
Thrane E V and Johannessen A 1975 *J. Atmos. Terr. Phys.* **37** 655
Thrane E V, Grandal B, Hogen O, Uglet Veit F, Bangert W, Beran D, Friedrich M, Loidl A, Schwentek H and Torkar K M 1979 *J. Atmos. Terr. Phys.* **41** 1097
Vaidyanathan S, Raghava Reddi C and Krishna Murthy B V 1978 *Nature (London)* **271** 41
Vidal-Madjer A 1975 *Solar Phys.* **40** 69

Application of a semi-implicit scheme of integration to barotropic prediction

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Abstract. Application of a semi-implicit version of primitive equation barotropic model to predict monsoon depressions is proposed. Forecasts upto 48 hr have been quite satisfactory. The results are compared with those obtained from an explicit version of primitive equation barotropic model developed earlier. The flow patterns from these two versions are similar with few exceptions in isolated pockets, however, the forecast movement obtained from the semi-implicit version is comparable or slightly superior to those obtained from the explicit version. The computational time in semi-implicit and explicit schemes is approximately in the ratio of 1:2.

Keywords. Semi-implicit scheme; barotropic model; monsoon depression.

1. Introduction

Systematic studies on the development of numerical weather prediction models in India were taken up nearly a decade back. Shukla and Saha (1970) used a non-divergent barotropic model with the wind at 500 mb as input to predict the movement of a tropical cyclone in the Bay of Bengal. The explicit version (Singh and Saha 1976) of the primitive equation barotropic model were applied for the prediction of monsoon depressions and tropical cyclones in the Indian region and was applied extensively to study the impact of additional data collected during MONEX-79, on the barotropic prediction of monsoon depressions by Sikka *et al* (1980, 1981).

In the present study, a semi-implicit scheme of integration to a primitive equation barotropic model has been applied to predict monsoon depressions. This scheme is similar to those by Kwizak and Robert (1971), McPherson (1971) and Halberstam (1974) in the integration of finite difference primitive equation barotropic model and the terms relevant to meteorological waves were approximated explicitly and gravitational oscillations evaluated implicitly. Thus, the restriction in time step may depend on the velocity of meteorological waves and horizontal grid resolutions, and a large time step (say 30–40 min) could be used for a grid length of 100–200 km, whereas in the explicit versions a maximum time step of 10 min could be used for the same grid length. The semi-implicit scheme of integration is now being used with increasing acceptance to integrate multilevel primitive equation models (BurrIDGE 1975; Gauntlett *et al* 1978; McGregor *et al* 1978) due to considerable saving of computational time. In the study of simulation of monsoon circulation and prediction of movement of a monsoon depression, Das and Bedi (1978a, b; 1979) have applied a semi-implicit scheme of integration in a multilevel primitive equation model. The purpose of the present experiment is to extend this scheme to the multilevel primitive equation model currently being developed by our group.

2. Data

For both the versions of the model, five typical synoptic situations (4, 5 and 6 August 1968 and 5 and 6 July 1979) dominated by monsoon depression have been selected. The free surface is assumed at 700 mb and as such the mean height of free surface is taken 2000 gpm. The stream-line isotach for all synoptic situations at 700 mb has been analysed manually and the wind speed and direction were picked up at 2 degree latitude-longitude intersections. A time-step of 30 min for the first version and 5 min for the second has been chosen.

3. Method of computations

3.1 Model equations

The prediction equation in Cartesian coordinate system on mercator projection are:

$$\frac{\partial u}{\partial t} + mu \frac{\partial u}{\partial x} + mv \frac{\partial u}{\partial y} - f'v + gm \frac{\partial h}{\partial x} = 0, \quad (1)$$

$$\frac{\partial v}{\partial t} + mu \frac{\partial v}{\partial x} + mv \frac{\partial v}{\partial y} + f'u + gm \frac{\partial h}{\partial y} = 0, \quad (2)$$

$$\frac{\partial h}{\partial t} + mu \frac{\partial h}{\partial x} + mv \frac{\partial h}{\partial y} + hm \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - hv \frac{\partial m}{\partial y} = 0, \quad (3)$$

where u and v are the component of the horizontal velocity vector (u -along the x -coordinate pointing eastward and v -along the y -coordinate pointing northward), h the height of free surface ($h = 0$ is the mean sea level), t the time, g the acceleration due to gravity, m the map factor (secant of latitudes) and f the Coriolis parameter.

3.2 Semi-implicit scheme

Equations (1) to (3) can be written using Leapfrog time differencing, and averaging in time the approximate linear terms:

$$\bar{u}^{2t} = u^{t-\Delta t} - \Delta t \left(mu \frac{\partial u}{\partial x} + mv \frac{\partial u}{\partial y} - f'v \right) + f_0 \bar{v}^{2t} \Delta t - gm \frac{\partial \bar{h}^{2t}}{\partial x} \Delta t, \quad (4)$$

$$\bar{v}^{2t} = v^{t-\Delta t} - \Delta t \left(mu \frac{\partial v}{\partial x} + mv \frac{\partial v}{\partial y} + f'u \right) - f_0 \bar{u}^{2t} \Delta t - gm \frac{\partial \bar{h}^{2t}}{\partial y} \Delta t \quad (5)$$

$$\bar{h}^{2t} = h^{t-\Delta t} - \Delta t \left(mu \frac{\partial h}{\partial x} + mv \frac{\partial h}{\partial y} \right) - mh\Delta t \left(\frac{\partial \bar{u}^{2t}}{\partial x} + \frac{\partial \bar{v}^{2t}}{\partial y} \right) + hv\Delta t \frac{\partial m}{\partial y} \quad (6)$$

where $f = f_0 + f'$, f_0 being the value of the Coriolis parameter at the centre of the domain (22° N) and the time-average operator is defined as

$$F^{2t} = 1/2 [F(t+\Delta t) + F(t-\Delta t)]$$

Eliminating $\bar{v}^{2t}$ from (4) and $\bar{u}^{2t}$ from (5), explicit equation for $\bar{u}^{2t}$ and $\bar{v}^{2t}$ in terms of h^{2t} can be written as

$$\lambda \bar{u}^{2t} = A - gm\Delta t \left(\frac{\partial \bar{h}^{2t}}{\partial x} + f_0 \Delta t \frac{\partial \bar{h}^{2t}}{\partial y} \right) \quad (7)$$

$$\lambda \bar{v}^{2t} = B - gm\Delta t \left(\frac{\partial \bar{h}^{2t}}{\partial y} - f_0 \Delta t \frac{\partial \bar{h}^{2t}}{\partial x} \right) \quad (8)$$

where $\lambda = 1 + f_0^2 \Delta t^2$ and A and B are given as follows:

$$A = u^{t-\Delta t} + f_0 \Delta t v^{t-\Delta t} - \Delta t \left[\left(mu \frac{\partial u}{\partial x} + mv \frac{\partial u}{\partial y} - f'v \right) + f_0 \left(mu \frac{\partial v}{\partial x} + mv \frac{\partial v}{\partial y} + f'u \right) \right]$$

$$B = v^{t-\Delta t} - f_0 \Delta t u^{t-\Delta t} + \Delta t \left[f_0 \Delta t \left(mu \frac{\partial u}{\partial x} + mv \frac{\partial u}{\partial y} - f'v \right) - \left(mu \frac{\partial v}{\partial x} + mv \frac{\partial v}{\partial y} + f'u \right) \right]$$

Finally differentiating (7) with respect to x and (8) with respect to y (neglecting the latitudinal variation of m) and substituting the velocity divergence into (6) the continuity equation yields:

$$\nabla^2 h^{2t} - \lambda \bar{h}^{2t} (\Delta t^2 gh)^{-1} = \lambda Q^N (\Delta t^2 gh)^{-1}, \quad (9)$$

where

$$Q^N = \frac{h \Delta t}{\lambda} \left(m \frac{\partial A}{\partial x} + m \frac{\partial B}{\partial y} \right) + \Delta t \left(mu \frac{\partial h}{\partial x} + mv \frac{\partial h}{\partial y} \right) - \Delta t h v \frac{\partial m}{\partial y} - h^{t-\Delta t}$$

Equation (9) can be solved for $\bar{h}^{2t}$ numerically by the method of relaxation with appropriate boundary condition, whilst $\bar{u}^{2t}$ and $\bar{v}^{2t}$ may be obtained from (7) and (8) respectively. Finally the values of all primary variables at $t + \Delta t$ are obtained by using relations:

$$\begin{aligned} u^{t+\Delta t} &= 2\bar{u}^{2t} - u^{t-\Delta t}, \\ v^{t+\Delta t} &= 2\bar{v}^{2t} - v^{t-\Delta t}, \\ h^{t+\Delta t} &= 2\bar{h}^{2t} - h^{t-\Delta t}. \end{aligned}$$

3.3 Difference scheme, initialisation and boundary conditions

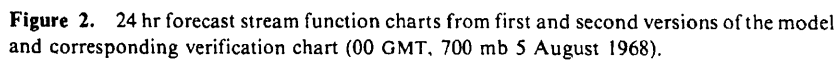
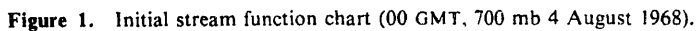
The space difference scheme is adapted from McPherson (1971). The difference equations are formulated on an offset grid lattice; in other words the geopotential height field is defined on all grid points and u and v components are defined at the centre of the squares.

For obtaining the initial balance between mass and wind fields, the nonlinear balance equation is solved for geopotential height with appropriate boundary conditions using non-divergent wind as basic input.

Cyclic continuity in the east-west and no flow across the north-south boundaries are specified as the lateral boundary conditions. A detailed description of initialization and boundary conditions has been given by Singh *et al* (1980).

4. Comparison of forecast results

Since the results of the initialization and forecast experiments are similar for all the synoptic situations, the figures for 4 August only will be presented and wherever necessary results of other synoptic situations will be discussed. Figure 1 shows the initial stream function chart (00 GMT 700 mb 4 August 1968) and figure 2 presents the 24 hr forecast from both versions together with the verification chart (00 GMT 700 mb 5 August 1968). The 48 hr forecast and corresponding verification (6 August 1968) charts are shown in figure 3. Tables 1 and 2 present the positional error of the forecast from Schemes I and II respectively for the input of 4, 5 and 6 August 1968 and 5 and 6 July 1979. The verification statistics computed separately for u - and v -fields are also presented in tables 1 and 2. It gives the root mean square errors and correlation coefficients between actual and forecast for the whole forecast domain. It may be seen that on average (of five cases) the forecast movement appears to be slightly better in the semi-implicit version than those obtained from the explicit version. The verification statistics also reflects the above conclusion *i.e.*, the prediction of u - and v -fields by semi-implicit version is slightly better or comparable than those obtained



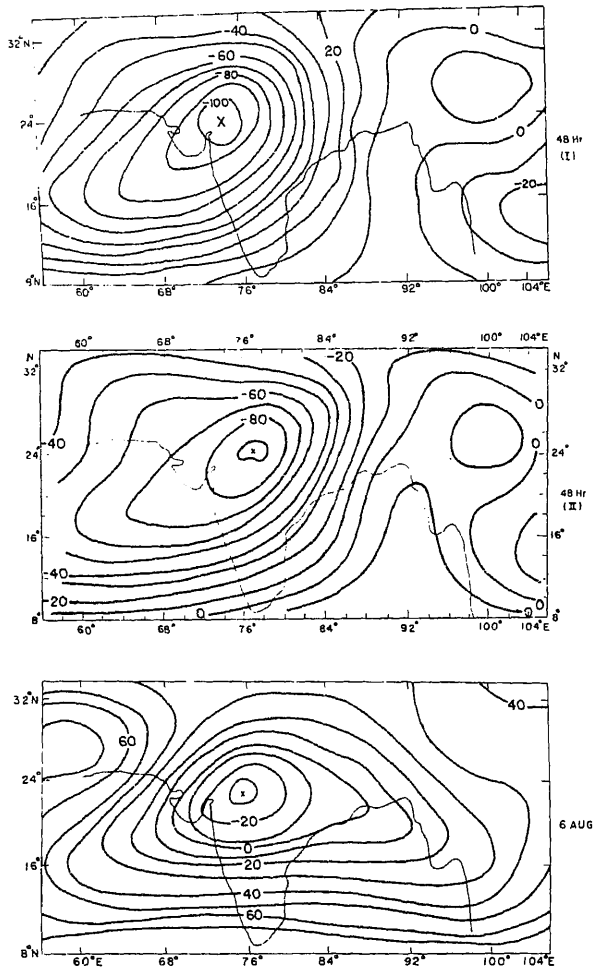


Figure 3. Same as figure 2 except for 48 hr forecast charts and corresponding verification chart (00 GMT, 700 mb 6 August 1968).

from the explicit version. The computational time required by two schemes is in the ratio of 1:2. As expected, the semi-implicit scheme reduces the computational requirement considerably.

It may be pointed out that both the versions not only differ in time integration schemes but there are a number of other differences in the formulations. The semi-implicit model is formulated on offset grid rather than the regular grid as is the case with the second version and as such the space derivatives are approximated differently in two models. Thus, the difference in the forecast is not due to different integration schemes alone but due to other factors as well.

Table 1. Forecast position vector error of the centre of the monsoon depressions. RMS errors and the correlation coefficient between the observed and forecast fields (semi-implicit)

Input	24 hour				48 hour					
	Position vector error (km)	RMS-error (m sec ⁻¹)		Correlation coefficient		Position vector error (km)	RMS-error (m sec ⁻¹)		Correlation coefficient	
		<i>u</i> -field	<i>v</i> -field	<i>u</i> -field	<i>v</i> -field		<i>u</i> -field	<i>v</i> -field	<i>u</i> -field	<i>v</i> -field
4.8.1968 (00 GMT)	0	4.6	3.0	0.8	0.6	5.7	3.6	0.6	0.4	
5.8.1968 (00 GMT)	220	4.1	3.4	0.8	0.4	5.4	3.8	0.6	0.3	
6.8.1968 (00 GMT)	330	4.8	3.4	0.6	0.4	5.7	4.0	0.4	0.5	
5.7.1979 (12 GMT)	254	3.5	3.6	0.8	0.4	3.7	4.6	0.8	0.0	
6.7.1979 (12 GMT)	254	3.6	3.9	0.8	0.4	4.3	4.4	0.8	0.3	
Average	212	4.1	3.5	0.8	0.4	5.0	4.1	0.6	0.3	

Table 2. Forecast position vector error of the centre of the monsoon depression, RMS errors and the correlation coefficient between the observed and forecast fields (explicit).

Input	24 hour				48 hour			
	Position vector error (km)	RMS-error (m sec ⁻¹)		Correlation coefficient	Positive vector error (km)	RMS-error (m sec ⁻¹)		Correlation coefficient
		u-field	v-field			u-field	v-field	
4.8.1968 (00 GMT)	55	4.6	3.0	0.8	0.6	6.1	4.2	0.5
5.8.1968 (00 GMT)	310	4.4	3.8	0.8	0.3	5.6	3.8	0.5
6.8.1968 (00 GMT)	250	4.7	3.3	0.6	0.4	5.6	4.1	0.4
5.7.1979 (12 GMT)	365	3.5	4.1	0.8	0.3	4.0	5.0	0.7
6.7.1979 (12 GMT)	343	3.9	4.3	0.8	0.3	4.1	4.8	0.8
Average	265	4.2	3.7	0.8	0.4	5.1	4.4	0.6

5. Conclusions

A semi-implicit scheme of integration has been successfully applied to predict monsoon depressions. The results are comparable or slightly superior to those obtained from the explicit version. The computational time as expected is least in this version as compared to the explicit version. A successful application of the semi-implicit analogue in the present study has encouraged the authors to apply this scheme to the multilevel primitive equation model currently under development.

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References

- Burridge D M 1975 *Q. J. R. Meteorol. Soc.* **101** 777
Das P K and Bedi H S 1978a *Indian J. Meteorol. Geophys.* **29** 375
Das P K and Bedi H S 1978b *Monsoon dynamics* (Cambridge: University Press) p. 351
Das P K and Bedi H S 1979 *Proc. Indian Acad. Sci. (Engg. Sci.)* **2** 17
Gauntlett D J, Leslie L M and McGregor J L 1978 *Q. J. R. Meteorol. Soc.* **104** 103
Halberstam I 1974 *J. Atmos. Sci.* **31** 1964
Kwizak M and Robert A J 1971 *Mon. Weath. Rev.* **99** 32
McGregor J L, Leslie L M and Gauntlett D J 1978 *Mon. Weath. Rev.* **106** 427
McPherson R D 1971 *Mon. Weath. Rev.* **99** 242
Shukla J and Saha K R 1970 *J. Meteorol. Soc. Jpn.* **48** 405
Sikka D R, Singh S S and Rajamani S 1980 *Results of the summer MONEX Field Phase Research, Part A, FGGE Operations Report*, 9 151
Sikka D R, Sinha S, Singh S S, Kulkarni P L and Kulkarni A A 1981 *Early results of FGGE and monsoon experiments*, Proc. Int. Conf., Tallahassee, USA
Singh S S and Saha K R 1976 *J. Appl. Meteorol.* **15** 805
Singh S S, Kulkarni A A and Sikka D R 1980 *Mon. Weath. Rev.* **108** 1315

Stability of a stationary Rossby wave embedded in barotropic monsoon zonal flow

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Abstract. Barotropic stability of a stationary Rossby wave of wavelength 30° longitude superposed on the uniform monsoon zonal flow has been examined. The wave is unstable to perturbations and the growth rate depends on the meridional scale. These perturbations grow by drawing on the kinetic energy of the stationary Rossby wave.

Keywords. Barotropic instability; baroclinic instability; beta-plane; zonal flow; meridional wind; Rossby wave.

1. Introduction

The barotropic instability of a zonal flow with superposed Rossby wave was investigated by Lorenz (1972). Dash and Keshavamurty (1982) have shown that the realistic winds in the monsoon atmosphere can sustain a finite amplitude baroclinic stationary Rossby wave of wavelength about 30° longitude and the amplitude of the Rossby wave is mainly confined to the lower troposphere. Such a stationary wave is observed over the Bay of Bengal and it may be induced by the topography of peninsular India (Gadgil 1977). It was found that such a stationary Rossby wave is unstable to perturbations (Dash and Keshavamurty 1982).

In the present study, we intend to perform the stability analysis by superposing a stationary Rossby wave of wavelength 30° longitude on the monsoon zonal flow having no vertical wind shear. The zonal flow corresponding to 18° N latitude is considered as the basic flow. The rate of conversion of kinetic energy is also calculated.

2. Barotropic zonal flow sustaining the stationary Rossby wave

First we calculate the barotropic zonal wind which can sustain a stationary Rossby wave of wavelength 30° longitude along the zonal direction. For that we consider the Rossby wave speed formula

$$c = U - \frac{\beta}{k_0^2}, \quad (1)$$

where k_0 is the zonal wavenumber of the Rossby wave.

Considering beta-plane centred at 18° latitude and substituting $k_0 = 1.84 \times 10^{-6} \text{ m}^{-1}$, which corresponds to the stationary wave of wavelength 30° longitude, in (1) we calculate,

$$U_s \equiv \frac{\beta}{k_0^2} = 6.5 \text{ m sec}^{-1}.$$

Thus a zonal wind $U_s = 6.5 \text{ m sec}^{-1}$ can sustain a stationary Rossby wave of wavelength 30° longitude.

3. The stability analysis

The vorticity equation is linearised by expressing the field variables as

$$X = [\bar{X}] + \bar{X}^* + X' \quad (2)$$

where the transient eddy $X' = X - \bar{X}$ and the stationary eddy $\bar{X}^* = \bar{X} - [\bar{X}]$. Here the bar and the square brackets represent time average and zonal average respectively. Thus the perturbation vorticity equation can be written as

$$\left(\frac{\partial}{\partial t} + U_s \frac{\partial}{\partial x} + \nabla^* \frac{\partial}{\partial y} \right) \zeta' + u' \frac{\partial \bar{\zeta}^*}{\partial x} + \beta v' = 0. \quad (3)$$

The basic flow is taken as

$$\Phi = -f_0 U_s Y + f_0 A \sin k_0 x, \quad (4)$$

where the meridional wind component is given by,

$$\bar{v}^* = k_0 A \cos k_0 x = \bar{v} \cos k_0 x. \quad (5)$$

Here $\bar{v}$ is substituted for $k_0 A$.

We propose to do normal mode analysis and following Lorenz (1972) assume solutions of the type

$$\phi' = \sum_{n=-\infty}^{\infty} \phi_n \exp \{ i(nk_0 x + ly + \lambda t) \} \quad (6)$$

where k_0 and l are real and λ may be complex. Here l is the meridional wavenumber of the perturbation. Equation (6) possesses solutions, which amplify with time if the imaginary part of λ were negative. For side boundary conditions we impose cyclic continuity. The zonal wind U_s is taken to be independent of y . Substituting (4), (5) and (6) in (3) and collecting the coefficients of $\exp \{ i(nk_0 x + ly + \lambda t) \}$ we get the following infinite system of linear, homogeneous algebraic equations while n is varied from $-\infty$ to $+\infty$.

$$\begin{aligned} & -1/2 l \bar{v} (b_{n-1} - k_0^2) \phi_{n-1} + a_n (\beta - b_n U_s) \phi_n \\ & -1/2 l \bar{v} (b_{n+1} - k_0^2) \phi_{n+1} - \lambda b_n \phi_n = 0, \end{aligned} \quad (7)$$

where

$$a_n \equiv nk_0 \quad \text{and} \quad b_n \equiv (n^2 k_0^2 + l^2).$$

Generally, one is able to obtain a good approximation by truncating the series (6) at a finite number $n = N$. When N is large enough such that λ_N has converged, a good approximation to the 'true' solution is obtained. In our analysis, we have taken $N = 10$. The 21 homogeneous algebraic equations can be written in matrix form as

$$(\mathbf{P} \cdot \mathbf{Q}^{-1} - \lambda \mathbf{I}) (\mathbf{Q} \cdot \boldsymbol{\phi}) = 0, \quad (8)$$

where $\mathbf{P}$ and $\mathbf{Q}$ are real square matrices and $\mathbf{I}$ is unit matrix. The eigenvalues λ and the eigenfunctions $(\mathbf{Q} \cdot \boldsymbol{\phi})$ are in general complex. The frequency of a disturbance and its doubling time are obtained from the real and imaginary parts of λ respectively. Similarly the amplitudes of different Fourier components ϕ_n are obtained from the eigenfunctions.

In another study (Dash and Keshavamurty 1982), it is found that for Rossby wave amplitude $\bar{v} = 10 \text{ m sec}^{-1}$ at the lower troposphere the growth rate of disturbances reasonably agree with those of monsoon disturbances. Hence, we have conducted the present stability analysis for $\bar{v} = 10 \text{ m sec}^{-1}$ only. The meridional wavenumber l is varied from $k_0/12$ to k_0 , satisfying the cyclic boundary conditions. This is done by putting $l = J k_0/12$ and varying J from 1 to 12 by steps of 1.

The stability analysis yields that the stationary Rossby wave is unstable to perturbations. The growth rate depends on the meridional scale of the disturbance and the fastest growing mode has a doubling time of about 1.5 days (figure 1). Also, the growth rate is maximum for $l \leq k_0/2$ which agrees with the results of barotropic stability analysis of Lorenz (1972). The growing disturbances are found to be stationary. The Fourier coefficients of geopotential perturbation corresponding to the fastest growing mode are shown in figure 2. It is seen that the amplitudes are mainly confined to the first few low order harmonics. This justifies the truncation of the series (6) for $N = 10$.

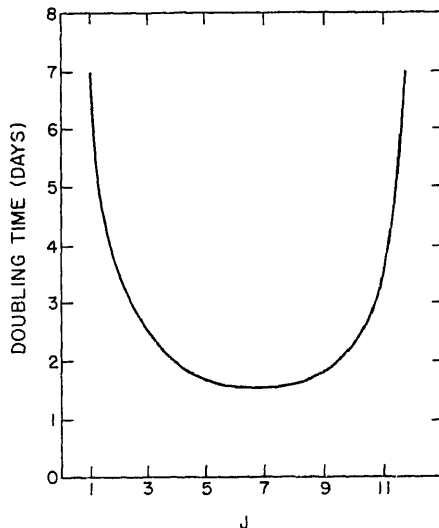


Figure 1. Meridional wavenumber ($l = Jk_0/12$) dependence of the growing modes.

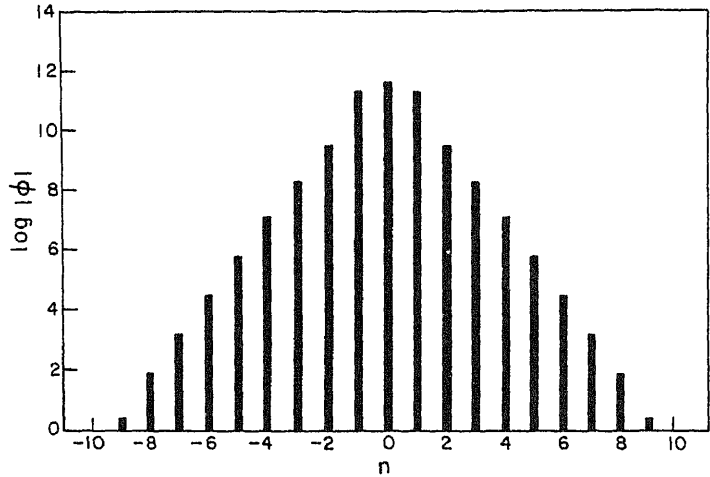


Figure 2. Fourier components of geopotential perturbation of the fastest growing mode in figure 1.

4. Energy conversion

As there is no vertical wind shear in the zonal flow and again the latter is independent of Y , the only source of energy for a perturbation to grow is the kinetic energy of the basic Rossby wave. The rate of conversion from kinetic energy of the basic wave to perturbation kinetic energy, $C(K_w, K')$ is calculated from

$$C(K_w, K') = - \frac{1}{g} \iiint u'v' \frac{\partial \bar{v}^*}{\partial x} dx dy dp, \quad (9)$$

where the integrations are over one wavelength in both x and y directions and from $p = 0$ to $p = p_0$. In terms of eigenfunctions, (9) can be expressed as

$$C(K_w, K') = i \frac{2\pi^2 k_0 p_0}{g f_u^2} \exp(-2\lambda_i t) \sum_{n=-\infty}^{\infty} [\bar{v} \{(n+1) \times (\phi_n \tilde{\phi}_{n+1} - \tilde{\phi}_n \phi_{n+1}) - (n-1) (\phi_n \tilde{\phi}_{n-1} - \tilde{\phi}_n \phi_{n-1})\}] \quad (10)$$

where ϕ is complex, $\tilde{\phi}$ is its complex conjugate and λ_i is the imaginary part of λ . In practice, the summation is carried out from $n = -N$ to N . Calculations show that the perturbation grows by drawing on the kinetic energy of the stationary Rossby wave.

5. Conclusions

In this paper we have carried out the barotropic stability analysis of a stationary Rossby wave superposed on the monsoon zonal flow having no vertical wind shear. It is found that the stationary Rossby wave of wavelength 30° longitude is unstable to

perturbations. The growth rate depends on the meridional scale. The fastest growing mode has a doubling time of 1.5 days for Rossby wave amplitude $\bar{v} = 10 \text{ m sec}^{-1}$. These perturbations grow by drawing on kinetic energy of the stationary Rossby wave.

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References

- Dash S K and Keshavamurty R N 1982 *Beitr. Phys. Atmosph.* **55** 311
Gadgil S 1977 *Pure Appl. Geophys.* **115** 1413
Lorenz E N 1972 *J. Atmos. Sci.* **29** 258

Recent foraminifers from the inner shelf of the central West Coast, India: A reappraisal using factor analysis

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Abstract. Setty and Nigam (1980) had described 72 species of benthonic foraminifers from 25 inner shelf stations off central West Coast of India and the results showed somewhat patchy and anomalous distributions. This paper presents the results of *Q*-mode factor analysis, which was applied to reduce the number of variables into assemblages. The analysis reveals 4 important foraminiferal assemblages, *Ammonia beccarii* — *Ammonia annectens* assemblage, *Nonion boueanum* — *Florilus scaphum* assemblage, *Trochammina inflata* assemblage and *Bulimina exilis* assemblage. They can be related to freshwater run-off and organic matter contents of the sediment.

Keywords. Recent foraminifera; factor analysis; inner shelf; central West Coast.

1. Introduction

Setty and Nigam (1980) had reported the benthonic foraminifers from the inner shelf area between Dabhol-Vengurla sector of the Arabian Sea and a total of 72 species were recorded from 25 stations. The results showed a somewhat patchy and anomalous distribution of benthonic foraminifers. It was, therefore, found desirable to reduce the number of variables by applying statistical techniques.

This paper presents the results of factor analysis of the foraminiferal data from the central West Coast of India. The other aspects of foraminiferal studies of this region like details of taxon, stationwise distribution of species and ecology, have already been reported (Nigam *et al* 1979; Setty *et al* 1979; Setty and Nigam 1980, 1982; Nigam and Sarupria 1981).

Among the various multivariate statistical techniques, factor analysis is a method which can reduce a great number of species to a few assemblages (Imbrie and Van Andel 1964). These assemblages can be plotted to study the geographical distribution and the resulting pattern can be compared to any environmental parameter or group of parameters about which information is available. Factor analyses are being applied extensively to different fields of micropalaeontology *e.g.* radiolarian studies (Sachs 1973a, b), diatoms (David and Sherwood 1981), Nannoplankton (Geitzenaur *et al* 1976) and foraminiferal studies (Howarth and Murray 1969; Imbrie and Kipp 1971; Streeter 1972; Thiede 1975). However, no significant attempts have been made to apply this technique to the voluminous collections of foraminiferal data from the Indian region.

The region between Dabhol (Lat. 17° 35' N, Long. 73° 06' E) in the north to Vengurla (Lat. 15° 50' N, Long. 73° 23' E) in the south is characterised by Shastri and Vashishti in the northern half and Rajapur and Vaghotan rivers in southern half (with several streams and streamlets in between). These rivers and streams are subjected to periodic tidal and seasonal changes, thus where these rivers discharge their load into the sea, dilution of seawater takes place and salinity cannot remain consistent.

The sea floor of the entire study area is covered by clayey substrate except sandy clay at St. No. 3, clayey sand at St. No. 15, silty clay at St. Nos. 16 and 22 and clayey silt at St. No. 27. Organic matter concentrations in terms of percentage in dry sediments ranged from 2.17 to 6.18 (table 1).

3. Materials and methods

The data which form the basis of the present study are in the form of percentage counts and have already been published (Setty and Nigam 1980). The samples for this study were collected during March 1977 as a part of the 17th cruise of *RV Gaveshani* within depth range of 18–51 m in the sector of the Indian continental shelf between 16 and

Table 1. Texture and organic carbon/matter concentration in sediments within study area.

Station Number	Texture	Organic carbon percent Dry weight	Organic matter* percent Dry weight
G 17/1	Clay	1.771	3.05
2	Clay	1.978	3.41
3	Clayey sand	2.162	3.73
5	Clay	1.978	3.41
6	Clay	1.95	3.37
7	Clay	1.84	2.17
8	Clay	2.599	4.48
9	Clay	3.128	5.39
10	Clay	2.99	5.15
12	Clay	2.346	4.04
13	Clay	2.3	3.96
14	Clay	2.806	4.84
15	Sandy clay	1.932	3.33
16	Silty clay	2.852	4.92
17	Clay	2.944	5.07
18	Clay	2.024	3.49
19	Clay	2.852	4.92
20	Clay	3.312	5.71
21	Clay	3.358	5.79
22	Silty clay	3.22	5.55
23	Clay	3.542	6.11
24	Clay	3.312	5.71
25	Clay	3.036	5.23
26	Clay	3.45	5.95
27	Clayey silt	3.588	6.18

* Organic matter = Organic carbon $\times$ 1.724.

18° N. The samples were collected with a Van Veen Grab (area 400 cm²) and La Fond Dietz snapper (area 104 cm²). This data subjected to a *Q*-mode factor analysis using Fortran IV program CABFAC (Klovan and Imbrie 1971) was analysed and print-out obtained from the computer centre of University of Oslo, Norway.

4. Results

The percentage was initially calculated for all the 72 species from 25 stations. Of these, 24 species occurring rarely within the study area were eliminated from the original matrix and the percentage of all such species was summed for each sample (total not exceeding 4.5% in any sample). These species were termed 'other species'. A final matrix of 49 variables (48 species + 'other species') was then subjected to *Q*-mode factor analysis. These 6 factors explain almost 93.49% of the information given by original data matrix. The variance and cumulative variance for these factors is given in table 2. Figure 1 summarises the geographical distribution of various factors/

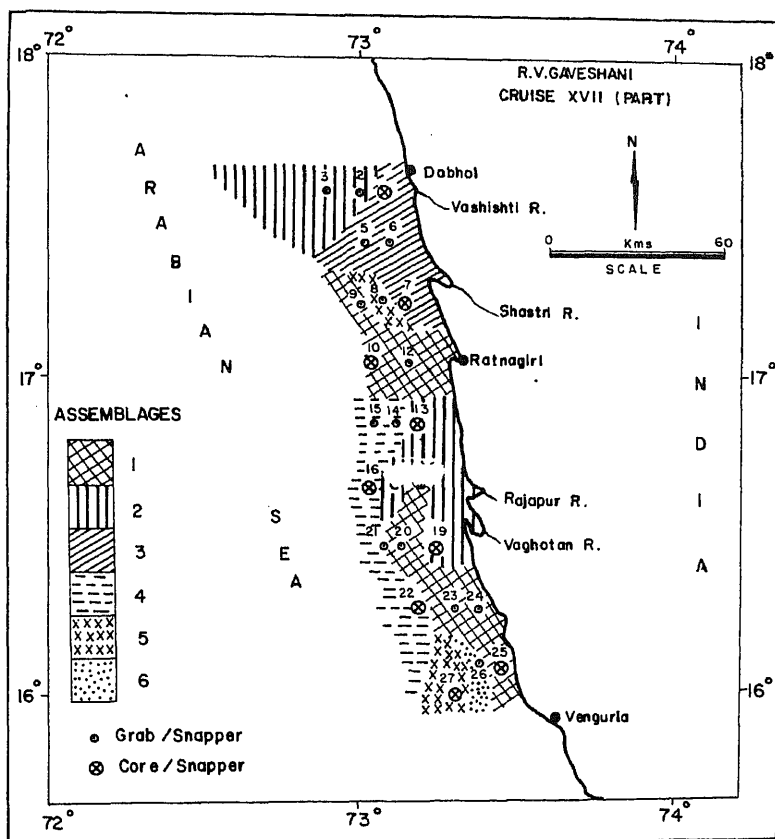


Figure 1. Sample locations and distribution of assemblages of benthonic foraminifera in the innershell of Central West Coast, India. 1 = *Ammonia beccarii* — *Ammonia annectens* assemblage; 2 = *Nonion boueanum* — *Florilus scaphum* assemblage; 3 = *Trochammina inflata* assemblage; 4 = *Bulimina exilis* assemblage; 5 = *Ammonia annectens* — *A. beccarii* — *Nonion boueanum* — *F. scaphum* assemblage; 6 = *Trochammina globigeriniformis* — *Ammobaculites agglutinans* assemblage.

1	23.247	23.247
2	22.716	45.963
3	17.448	63.411
4	17.130	80.541
5	8.740	89.280
6	4.206	93.486

assemblages. The boundaries are marked by considering the important factor/ factors at every station. Figure 2 shows the contours corresponding to factor-loading at different stations for four important factors/assemblages with reference to depth and location. Varimax factor score matrix (table 3) shows contribution of each species to all factors. The factors are further described as assemblages and named for one or few common species in each factor as determined from table 3.

Factor 1 (23.7% of the variance): *Ammonia beccarii* — *Ammonia annectens* assemblage.

Factor 2 (22.7%): *Nonion boueanum* — *Florilus scaphum* assemblage.

Factor 3 (17.45%): *Trochammina inflata* assemblage.

Factor 4 (17.13%): *Bulimina exilis* assemblage.

Factor 5 (8.7%): *Ammonia annectens* — *Ammonia beccarii* — *Nonion boueanum* — *Florilus scaphum* assemblage.

Factor 6 (4.2%): *Trochammina globigeriniformis* — *Ammobaculites agglutinans* assemblage.

5. Discussion

Ammonia beccarii — *Ammonia annectens* assemblage is dominated by two species of *ammonia* viz *A. beccarii* and *A. annectens*. These species are typical for transitional fauna between the marginal-marine and open-marine faunal groups (Walton 1964; Seibold 1971). *Ammonia beccarii* assemblages are found worldwide in shallow water tropical and temperate areas, particularly those areas influenced by freshwater run-off. Variable environmental conditions including periodic hyposalinity are often mentioned as favourable for this fauna (Boltovskoy and Wright 1976). Hence this assemblage shows mixing of hyposaline water to normal marine water.

Nonion boueanum — *Florilus scaphum* assemblage is a widely reported assemblage from inner shelf off West Coast of India (Antony 1968; Setty 1974; Setty and Nigam 1980, 1981).

Ammonia annectens — *Ammonia beccarii* assemblage and *Florilus scaphum* — *Nonion boueanum* assemblage are transitional assemblages showing mixing of fresh-water from the nearby rivers to open seawater, but these assemblages are grouped

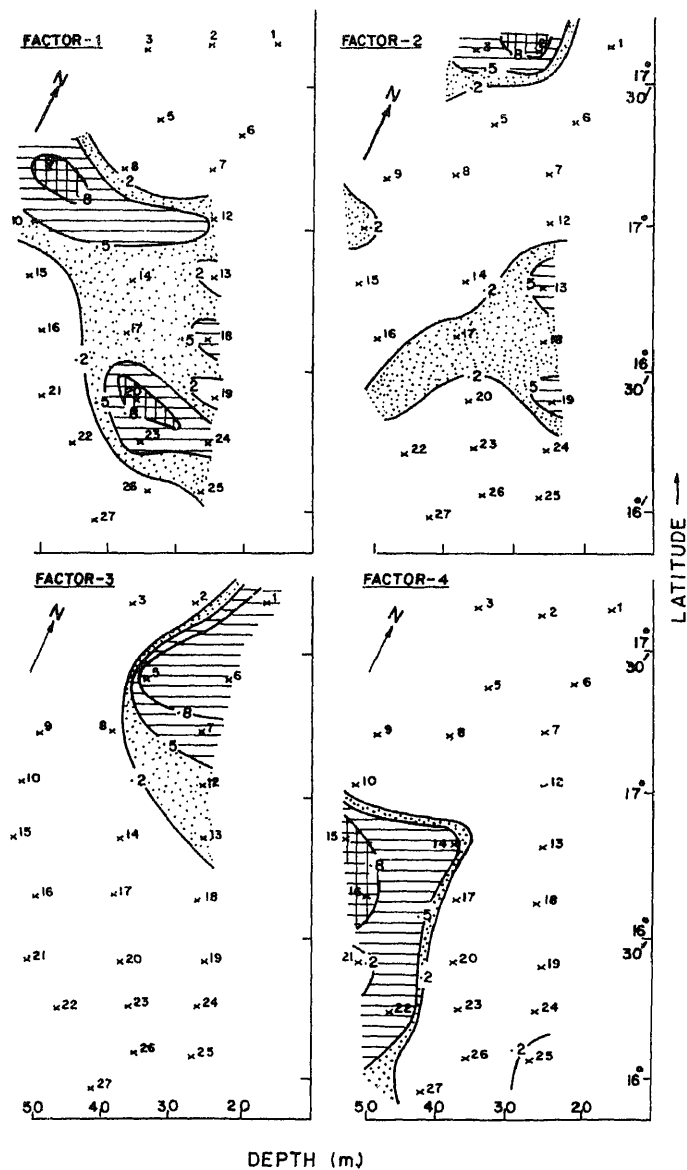


Figure 2. Contours corresponding to factor loading at different stations for four important factors/assemblages with reference to depth and location. Factor 1: *Ammonia beccarii* — *Ammonia annectens* assemblage; Factor 2: *Nonion boueanum* — *Florilus scaphum* assemblage; Factor 3: *Trochammina inflata* assemblage; Factor 4: *Bulimina exilis* assemblage.

Table 3. Varimax factor score matrix

Species	1	2	3	4	5	6
<i>Ammobaculites agglutinans</i>	-0.003	-0.007	0.005	-0.007	0.026	0.501
<i>A. dilata</i>	-0.007	0.004	-0.002	0.010	0.005	0.003
<i>Trochammina globigeriniformis</i>	-0.064	-0.061	-0.066	0.013	0.021	0.846
<i>T. inflata</i>	0.110	0.18	-0.985	0.006	-0.041	-0.043
<i>Eggerella advena</i>	-0.011	-0.000	-0.004	0.009	-0.008	-0.005
<i>E. bradyi</i>	-0.003	-0.012	0.000	0.049	0.025	-0.003
<i>Spiroloculina corrugata</i>	-0.005	0.006	0.002	-0.003	0.007	-0.001
<i>S. depressa</i> var. <i>rotundata</i>	-0.001	-0.000	-0.002	-0.001	0.009	-0.003
<i>Quingueloculina seminulum</i>	-0.024	0.027	0.008	-0.013	0.033	-0.006
<i>Bolivina attica</i>	0.013	0.025	0.004	0.063	0.042	-0.014
<i>B. lanceolata</i>	0.005	-0.024	-0.000	0.064	0.002	-0.009
<i>B. striatula</i>	0.029	0.181	0.024	0.037	0.091	-0.045
<i>B. subaenariensis</i>	0.003	0.069	0.011	0.012	0.041	-0.021
<i>Bulimina exilis</i>	0.051	-0.187	0.012	0.889	0.100	0.018
<i>B. marginata</i>	0.002	-0.055	-0.018	0.298	-0.005	-0.035
<i>Reussella pacifica</i>	-0.009	-0.001	0.001	0.002	0.004	-0.001
<i>Uvigerina senticosa</i>	-0.005	-0.006	0.000	0.007	-0.002	-0.000
<i>Cancris auricula</i>	0.006	-0.017	0.001	0.063	-0.006	-0.009
<i>C. sagara</i>	0.003	-0.015	0.000	0.052	-0.000	-0.007
<i>Ammonia annectens</i>	-0.518	-0.230	-0.012	0.062	-0.759	-0.022
<i>A. beccarii</i>	-0.824	0.269	0.112	0.032	0.451	-0.036
<i>A. dentata</i>	-0.008	0.083	0.019	0.003	-0.023	-0.012
<i>A. papillosus</i>	-0.073	0.001	0.035	0.010	-0.106	-0.009
<i>A. sp.</i>	0.007	0.010	-0.024	-0.012	-0.016	-0.011
<i>Elphidium falunicum</i>	-0.043	-0.030	-0.010	0.048	-0.091	0.015
<i>E. indicum</i>	-0.045	-0.032	-0.001	0.021	-0.032	0.004
<i>E. minutum</i>	0.004	0.040	0.006	0.023	0.005	-0.006
<i>E. simplex</i>	-0.007	0.001	-0.008	0.011	-0.003	-0.001
<i>Nummulites ammonoides</i>	0.007	-0.004	-0.020	0.005	-0.004	-0.004
<i>Eponides repandus</i>	0.007	-0.009	-0.028	0.001	-0.013	0.010
<i>Amphistegina radiata</i>	0.006	-0.012	0.031	0.004	-0.014	0.009
<i>Cibicides basiloba</i>	0.007	-0.017	-0.001	0.037	-0.005	-0.005
<i>C. kullenbergi</i>	0.001	-0.007	-0.000	0.014	-0.002	-0.002
<i>C. lobatulus</i>	0.002	-0.010	-0.000	0.023	-0.002	-0.003
<i>C. sp.</i>	0.006	-0.005	-0.002	-0.000	-0.011	-0.002
<i>Fursenkoina pontoni</i>	0.002	-0.001	-0.002	-0.002	0.007	-0.005
<i>F. sp.</i>	-0.005	0.005	0.004	-0.004	-0.010	-0.003
<i>Virgulinea pertusa</i>	0.019	0.050	-0.011	0.041	0.024	-0.015
<i>Nonion boueanum</i>	0.141	0.738	0.062	0.112	-0.238	0.134
<i>N. elongatum</i>	0.003	0.434	-0.035	0.175	-0.313	-0.035
<i>N. sp. A.</i>	0.007	0.040	-0.012	-0.004	-0.038	-0.016
<i>Florilus grateloupi</i>	0.005	0.084	-0.005	0.015	0.002	0.015
<i>F. scaphum</i>	-0.041	0.177	0.023	0.057	0.016	0.020
<i>Nonionella basispinata</i>	-0.004	0.048	0.013	0.054	0.082	-0.048
<i>N. auris</i>	0.007	0.036	0.009	0.096	0.051	-0.027
<i>N. auricula</i>	-0.006	0.040	0.009	0.061	0.071	-0.023
<i>N. turgida</i>	-0.006	0.013	0.008	0.041	0.019	-0.014
Other sp.	-0.017	0.014	-0.000	0.071	0.029	-0.018

separately. They might be discernible because of their different relationship to organic contents. Total organic matter content varies from 4% to 6.1% in factor 1 and 3.4% to 5.0% in factor 2 (table 1).

Trochammina inflata assemblage is confined to nearshore stations of the northern half of the study area. Because this species is known to occur abundantly in estuarine environment (Bhatia and Kumar 1976), its abundance in the stations near river mouths of Shastri and Vashishti possibly indicating high influx of freshwater. However, stationwise salinity data are not available to check this possibility. The organic matter values are low (3.06% to 3.37%) at sample locations where the assemblage is important.

Bulimina exilis assemblage has its maximum loading in stations deeper than 40 m depths. *Bulimina* is typically an open-marine fauna with depth range 40–100 m (Murray 1973). This species prefers to sandy clay and silty clay substrate. Ecologically, this factor seems to be least effected by freshwater run-off. Organic matter values for this factor ranging from 3.3–5.5%.

Ammonia annectens — *A. beccarii* — *Nonion boueanum* — *Florilus scaphum* assemblage has maximum loading at only two stations and assemblage in a mixture of factors 1 and 2.

Trochammina globigeriniformis — *Ammobaculites agglutinans* assemblage is dominant at one station only. This factor may have a extension further south of 16° latitude and showing freshwater influence as both species prefer hyposaline conditions.

In view of their low loading and short geographical distribution, the last two assemblages are less important.

6. Conclusions

Q-mode factor analysis of 72 foraminiferal species from 25 stations reveals six assemblages within the study area. These assemblages were not apparent in the original data matrix due to the patchy distribution of species. Out of these six, 4 assemblages are prominent. These assemblages can be mapped and compared to any environmental factor or variable for which data are available. In the present study area, factor 3 represents fauna of hyposaline and low organic matter conditions, whereas factor 4 represents a normal marine environment and prefers coarse sediments. Factors 1 and 2 represent transitional fauna indicating mixing of fresh and marine waters. However, organic matter values were relatively high for factor 2. Factors 5 and 6 are less important as factor 5 is a mixture of factors 1 and 2 and occurs as a major factor at only two stations and factor 6 is confined to only one station.

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References

- Antony A 1968 *Bull. Dept. Mar. Biol. Oceanogr. Univ. Kerala*, 4 11
- Bhatia S B and Kumar S 1976 *1st Int. Symp. Benthonic Foram. Cont. Margin Part A Ecology and Biology, Maritime Sediment spec. pub.* 1 239

- Boltovskoy E and Wright R 1976 *Recent foraminifera* (The Hague: Dr W Jung b.v. — Publisher) 515
- David R D and Sherwood W W Jr 1981 *Mar. Micropal.* **6** 29
- Geitzenaur K R, Roche M B and McIntyre A 1976 *Geol. Soc. Am.* **45** 423
- Howarth R J and Murray J W 1969 *J. Palaeontol.* **43** 660
- Imbrie J and Van Andel T H 1964 *Geol. Soc. Am. Bull.* **75** 1131
- Imbrie J and Kipp N G 1971 In *Late Cenozoic glacial ages* (ed) K K Turckian (New Haven and London: Yale Univ. Press) 71
- Klovan J E and Imbrie J 1971 *J. Int. Assoc. Math. Geol.* **3** 61
- Murray J W 1973 *Distribution and ecology of living benthonic foraminiferids* (London: Heineman Educational Books) p. 274
- Nigam R and Sarupria J S 1981 *J. Geol. Soc. India* **22** 175
- Nigam R, Setty M G A P and Ambre N V 1979 *J. Geol. Soc. India* **20** 244
- Sach H M 1973a *Quat. Res.* **3** 73
- Sach H M 1973b *Quat. Res.* **3** 89
- Seibold I 1971 *Palaeontol. Z.* **45** 41
- Setty M G A P 1974 *Proc. VI Indian Collog. Micropal. and Stratigr. Varanasi* 225
- Setty M G A P and Nigam R 1980 *Revista Ital. Paleont.* **86** 417
- Setty M G A P and Nigam R 1981 *Proc. IX Ind. Colloq. Micropal. and Stratigr. Udaipur* (Abstr.) 32
- Setty M G A P and Nigam R 1982 *Indian J. Mar. Sci.* **11** 225
- Setty M G A P, Nigam R and Ambre N V 1979 *Mahasagar — Bull. Natn. Inst. Oceanogr.* **12** 195
- Streeter S S 1972 *Micropal.* **18** 64
- Thiede J 1975 *Nature (London)* **253** 712
- Walton W R 1964 In *Approaches to paleoecology* (eds) J Imbrie and N D Newell (New York: John Wiley) p. 151

Petrochemistry of granite and granophyres of the Ezhimala igneous complex, Kerala, India

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Abstract. Anorthosite-gabbro-granite-granophyre suite of rocks occurring in the Precambrian migmatitic gneiss of Cannanore district, Kerala and spatially related to the Bavali lineament, constitute the Ezhimala igneous complex (EIC). The granophyres exhibit typical cuneiform and radiating fringe-type intergrowths. Geochemistry indicates a tonalitic - trondhjemitic to granitic composition of the residual magma. Petrochemical characters and lithologic association suggest that the EIC represents a complete differentiation sequence derived by strong fractionation and crystal cumulation of a parent basaltic magma which followed a tholeiitic differentiation trend, with no significant crustal participation as implied by the low initial strontium isotope levels. Occurrence of the complex along the Bavali lineament, which is a major rift-zone, emphasises a rift-controlled magmatism in this part of the Indian shield.

Keywords. Granite; granophyre; lineament; fractional crystallization; rift-controlled magmatism.

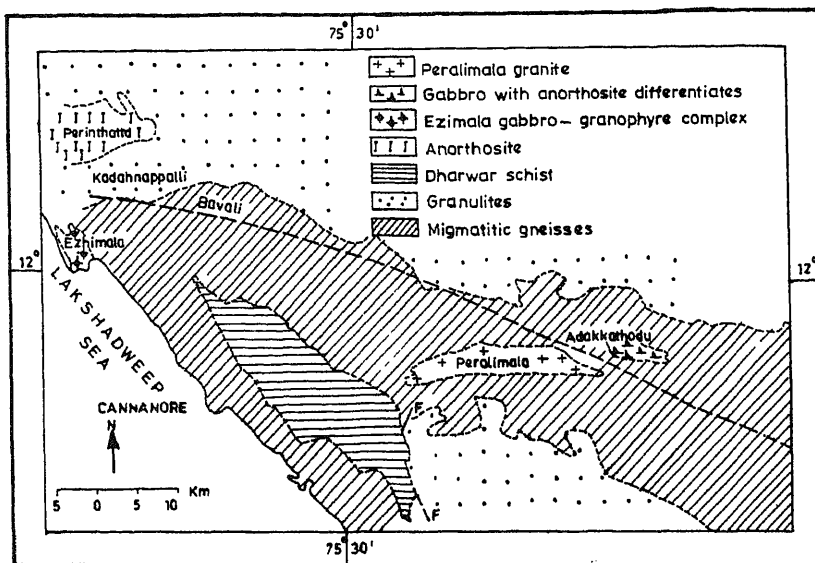
1. Introduction

Petrochemical studies on acid igneous rocks occurring in the Precambrian crystalline terrane of the Kerala region which forms the south-western part of the Indian shield, have been recently taken up (Santosh and Nair 1983) to decipher the tectonic environment of the rifted continental margin of India. The Ezhimala complex of Cannanore district, Kerala is of special interest, as rock types ranging from massive anorthosites through gabbros to granite and granophyres occur in spatial relation to a major fracture zone designated as the Bavali lineament (figure 1a).

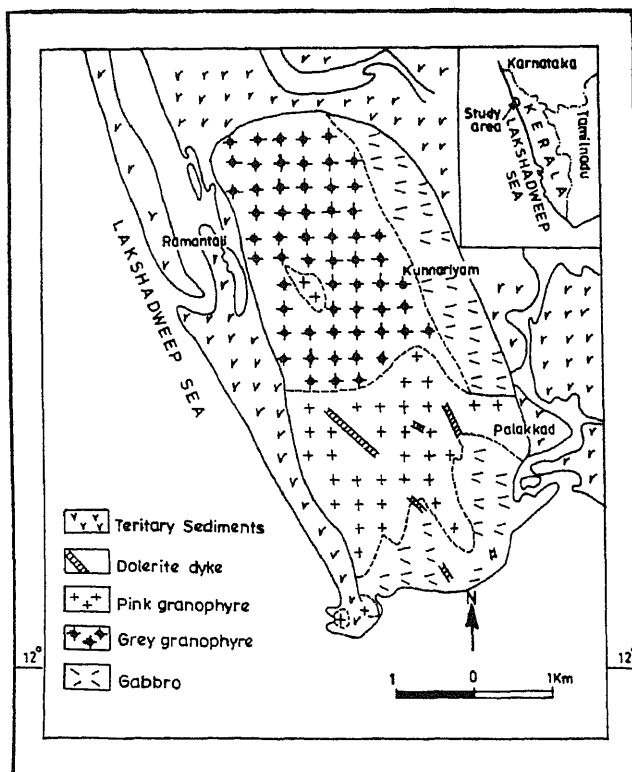
The occurrences of the granophyre (Varadan and Venkataramana 1976) and rapakivi texture in the associated granite (Nair and Vidyadharan 1982) in the area were earlier reported. The Bavali lineament is considered to be a deep-seated rift-zone based on detailed petrochemical studies on the perpotassic pluton of Peralimala which occurs along this lineament (Nair and Santosh 1983).

2. Geological setting

The anorthosites of Perinthatta and Kadannappalli and the gabbro, granite and granophyres of Ezhimala, occurring in the Precambrian migmatitic gneisses, together constitute the Ezhimala complex, hereinafter referred to as the Ezhimala igneous complex (EIC). There is a pronounced compositional variation from massive anorthosite through gabbroic anorthosite and gabbro to the gabbro-granophyre massif of Ezhimala. Eventhough correlation of the EIC with Deccan Trap volcanism has been attempted (Varadan and Venkataramana 1976), consanguinous age of 678 m.y. is



(a)



(b)

assigned to the granite and granophyres (Nair and Vidyadharan 1982) based on Rb-Sr dating.

The granophyre stock at Ezhimala exhibits intrusive relationship with the associated gabbro body (figure 1b). The gabbro is locally traversed by veins of granophyre. Granophyres of both pink and grey varieties are noted and the pink variety grades into pink granites towards south of the complex. The associated gabbro locally incorporates anorthosite differentiates. A number of dolerite dykes cut across the lithotypes.

Studies at different outcrops show that both pink and grey varieties are generally massive, with an overall mineral assemblage of feldspar, quartz, biotite and magnetite. The grey variety has a predominance of mafic laths exhibiting irregular orientation. Both varieties occasionally show porphyritic texture with euhedral to subhedral feldspar megacrysts ranging up to 2.5×1 cm in size.

3. Mineralogy

K-feldspar, which is rarely microperthitic, forms the dominant mineral constituent and occurs both as megacrysts in porphyritic varieties and in the mesostasis. The megacrysts are generally orthoclase and locally show inversion to a more ordered state with faint microcline cross-hatching. Eventhough Nair and Vidyadharan (1982) observed plagioclase rims around the orthoclase megacrysts in a single quarry at Palakkad, the present study, based on a number of samples, did not substantiate this. K-feldspar is also found in a hypidiomorphic granular fabric with quartz and minor plagioclase of albite-oligoclase range. The plagioclase laths of the grey granophyre are generally euhedral and show pronounced zoning with a more calcic core and a sodic margin indicative of the rapid change in composition of the fractionating magma. All the feldspars invariably show varying degree of sericitization or saussuritization.

Typical granophyric intergrowths of quartz and feldspar are noticed in some sections. Generally, the intergrowths are of cuneiform and radiating fringe types (Leighton 1954). The cuneiform type resembles graphic texture and in the radiating fringe type, micrographic intergrowth of quartz and feldspar radiate outward from anhedral grains of albitic plagioclase (figure 2). A minor insular variety, with islands and blebs of quartz set in a polygonal ground mass of alkali feldspar is also observed. In a few cases, fine-grained plumose types of textural intergrowths are also seen. In addition to the occurrence as a component in granophyric intergrowths, quartz is found also as a subhedral grains along with feldspars in normal granitic textures.

Augite showing varying degrees of resorption forms the dominant mafic constituent in the grey variety. Here biotite forms a minor accessory along with apatite and zircon. Euhedral biotite flakes with pleochroism and absorption as X = brownish yellow, Y = yellowish brown and Z = dark brown form the major mafic component in the pink variety. Locally, the biotite flakes show resorption to an aggregate of chlorite and magnetite. The magnetite grains are frequently bordered by a rim of epidote.

4. Geochemistry

Analytical results of major and trace elements in fifteen representative samples of the granite and granophyres of EIC are given in table I where the data are compared with those of similar rocks. Major elements were analysed by conventional wet methods and trace elements by atomic absorption spectrophotometer (Perkin-Elmer 4000).



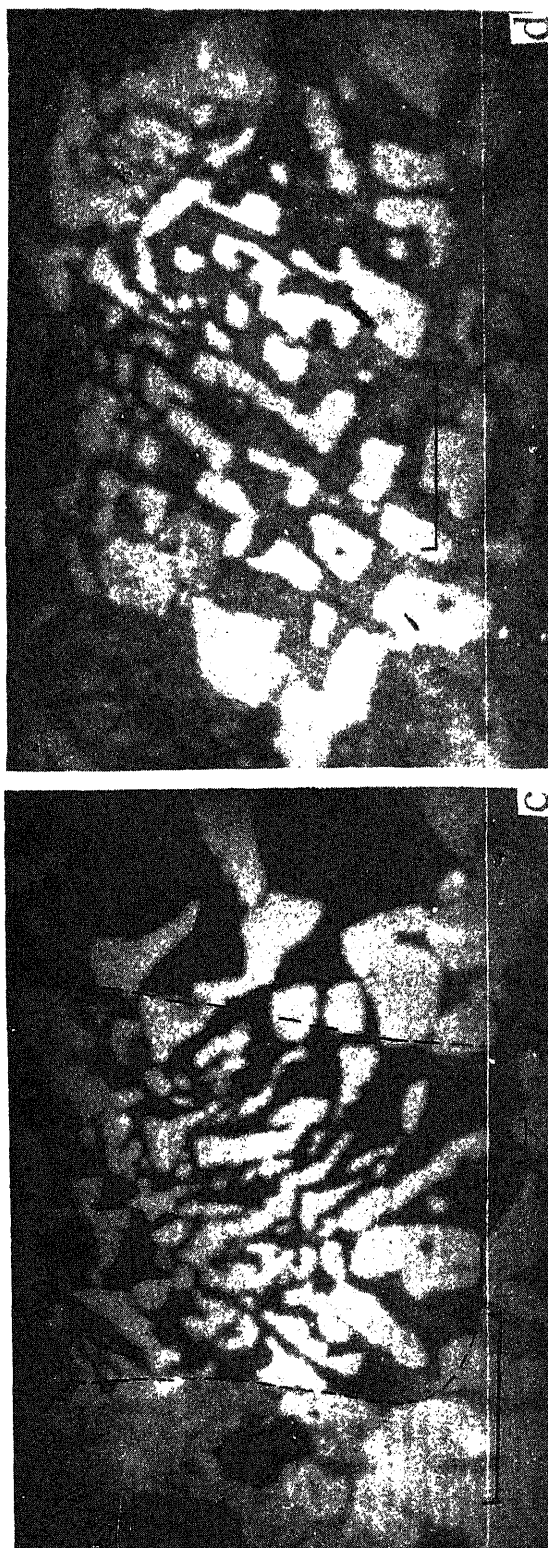


Figure 2. Photomicrographs showing quartz-plagioclase intergrowths in Ezhimala Granophyre Bar scales represent 1 mm. (a) Granophyric intergrowth of quartz and plagioclase. (b) Vermicular intergrowths of quartz and plagioclase diverging from grain boundary of a subhedral plagioclase. (c) Graphic intergrowth partially pseudomorphing a subhedral normally zoned plagioclase (grain boundary shown in dotted lines). (d) Coarse 'cuneiform' intergrowth with quartz grains showing semi-regular boundaries.

The mineralogical variation from grey to pink granophyre and granite is reflected in the SiO_2 range of 64.57–75.81 with the mean value (71.21) falling intermediate between the Skaergaard granophyre and the Red Sea granophyre. With low SiO_2 content, there is a corresponding increase in the FeO^* , MgO and CaO . The mean Al_2O_3 , Fe_2O_3 , Na_2O and K_2O values (13.47, 1.73, 3.96 and 2.98 respectively) are comparable with those of acid granophyre of Skaergaard (13.17, 1.56, 4.24 and 3.85 respectively).

Plots in SiO_2 vs K_2O diagram (Coleman and Peterman 1975) fall in the field of continental granophyre (figure 3). Even though variations in $\text{K}_2\text{O}/\text{Na}_2\text{O}$ and $\text{K}_2\text{O}/\text{CaO}$ ratios indicate that the residual liquid varied through tonalitic-trondhjemitic to granitic in composition, in an A–F–M diagram (figure 4a) the plots closely follow a tholeiitic differentiation trend as defined by Barker and Arth (1976). This trend is identical to those observed in the case of continental granophyres (Coleman and Donato 1979) and is comparable with the trend of Skaergaard granophyre. Mutual relationship of alkalies is brought out in figure 4b where K and Na exhibit a more or less equal accumulation with respect to Ca and is different from the pronounced Na-enrichment observed in tonalite-trondhjemite series. Alkali enrichment with respect to alumina towards the residual phase of fractionation is indicated by the occurrence of normative diopside (table 2) in some analyses.

Harker variation diagrams of the major element oxides (figure 5) show discernible magmatic trends. Al_2O_3 exhibit an overall depletion consistent with feldspar fractionation. K_2O shows concentration whereas all the other oxides indicate moderate to sharp trends of depletion with progressive fractionation. The sharp decrease in FeO , MgO , CaO and TiO_2 till the SiO_2 level of 70 indicate continuous removal of mafic phases. However, there is only limited variation in MgO levels (0.32–1.6) and the magnesium number ($\text{Mg}/\text{Mg} + \text{Fe}^*$) is less than 0.4.

The behaviour of trace elements is overall consistent with the empirical laws governed during magmatic differentiation (figure 6). Ba levels are initially rather high as Ba is not partitioned in the earlier phases of crystallization of the parent magma. As K-bearing minerals (biotite and K-feldspar) started crystallizing, Ba is accommodated resulting in a decreasing trend. The concentration of Pb with increasing differentiation index is also in agreement with the same. Sr, which follows Ca, shows a decreasing trend. Cr and V show depletion and Cr/V ratio fall with fractionation. The initial higher levels of Ni, Cu and Zn with sudden drop followed by consistent levels and absence of significant variations imply that crystallization of pyroxenes continued till the differentiation index value of 18. Li shows a steady concentration.

5. Discussion

The major element levels in the present case correspond to a tonalitic-trondhjemitic to granitic composition of the residual magma. Crystallization of such a final residuum of a parent basaltic magma would lead to a eutectic mixture of quartz and feldspar which can either form normal granites or granophyres depending on the P–T conditions. The spatial and genetic association of rocks with granitic and granophyric textures imply a sudden change in the P–T regimes during the crystallization of the residual fraction. The typical occurrences of granophyric intergrowths indicate rapid simultaneous late-stage growth of quartz and feldspar under cotectic conditions (Cerny 1971). The irregular blebs of sodic feldspar in optical continuity with the

Table 1. Major and trace element analyses of granites and granophyres from Ezhimala compared with similar analyses.

	EM-1	EM-2	EM-3	EM-4	EM-5	EM-6	EM-8	EM-9	EM-10	EM-12	EM-13	EM-14	EM-15	EM-16	EM-17	Mean (1)	Mean (2)	AG	RSG
Major elements (wt %)																			
SiO ₂	75.35	64.65	64.57	72.93	74.56	72.86	74.13	71.91	73.95	75.81	67.45	75.43	67.33	69.58	67.71	71.21	68.57	75.03	67.60
TiO ₂	0.12	1.00	0.90	0.25	0.20	0.28	0.25	0.23	0.15	0.18	0.97	0.15	0.86	0.78	0.67	0.47	0.73	0.31	0.76
Al ₂ O ₃	12.24	15.26	14.77	12.75	13.26	13.26	12.28	13.26	13.26	12.28	13.77	12.24	13.77	12.75	16.83	13.47	12.83	13.17	13.00
Fe ₂ O ₃	0.74	3.46	3.46	1.24	1.20	1.41	0.48	1.36	0.76	0.98	3.91	0.88	2.95	1.88	1.21	1.73	3.42	1.56	4.20
FeO	0.20	3.14	3.07	0.90	0.50	0.75	1.08	2.69	0.40	0.41	2.59	0.52	2.95	2.98	1.22	1.56	2.81	0.58	2.70
MnO	0.04	0.11	0.10	0.05	0.04	0.05	0.04	0.03	0.02	0.04	0.10	0.03	0.06	0.09	0.06	0.05	0.14	0.01	0.17
MgO	0.64	1.60	1.44	0.64	0.80	0.80	0.64	0.64	0.81	0.32	0.80	0.64	0.80	0.64	0.80	0.80	1.03	0.15	0.64
CaO	0.90	3.14	3.36	2.02	1.12	1.79	1.57	1.34	1.70	0.61	3.36	1.57	2.69	3.14	3.81	2.15	3.27	0.69	2.60
Na ₂ O	4.29	3.48	4.45	3.88	3.69	3.64	3.86	3.83	3.88	3.81	4.14	3.71	5.03	3.96	3.79	3.96	3.21	4.24	4.30
K ₂ O	3.61	2.08	2.13	3.42	3.02	3.24	3.89	3.82	3.66	3.6	2.05	2.98	2.24	1.99	2.65	2.98	2.19	3.85	2.50
P ₂ O ₅	0.05	0.27	0.28	0.18	0.06	0.17	0.14	0.15	0.16	0.17	0.34	0.13	0.24	0.26	0.28	0.19	0.32	—	—
Moisture	0.48	0.41	0.36	0.42	0.39	0.46	0.34	0.66	0.45	0.18	0.21	0.37	0.42	0.46	0.21	0.40	—	—	—
LOI	0.59	0.46	0.48	0.36	0.57	0.48	0.42	0.32	0.58	0.30	0.39	0.48	0.52	0.67	0.34	0.46	—	—	—
Trace elements (ppm)																			
Ba	149	331	32	198	215	403	438	202	180	273	396	213	410	556	382	292	—	—	—
Cr	90	160	181	98	87	65	76	96	58	59	55	60	51	48	61	83	—	—	—
Cu	5	20	6	4	8	4	5	5	2	6	3	2	6	9	2	6	—	—	—
Li	17	13	12	20	16	19	16	14	19	19	14	17	12	5	18	15	—	—	—
Ni	53	126	131	64	52	42	46	54	28	25	49	17	55	40	32	54	—	—	—
Pb	51	6	7	38	26	22	29	25	32	25	7	31	16	23	15	24	—	—	—
Sr	75	354	290	268	276	126	236	97	69	101	232	86	180	503	155	203	—	—	—
V	10	40	30	10	20	20	10	20	20	20	30	10	30	50	10	22	—	—	—
Zn	36	133	141	30	42	31	29	29	11	5	200	13	79	36	115	62	—	—	—

Analyst: M Santosh

EM-1 to EM-17 — Present study; Mean (1) — Average of present study; Mean (2) — Average of analyses given in Nair and Vidyadharan (1982); AG — Acid granophyre (Wager and Deer 1939, quoted in Turner and Verhoogen 1961, Table 30); RSG — Red sea granophyre (Coleman *et al* quoted in Coleman and Donato 1979, table 2).

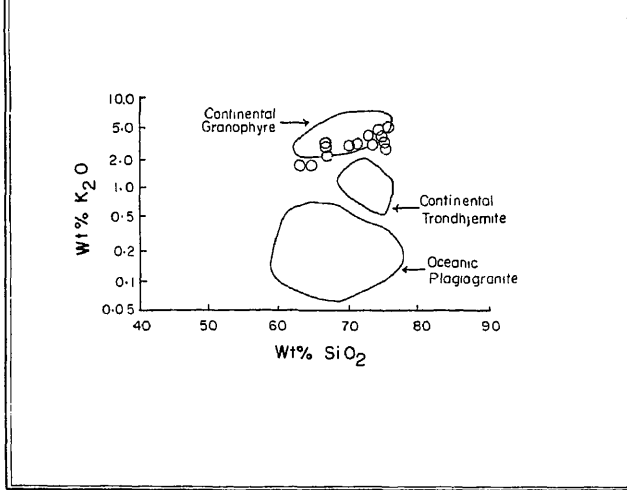


Figure 3. SiO_2 vs K_2O plots. The field boundaries are after Coleman and Peterman (1975).

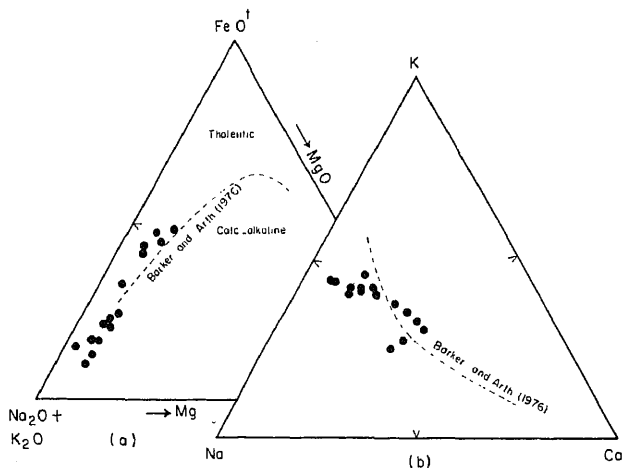


Figure 4. (a) A-F-M plots showing alkali enrichment trend, (b) K-Na-Ca plots. The boundary lines are after Barker and Arth (1976).

normally zoned plagioclase and the vermicular blebs of quartz and plagioclase making up much of the mesostasis between the plagioclase grains imply a late-stage supercooling of the liquid (Dunham 1968) brought about by a reduction of pressure below that which yields a normal granitic texture (Parsons 1972).

Variations in granophyric intergrowths in a single intrusive (Barker 1970) are conceived as a scale of the rate of cooling. From the bow-tie shaped plumose texture, which was experimentally produced by Lofgren (1971) at 700°C , to the coarse cuneiform intergrowth indicate a rapid lowering of temperature and pressure brought about by the high level emplacement of the granophyres, probably lineament-controlled. The presence of vugs/druses in the upper levels of the granophyre substan-

Table 2. Normative composition of Ezhimala granite and granophyres.

	EM-1	EM-2	EM-3	EM-4	EM-5	EM-6	EM-8	EM-9	EM-10	EM-12	EM-13	EM-14	EM-15	EM-16	EM-17
Q	34.14	21.46	21.36	32.70	37.80	33.90	33.48	29.52	32.76	36.48	28.80	38.34	73.22	31.26	26.6
Or	21.13	12.23	12.79	20.02	17.79	19.46	22.80	22.80	21.68	23.35	12.23	17.79	13.34	12.23	16.12
Ab	36.16	29.34	37.73	33.01	31.44	30.92	32.49	32.49	33.01	32.49	35.11	31.44	42.44	35.54	31.96
An	3.61	13.90	13.90	7.23	5.56	8.06	4.73	5.84	7.51	3.34	12.79	6.95	5.56	10.84	17.74
Al	0.65	—	0.86	1.54	—	—	1.86	—	—	—	1.73	—	4.62	2.67	—
Hy	0.13	1.59	4.52	1.13	2.13	2.00	1.73	4.90	1.13	0.80	1.20	1.60	2.79	2.59	2.13
Ms	0.23	5.10	5.10	1.86	0.70	1.39	0.70	2.09	—	0.93	5.57	1.16	4.18	2.78	1.86
Fe	0.64	—	—	—	—	0.48	—	—	—	0.32	—	0.16	—	—	—
Il	0.30	1.98	1.67	0.46	0.46	0.61	0.46	0.46	0.76	0.30	1.82	0.30	1.67	1.52	1.22
Ap	—	0.67	0.67	0.33	—	0.34	0.34	0.34	0.34	—	0.67	0.34	0.67	0.67	0.67
C	—	2.24	—	—	1.84	0.71	—	0.61	0.10	0.41	—	0.31	—	—	1.33

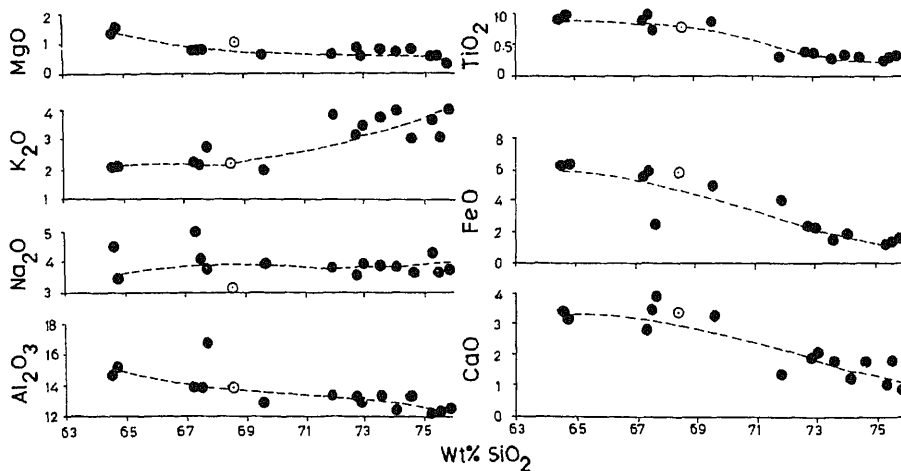


Figure 5. Harker variation diagrams of major elements in fifteen representative samples of the granite and granophyres of Ezhimala. The open circles represent average of values presented in Nair and Vidyadharan 1982 (table I).

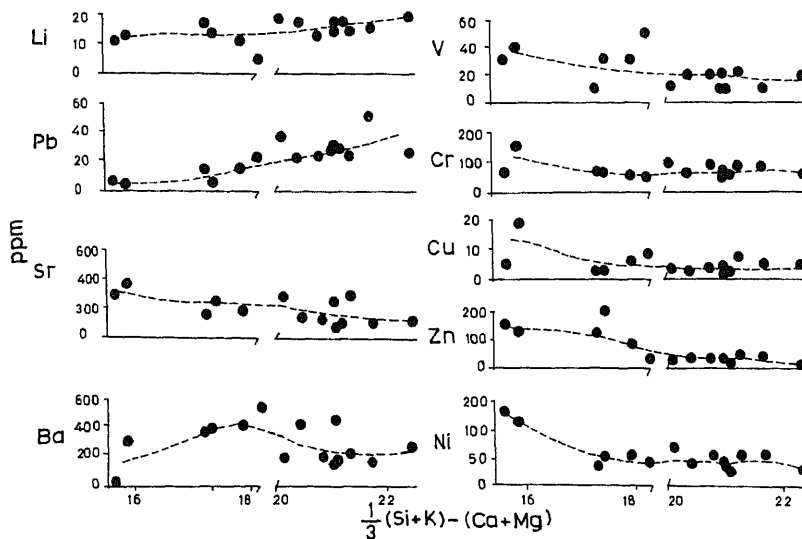


Figure 6. Trace element variation.

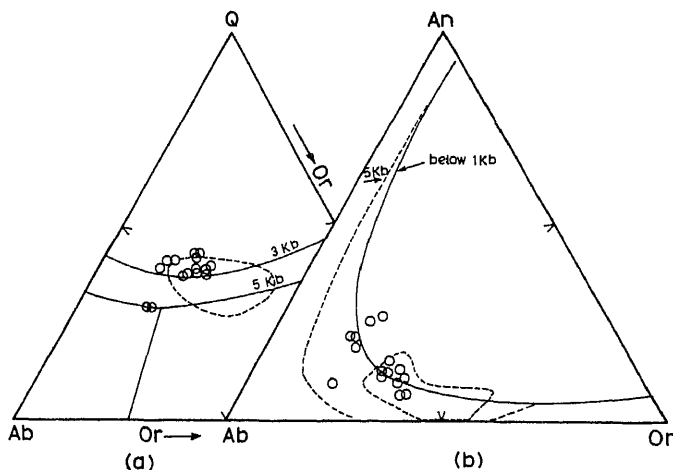


Figure 7. (a) Q-Ab-Or plots (after Tuttle and Bowen 1958), (b) Normative feldspar plots. The Plagioclase boundaries are after Glikson and Sheraton (1972). Dotted line represents field of granophyres as compiled by Barker (1970).

tiates the view that the rock was emplaced at high level and near to the surface (Varadan *et al* 1976). The normative Q-Ab-Or plots (figure 7a) cluster towards the granite minimum at 3 kb. Plots in a feldspar ternary (figure 7b) also fall in the low pressure field, consistent with similar observations of Coleman and Donato (1979). In both the diagrams, the plots lie within or close to the field of granophyres as compiled by Barker (1970).

The sericitization and saussuritization of the feldspars with associated alteration products like epidote imply the activity of post-magmatic hydrothermals (Marmo 1971) generated here probably during the later emplacement of the mafic dykes. Such hydrothermal alterations are observed elsewhere also (Coleman and Donato 1979).

With the association of lithotypes showing compositional variation from anorthosite through gabbro to granite and granophyre, the EIC represents a complete differentiation sequence resulting from strong fractional crystallization and crystal cumulation of a parent basaltic magma which followed a general tholeiitic differentiation trend with no significant crustal participation as implied by the low initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratios for the granite and granophyres obtained from the isochron plot (Nair and Vidyadharan 1982). Separation of early mafic phases initiated precipitation of intercumulus plagioclase. Flotation and segregation of discrete plagioclase led to the cumulus anorthosite (Goode 1977). With the enrichment of silica and alkalis, the ultimate fraction attained tonalitic-trondhjemitic to granitic composition. Further fractionation was largely controlled by K-feldspar as indicated by the depletion of Ba (Hanson 1978).

Spatial associations of acid intrusives with regional lineaments representing probable rift-settings are typical in Kerala and are recently deriving much attention. Even though no geophysical data are available, the Bavali lineament is considered to be a major rift-zone as inferred from the petrochemical characters of the perpotassic pluton of Peralimala which occur along this lineament (Nair and Santosh 1983). Occurrence of the EIC close to the rifted continental margin and distribution of its litho-types along the Bavali lineament emphasise a rift-controlled magmatism in this part of the Indian shield during the Late-Precambrian times.

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References

- Barker D S 1970 *Bull. Geol. Soc. Am.* **81** 3339
Barker F and Arth J G 1976 *Geology* **4** 596
Cerny P 1971 *Contr. Mineral. Petrol.* **30** 343
Coleman R G and Donato M M 1979 in *Developments in petrology* (ed.) F Barker (Amsterdam: Elsevier) **6** 149
Coleman R G and Peterman Z E 1975 *J. Geophys. Res.* **80** 1099
Dunham A C 1968 *Q. J. Geol. Soc. London* **123** 327
Glikson A Y and Sheraton J W 1972 *Earth Planet. Sci. Lett.* **17** 227
Goode A D T 1977 *Earth Planet. Sci. Lett.* **34** 375
Hanson G N 1978 *Earth Planet. Sci. Lett.* **38** 26
Leighton M W 1954 *Bull. Geol. Soc. Am.* **65** 401
Lofgren G 1971 *Bull. Geol. Soc. Am.* **82** 111
Marmo V 1971 *Granite petrology and the granite problem* (Amsterdam: Elsevier) p. 244
Nair M M and Vidyadharan K T 1982 *J. Geol. Soc. India* **23** 46
Nair N G K and Santosh M 1983 *J. Geol. Soc. India* (in press)
Parsons I 1972 *Medd. om Gronland* **195** 73
Santosh M and Nair N G K 1983 *J. Geol. Soc. India* **24** 291
Turner F J and Verhoogen J 1961 *Igneous and metamorphic petrology* (New York: McGraw Hill) 694
Tuttle O F and Bowen N L 1958 *Geol. Soc. Am. Mem.* **74** 1
Varadan G N and Venkataramana B 1976 *Ind. Minerals* **30** 96
Varadan G N, Venkataramana B and Viswanathan T V 1976 *Symp. Paleovolcanicity, associated tectonism and metallogeny in India* (Ind. Acad. Geosciences, Hyderabad). p. 31

Lithological subdivision and petrology of the Great Himalayan Vaikrita Group in Kumaun, India

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Abstract. The Vaikrita Group made up of coarse mica-garnet-kyanite and sillimanite-bearing psammitic metamorphics constituting the bulk of the Great Himalaya in Kumaun is divisible into four formations, namely the *Joshimath* comprising streaky, banded psammitic gneisses and schists, the Pandukeshwar consisting predominantly of quartzite with intercalations of schists, the *Pindari* made up of gneisses and schists with lenses of calc-silicate rocks and overwhelmingly injected by Tertiary pegmatites and granites (Badrinath Granite) leading to development of migmatites, and the *Budhi Schist* comprising biotite-rich calc-schists. The Vaikrita has been thrust along the Main Central Thrust over the Lesser Himalayan Munsiri Formation made up of highly mylonitized low- to meso-grade metamorphics, augen gneisses and phyllonites.

Petrological studies demonstrate contrasting nature of metamorphism experienced by the Vaikrita and the Munsiri rocks. Sillimanite-kyanite-garnet-biotite-muscovite ($\pm$ K-feldspar and $\pm$ plagioclase).—quartz metapelites and interbanded calc-schists and calc-gneisses with mineral assemblages of calcite-hornblende-grossular garnet, labradorite (An_{50} – An_{65}), ($\pm$ K-feldspar)-quartz ($\pm$ biotite), and hornblende-diopside $\pm$ labradorite $\pm$ quartz, suggest medium to high grade of metamorphism or indicate upper amphibolite facies experienced by the rocks of the Vaikrita Group. The associated migmatites, granite-gneisses and granites of the Pindari Formation were formed largely as a result of anatexis of metapelites and metapsammites. While, the sericite-chlorite-quartz and muscovite-chlorite-chloritoid-garnet-quartz, assemblages in metapelites and epidote-actinolite-oligoclase (An_{20})-quartz and epidote-hornblende-andesine (An_{29}) $\pm$ quartz in the metabasites suggest a low-grade metamorphism (greenschist facies) for the Munsiri Formation, locally attaining the lower limit of medium-grade (epidote-amphibolite) facies.

The inferred P–T conditions obtained from textural relations of various mineral phases and the stability relationship of different coexisting phases in equilibrium, suggest that the temperature ranged between 600° and 650° C and pressure was over 5 kb for the Vaikrita rocks. The mineral assemblages of the Munsiri Formation indicate comparatively lower P–T conditions, where the temperature reached approximately 450° C and pressure was near 4 kb.

The rocks of the two groups were later subjected to intense shearing, cataclasis and attendant retrograde metamorphism within the zone of the Main Central (= Vaikrita) Thrust.

Keywords. Vaikrita; Munsiri formations; Tertiary; mylonitized; lithology; petrology; Great Himalaya.

1. Introduction

The bulk of the snowy mountain range in the central belt of the Himalaya, represented by pre-eminent peaks of Nandadevi (7817 m), Trishul (7120 m), Dunagiri (7066 m) and Panch Chuli (6904 m), is constituted of Precambrian metamorphic rocks of upper amphibolite facies and extensively intruded in the northern part by late- to post-tectonic Tertiary granite. This Great Himalayan lithological assemblage was desig-

nated as the Vaikrita by Greisbach (1891, 1893) and Valdiya (1973, 1979, 1981) and as the "Central Crystalline Zone" by Heim and Gansser (1939). The thrust-bound homoclinal slab of huge dimension (6000 to 12,000 m thick) has been interpreted variously as the root of the crystalline nappe covering the sedimentary formations of the Lesser Himalaya (Auden 1937; Heim and Gansser 1939; Berthelsen 1951; Hagen 1958; Valdiya 1962) and as the remobilized Precambrian geanticlinal basement ridge that has been spectacularly uplifted (Pande and Saxena 1968; Saxena 1971; Mehdi *et al* 1972). According to Powell and Conaghan (1973), the Himalayan "Central Gneiss" represents regenerated older rocks within the Indian continental crust.

The easterly extension of the Vaikrita is named differently in different parts of Nepal such as "Tibetan Slab" (Dalle du Tibet) by Lombard (1958) and Bordet (1961), 'Annapurna Gneissic Complex' by Bodenhausen *et al* (1964), Himalayan Gneiss Zone by Hashimoto *et al* (1973), Khumbu Nappe by Hagen (1958) and Barun Group by Jaros and Kalvoda (1976). Further east, it is known as Darjeeling Gneiss in Sikkim Himalaya, as Thimpu Formation (Nautiyal *et al* 1964) or Chasilakha-Takhtsang Gneiss (Gansser 1964) in Bhutan, and as Sela Group (Das *et al* 1975) in Kameng District of Arunachal Himalaya. To the northwest, the extension of the Vaikrita has been observed east of Karcham in the Satluj valley. The Zaskar Range of north-eastern Kashmir probably represents the northwestern continuation of the Vaikrita. The Padar in Kishtwar (Jangpangi *et al* 1978) and Giambal in southeastern Ladakh (Srikantia and Bhargava 1978) are probably correlatable with the Vaikrita.

The present work embodies the petrological studies of the Vaikrita Group and the Munsiri (= Jutogh) Formation of the 'Central Crystalline Zone' of Heim and Gansser (1939) in the central sector of the Himalayan arc (figure 1). The lithological mapping and tectonic studies have been carried out by the first author (KSV) and the petrological studies by the second (OPG). An endeavour has been made through the present study to bring out the distinction in the petrological characters of two group of rocks, the Vaikrita and the Munsiri.

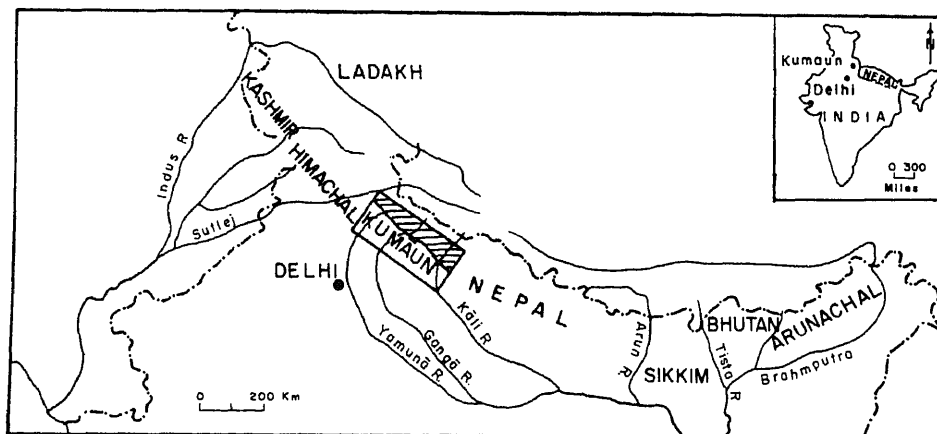


Figure 1. Location of the area of study in the Kumaun Himalaya — the central sector of the Himalayan arc.

Gansser (in Heim and Gansser 1939), describing in some detail the petrologic-mineralogic characters of the Great Himalayan rocks of Kumaun, attempted a petrological zonation of the sequence in the Kali valley in the eastern border. Dave and Rawat (1968), Kumar *et al* (1970) have also given a general account of these rocks. In recent years Powar (1972), Misra and Bhattacharya (1977) and Gupta (1978) have dealt with the petrogenesis of the rocks discernible in the Kali, Pindar and Alaknanda valleys, respectively. Detailed petrochemical studies of the Tibetan Slab in the Annapurna-Dhaulagiri massif have been carried out by Le Fort (in Bordet 1971; Bordet *et al* 1972; Pecher 1977).

2. Tectonic Setting

As already alluded to, the Great Himalaya is a huge homoclinal tectonic slab (figure 2) delimited by intracrustal thrusts (Valdiya 1979, 1980). The moderately dipping Main Central (= Vaikrita) Thrust separates the higher-grade Vaikrita rocks above from the low-grade metamorphics of the Lesser Himalayan Munsiri Formation. The Munsiri itself has been thrust over the sedimentaries and represents the root of the crystalline nappes that cover a large part of the Lesser Himalaya (Valdiya 1977, 1979). The Munsiri extends westwards to join up with the Jutogh Formation of Himachal Pradesh. The Vaikrita Thrust which Valdiya (1979, 1980) recognizes as the real Main Central Thrust, is a zone of extreme shearing, cataclastic deformation and metamorphic retrogression. The width of the thrust zone varies from a few metres to about a kilometre. The general strikes in the different formations of the Vaikrita Group are: WNW-ESE with gentle dip of 25° towards NNE of the Joshimath Fm., variable with 35° towards N or NNE in the Pandukeshwar and in E-W to WNW-ESE with dip varying from 35° to 40° towards N to NNE in the Pindari Fm. (Tara Chandra, Personal Communication).

In the north the steeply inclined Malari Fault/Thrust sharply cuts off the Vaikrita rocks from the sedimentaries of the Tethyan domain that stretches over the Indo-Tibetan borderland. In a cross-section (figure 3), the Malari appears to be a reverse fault that has truncated both the Vaikrita and Tethyan rock formations, attenuating and beheading the former and cutting the base of the latter. The emplacement of granite and attendant migmatization has taken place generally in the upper (northern) part, obliterating the structural relationship between the various units. All workers excepting Valdiya (1977, 1979, 1980) include the Munsiri Formation within the gamut of the "Central Crystalline Zone". The Munsiri metasediments indicate greenschist facies of metamorphism, and their gneissic components (mainly augen gneisses) exhibit pronounced cataclastic deformation and attendant retrograde transformations. Moreover, the style and orientation of the first-generation folds (isoclinal-reclined with axes trending NE-SW and having 25° plunge in the Munsiri rocks) are very different from those of the Vaikrita (also isoclinal to reclined having variable trend from E-W to ENE-WSW with 20 to 25° plunge towards E or ENE and superimposed with second-generation tight overfolds or isoclines trending NW-SE to NNW-SSE and plunging NW (Tara Chandra, Personal Communication). It is, therefore, difficult to subscribe to the contention of Hashimoto *et al* (1973) in case of Nepal sector, that the augen gneisses are derived from the Himalayan (Vaikrita) gneisses and metasomatically modified by the granitic material introduced along the shear zone. Nor the plea of Thakur (1977) seems plausible that the metamorphic grade decreases both upward and downward from the sillimanite-migmatite zone.

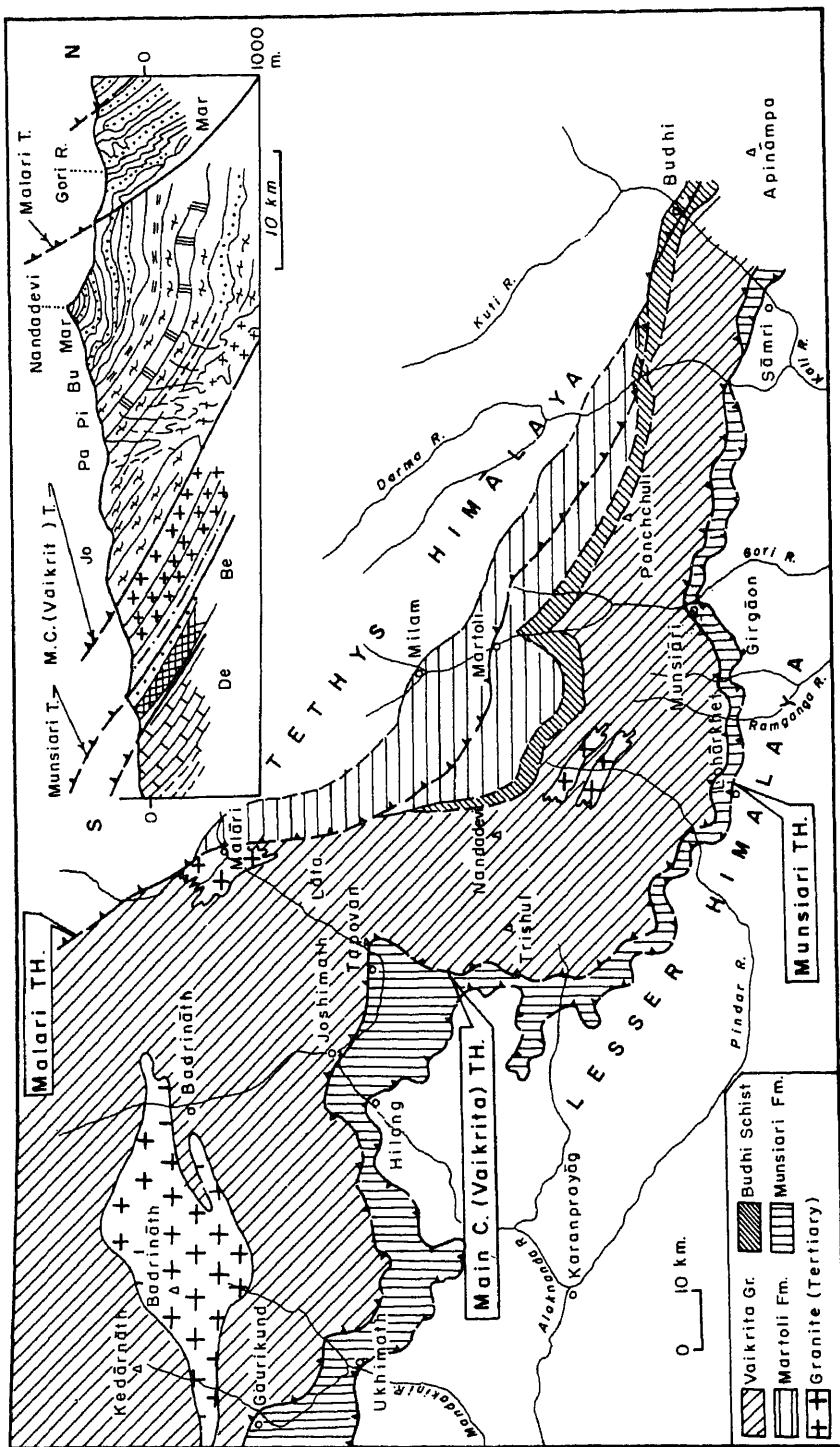


Figure 2. The tectonic map of the Great Himalaya in Kumaun (after Valdiya 1979). The inset demonstrates the Great Himalaya is a huge tectonic slab delimited by MC (Vaikrita) T and Malari Thrust-fault. (De = Deoban dolomites; Be = Bering quartzites; Mu = Munsiri Fm.; Jo = Joshimath Formation; Pa = Pandukeshwar Formation; Pi = Pindari Formation; Bu = Budhi Schist; Mar = Martoli flysch).

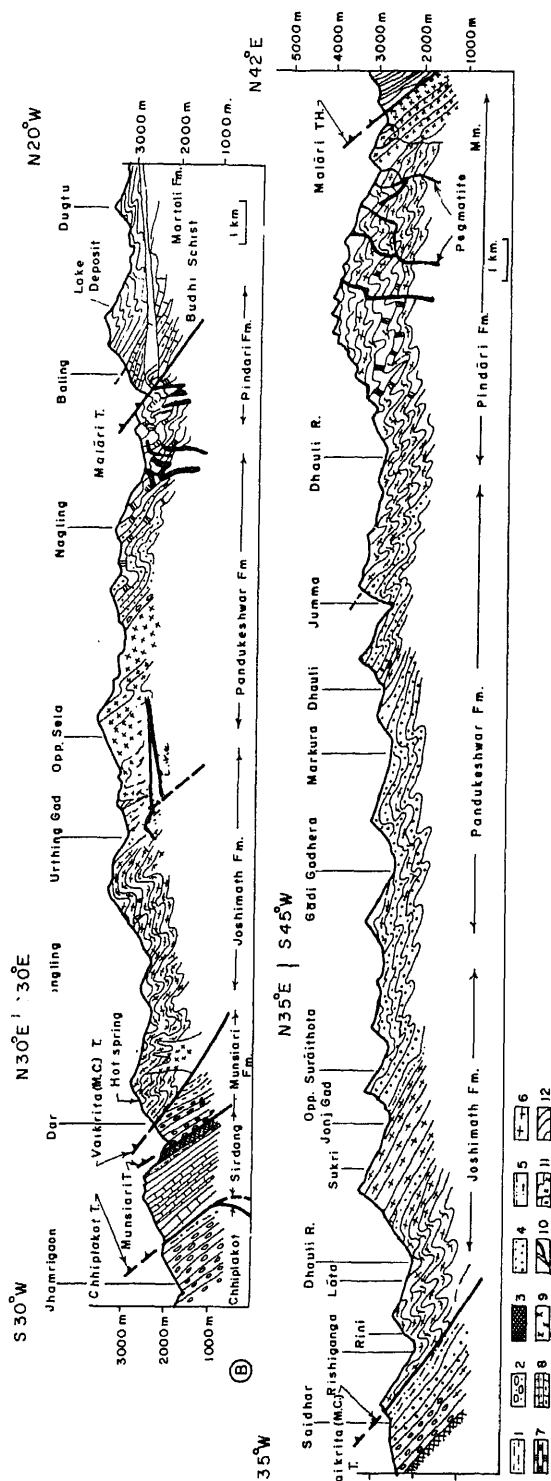


Figure 3. The cross sections across the Great Himalaya along the Eastern Dhaul (Darma) and Western Dhaul (Ganga) rivers. 1 – Chlorite-sericite schist with garnetiferous micaceous quartzite; 2 – Augen gneiss (mylonitized); 3 – Amphibolite; 4 – Quartzite (orthoquartzite) in the Berinag but biotite-garnet-bearing quartzite in the Pandukeshwar Fm; 5 – Garnetiferous mica schist; 6 – Kyanite-sillimanite psammitic gneiss; 7 – Calc-silicate marble (calc-gneiss) and calc-schist; 8 – Budhi Schist; 9 – Granite; 10 – Pegmatite; 11 – Migmatite; 12 – Martoli flysch.

3. Lithological Subdivision

The earliest attempt to give an account of petrological subdivision within the Vaikrita succession discernible in the Kali valley was made by Gansser (in Heim and Gansser 1939). Designating the Vaikrita as the "Central Crystalline Zone" Gansser identified eight zones (a to h), the first of which is a part of the Munsiri Formation and the last one being the basal member of the Garbyang Formation of the Tethyan facies. A four-fold classification of the "Tibetan Slab" in the Annapurna Range has been proposed by Le Fort (in Bordet 1971; Bordet *et al* 1972), but the sequence and content of the four units are somewhat different from those of Gansser. The present lithological subdivision (table 1) based on the traverses taken by Valdiya in the valleys of Kali, Darma (Eastern Dhauli), Gori, Pindar, Western Dhauli, Alaknanda, Mandakini, Bhagirathi and Tons, is an elaboration of the scheme outlined by Valdiya (1973).

Table 1. Lithological subdivision of the "Central Crystalline Zone" in Kumaun.

Tectonic Unit	Lithological units
Tethyan Facies	Martoli Formation of greywacke-slate alternation or Garbyang Formation of argillocalcareous sediments.
Malari Thrust/ Fault	
Vaikrita Group	(4) <i>Budhi Schist</i> : Biotite porphyroblastic calc-schist interbedded with micaceous schist, and phyllites toward the top. Locally, carbonaceous, pyritic, or staurolite-bearing. (= Gansser's zone g)
	(3) <i>Pindari Formation</i> : Predominantly banded lime-silicate gneiss and marble interbedded with subordinate biotite-psammitic gneisses and schists with or without kyanite and sillimanite. Extensively penetrated by network of dykes and veins of aplite, pegmatite and granite. Pervasive penetration and invasion of granite plutons (e.g. 18 m.y. old Badrinath) has brought about wide-spread migmatization and development of porphyroblastic augen gneisses, characterized by sillimanite and garnet. (= Zones e and f of Gansser)
	(2) <i>Pandukeshwar Formation</i> : Biotite- and/or muscovite-rich quartzite intercalated with kyanite-garnet-bearing mica schists and psammitic gneiss. Locally, lenses and subordinate layers of calc-silicate gneisses and garnet-bearing amphibolite. Where garnet has developed in abundance, the quartzite resembles leptynitic granulite. (= Gansser's zone d)
	(1) <i>Joshimath Formation</i> : Streaky and banded — and kyanite-rich biotite-psammitic gneiss. Interbedded with garnet-kyanite-biotite schists and at the base, with phyllonites. Very subordinate and local lenses of calc-silicate gneiss. (= Zone c of Gansser)
	Main Central (Vaikrita) Thrust
Munsiri Formation	Profoundly mylonitized gneisses of granodioritic, granitic and aplitic composition, grading locally into augen mylonite. Interbedded with chlorite-sericite schist, graphitic schist and crystalline blue-grey limestone. (= Zones a and b of Gansser)
Munsiri Thrust	

4. Petrography of the Vaikrita and Munsiri Metamorphics

The crystalline rocks of the Vaikrita Group in the valleys of Dhauli, Pindar and Darma show progressive metamorphism, reflected in the gradual coarsening of grain-size, decrease in the amount of muscovite and increase in quartz-feldspathic material which resulted in a gradual change from schists through migmatites to granite-gneisses and granites. The rocks are medium to coarse-grained, having schistose to gneissose textures. In the Pindari, the rocks show well-developed foliations and in the Budhi Schist compositional banding is very characteristic such as noticed near Malari.

The different stable assemblages as observed in various formations are given below:—

4.1 *Munsiri Formation*

A — Pelitic and psammitic compositions:

Quartz-sericite-chlorite

Quartz-muscovite-chlorite-biotite-garnet $\pm$ chloritoid-plagioclase $\pm$ epidote-tourmaline.

Quartz-plagioclase-biotite $\pm$ garnet $\pm$ muscovite.

Quartz-K-feldspar-muscovite $\pm$ biotite.

B — Basic composition:

Epidote-actinolite-plagioclase (An_7 – An_{15})-quartz.

4.2 *Joshimath Formation*

A — Pelitic composition:

Quartz-muscovite-biotite-garnet-kyanite $\pm$ plagioclase.

Quartz-muscovite-biotite-kyanite $\pm$ fibrolite ($-$ plagioclase).

B — Quartzofeldspathic composition:

Quartz-K feldspar-plagioclase-biotite $\pm$ muscovite.

4.3 *Pandukeshwar Formation*

A — Pelitic composition:

Quartz-biotite-kyanite-sillimanite-plagioclase $\pm$ K-feldspar.

Quartz-biotite-sillimanite-muscovite-K-feldspar $\pm$ garnet.

B — Quartzofeldspathic composition:

Quartz-plagioclase-microcline-biotite $\pm$ garnet ($-$ sillimanite)

4.4 *Pindari Formation*

A — Pelitic composition:

Quartz-biotite-garnet-staurolite-plagioclase.

Quartz $\pm$ muscovite-sillimanite-plagioclase ($\pm$ K-feldspar).

B — Quartzofeldspathic composition.

Quartz-muscovite-biotite $\pm$ garnet-plagioclase (myrmekite)-K-feldspar.
Quartz-biotite (-sillimanite)-plagioclase-K-feldspar-garnet (myrmekite).

4.5 *Pindari and Pandukeshwar Formations:*

C — Calcareous composition:

Epidote-calcite-sphene-quartz-biotite ($\pm$ muscovite).
Zoisite-actinolite-calcite-sphene (-quartz) grossularite $\pm$ amphibole.
Zoisite-actinolite-calcite-sphene (-quartz).
Talc-tremolite/actinolite (-quartz).
Calcite-hornblende-plagioclase (An_{50-60})-quartz-microcline ($\pm$ biotite).
Hornblende-diopside $\pm$ calcite-scapolite $\mp$ quartz $\pm$ labradorite.

4.6 *Budhi Schist*

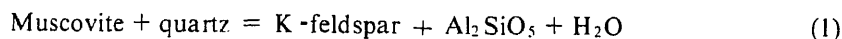
Chlorite-garnet-muscovite-biotite $\pm$ staurolite-K-feldspar-plagioclase-quartz.

The mineral assemblages recognized from Joshimath through Pandukeshwar, Pindari to Budhi Schist in the Vaikrita Group indicate progressive upward decline in the grade of metamorphism. In the Pindari Formation, the migmatites and gneisses show such varied features as veinlets, arterites and stromatic bands and even porphyroblastic to augen structures (figures 4a, b) which are often associated with tourmaline-rich muscovite-granite. The mineral assemblages of the rocks of the Munsiri, on the other hand, clearly indicate greenschist facies assemblage and indicate extensive retrogression of the feldspathic gneisses.

In the following pages the authors describe the salient features of the different mineral constituents of the rocks of the Vaikrita Group and the Munsiri Formation with special reference to the stability of the phase associations in the rocks. This is specially desired in view of the possible processes that might have occurred as a consequence of post-tectonic intrusions of granitic rocks, as stated earlier.

In contrast to the Munsiri rocks, the Pandukeshwar and the Pindari Formations show granoblastic mosaic of quartz grains. The triple-point junctions at approximately 120° angle with one another (figure 5a) indicate low-energy boundaries and thermodynamic equilibrium in the rocks (Kretz 1966; Vernon 1968). However, near the thrust plane (MCT) the Vaikrita Group of rocks (*e.g.* Joshimath Fm.) shows quartz in aggregation of minute crystals and occurs as elongated grains with undulose extinction (figure 5b).

Muscovite is a ubiquitous mineral in all the assemblages and is also found in migmatites, quartzofeldspathic gneisses and granites. The amount decreases in the higher grade metapelites in which kyanite and/or fibrolitic sillimanite and sillimanite + K-feldspar are the dominating phases in the rocks of the Pandukeshwar and Pindari, suggesting a reaction:



which demarcates the upper limit of amphibolite facies (Chatterjee and Johannes 1974). In the Munsiri muscovite is pleochroic in shades of green to pale green and is probably phengitic.

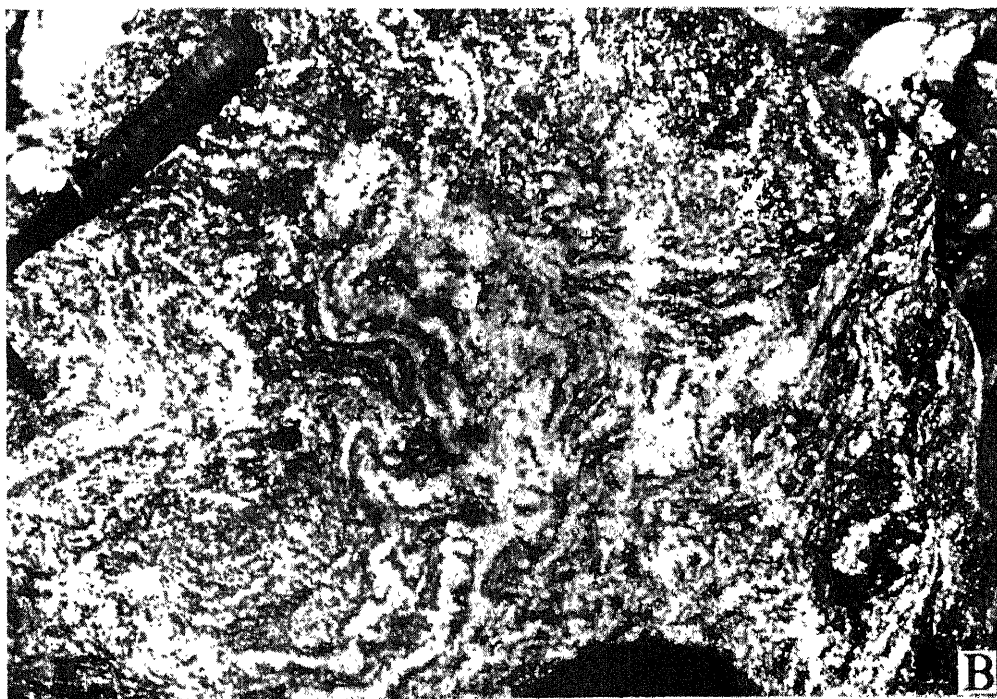
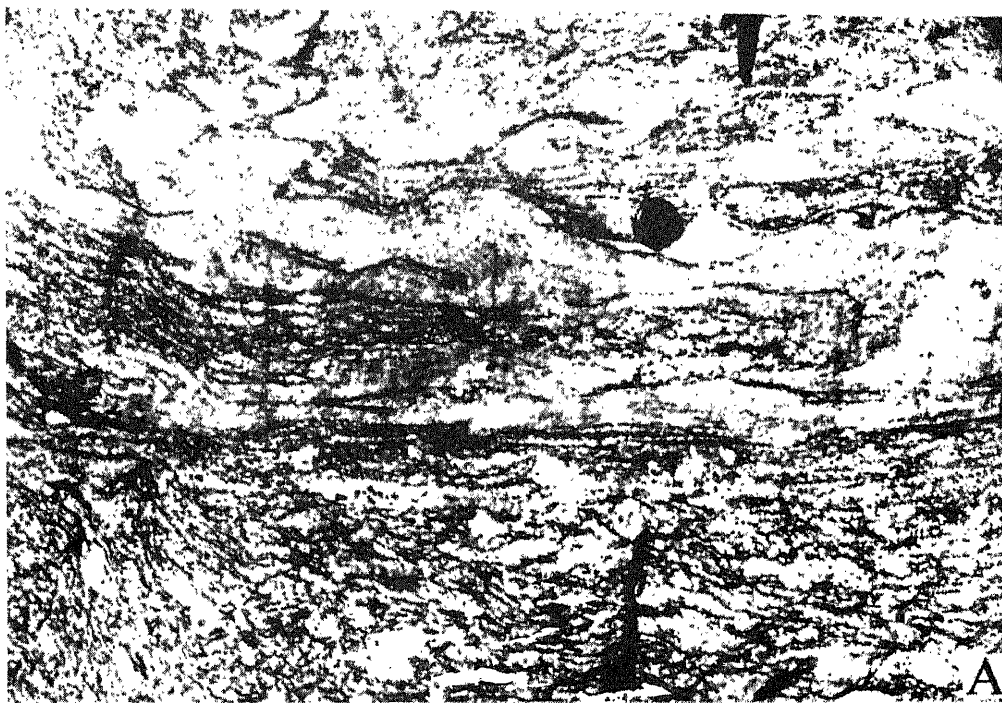


Figure 4. (a) Migmatite (phlebitic to arterite), Pindari Fm., Near Bogdiar, Gori valley.



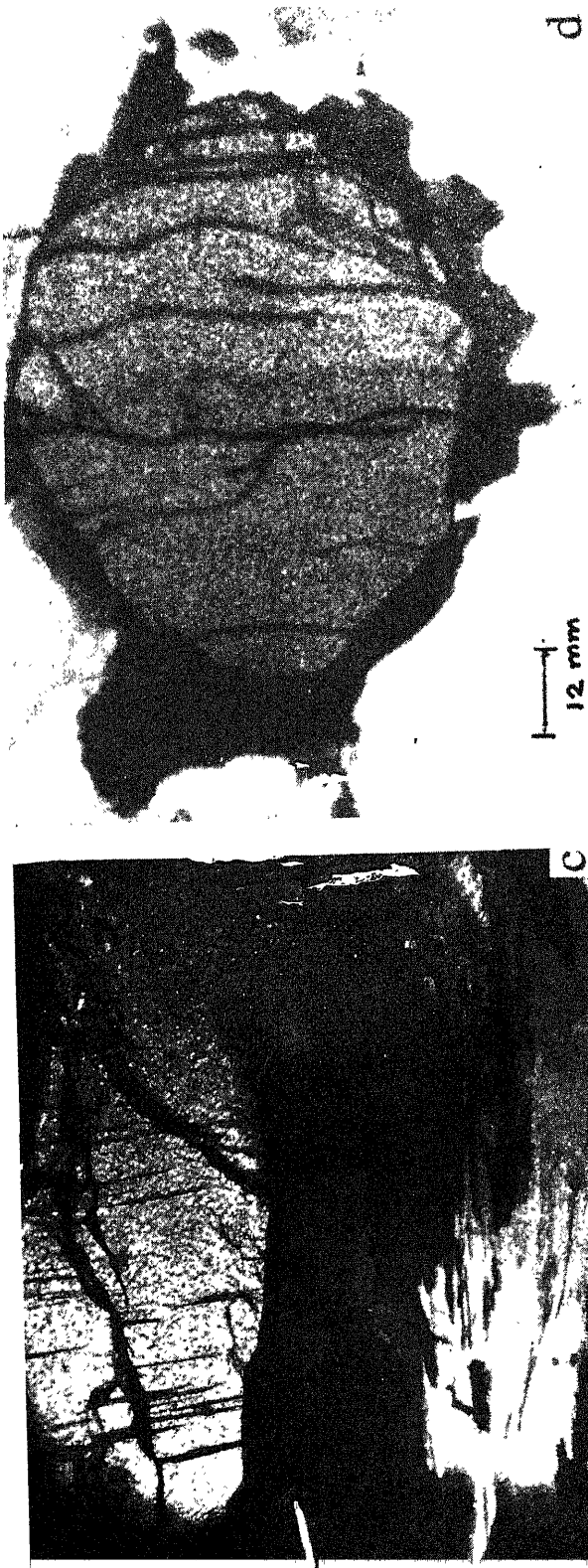
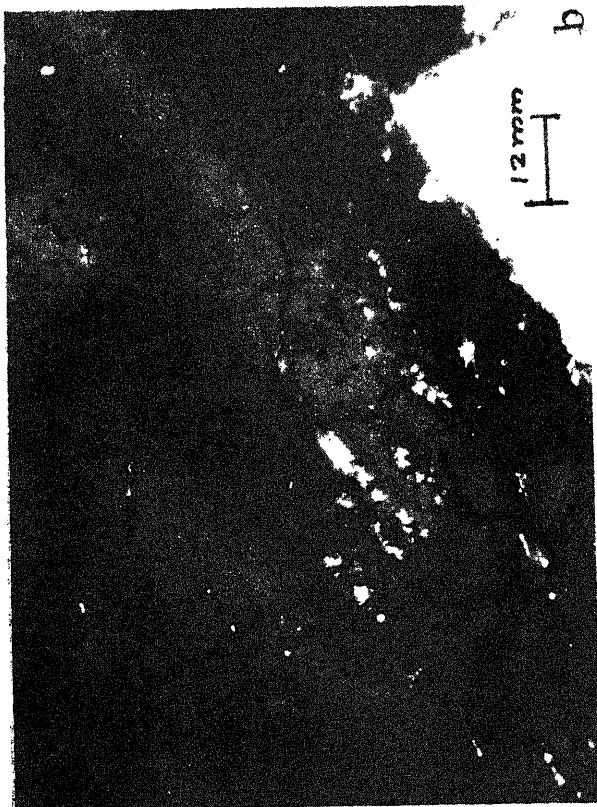


Figure 5. Vaikrita Group (a) Quartz forming triple-point junctions approximately at 120° ($100\times$). Locality: 3/4 km north of Bongling E. Dhuli. (Munsiri Fm.); (b) Coarsely crystalline quartz alternating with fine-grained ones associated with sericite ($120\times$). Locality: 1 km south of Khimtalla. E. Dhuli. (c) Development of kyanite within biotite and of fibrolite in muscovite. ($120\times$). Locality: 1 km N of Urthing (Pandukeshwar Fm.). (d) Fractured garnet (grey) rimmed by biotite (dark) around border. The fractures filled with chlorite. ($120\times$). Locality: 1 km E. of Dar. Eastern Dhuli.





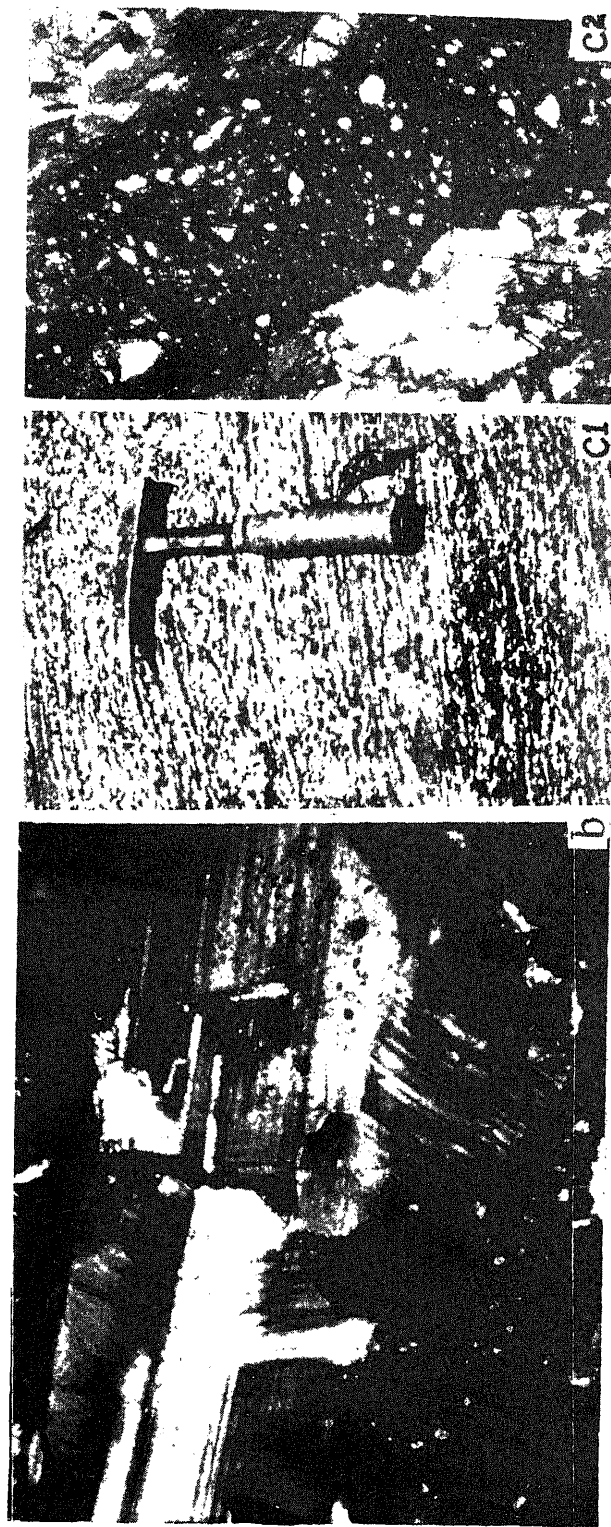


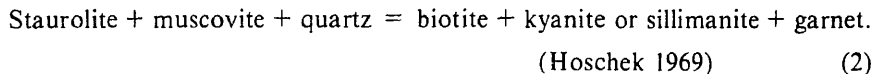
Figure 6. (a) Staurolite (light grey to grey with fractures) with inclusions of quartz and magnetite. (120 $\times$). Pindari Fm., Pindar valley. (b) Plagioclase showing "splochy" extinction (note the uneven and diffused portions dark grey) and simple Carlsbad twinning (120 $\times$). Pindari Fm., Pindar valley. (c) Mylonitized Munsiri gneiss characterized by streaked or drawnout fine laminae of feldspars. (Field photo with hammer, and towards left thin section C-2.) Locality: 1 km N of Kalamuni, Munsiri area.

In a progressive phase of metamorphism the development of kyanite and fibrolite-sillimanite seems to have taken place along the grain boundaries of muscovite flakes, (e.g. in rock sample No. D-50).

Primary chlorite is absent in rocks of the Vaikrita. In the Munsiri rocks, however, it occurs as thin, pleochroic greenish flakes in equilibrium with garnet and chloritoid.

Chloritoid occurs sparsely in the Munsiri rocks, as porphyroblasts showing pleochroism in the shades of bluish-green to pale-green and is characterized by prolific inclusions of magnetite and occasionally quartz. The rare occurrence of chloritoid in metapelites is not only suggestive of large Fe:Mg ratio, relatively high Al content and low amount of alkalis of the bulk composition, but its association with chlorite and muscovite is a strong indicator of low-grade metamorphism of green-schist facies.

Biotite exhibits a considerable variation in grain-size, shape and colour from one litho-unit to another. The porphyroblastic biotite of the Pindari metapelites contains inclusions of zircon, ilmenite and quartz. It is intimately associated with kyanite (figure 5c) and in some cases with fibrolitic sillimanite as seen in a Joshimath rock 3/4 km north of Bagling. Such an association indicates its formation by the following reaction:



Garnet occurs as small crystals in the Munsiri rocks and as large well-developed porphyroblasts, the diameter of which ranges from a few mm up to 20 mm, in the Vaikrita, particularly in the Pandukeshwar and the Pindari formations. Along the MCT zone in the Munsiri and basal part the Joshimath rocks, the garnet grains show evidence of strong deformation and attendant diaphoresis as borne out by chlorite, biotite and epidote formed extensively around the grains and along the fractures (figure 5d).

Kyanite is a common mineral in the rocks of the Vaikrita Group, but strikingly absent in the Munsiri rocks. In view of the coexistence of kyanite in equilibrium with biotite, garnet, muscovite and quartz and the absence of staurolite in these rocks, but its (staurolite) presence in the overlying Budhi Schist, the formation of kyanite is attributed to the reaction mentioned above in connection with biotite (e.g. see Sharma and Narayan 1975).

Sillimanite is less common than kyanite and occurs generally as a fibrolitic felted mass. Granular form is also observed in the Pindari metapelites. The textural relations between biotite, muscovite and quartz suggest its formation by the breakdown of muscovite, that is $\text{muscovite} + \text{quartz} = \text{sillimanite} + \text{K-feldspar} + \text{H}_2\text{O}$ (Chatterjee and Johannes 1974). The heat given out by post-tectonic Badrinath Granite seems partly responsible for this reaction as borne out by the widespread presence of sillimanite in the Pindari Fm. conspicuously intruded by granite and pegmatite.

The Vaikrita Group locally shows (e.g. sample No. D-57 of the Pandukeshwar Fm.) coexistence of kyanite and sillimanite, suggesting that either the P-T conditions were very close to the kyanite-sillimanite univariant curve or that the dehydration reaction (see e.g. No. 3, figure 7) was subsequently placed to higher temperature, whereby sillimanite was produced and kyanite continued to exist metastably.

Staurolite is of rare occurrence, and only in the Pindari rocks of the Vaikrita. It is completely absent in the Munsiri. The idioblastic and pleochroic porphyroblasts in the Pindari rocks (figure 6a) contain few inclusions of quartz and magnetite. In view of

staurolite got stabilized because of its higher zinc content (Guidotti 1970).

Hornblende is present in calc-schists and calc-gneisses of the Pindari Fm. and in the epidote-amphibolites of the Munsiri Fm. In the latter it occurs as prisms exhibiting pleochroism in shades of pale-green to bluish-green and showing small extinction angle ($C \wedge Z = 15 \pm 2^\circ$) and is actinolitic. In contrast, the hornblende of the Pindari Fm. occurs as porphyroblasts showing yellowish-green to greenish-brown colour. Such a colour has been taken to indicate a higher grade of metamorphism and higher Ti content (Gable and Smith 1975; Miyashiro 1973). It is also characterized by large extinction angle ($C \wedge Z = 28^\circ \pm 2^\circ$) and schiller inclusions of magnetite.

Diopside occurs sporadically in association with hornblende, biotite, plagioclase, scapolite and quartz in the assemblages of the calcareous rocks of the Pindari Fm. It is xenoblastic to idiomorphic, optically positive, feebly pleochroic in shades of pale bottle-green to green colour, has large extinction angle ($C \wedge Z = 44^\circ \pm 2^\circ$) and thus appears to be hedenbergitic.

Epidote occurs in equilibrium and is compatible with calcite, dolomite and quartz. This suggests that the upper limit of reaction — epidote + quartz = grossularite + magnetite, has not surpassed the temperature of equilibrium investigated by Liou (1973).

Scapolite is observed to occur in association and in equilibrium with diopside (hedenbergitic), hornblende, plagioclase, K-feldspar and quartz from the Pindari rocks (e.g. D-49).

Plagioclase is quite common in the Vaikrita rocks, particularly in quartzofeldspathic gneisses and migmatites. It occurs as porphyroblasts, characterized by numerous inclusions of quartz and sillimanite. The composition of plagioclase varies from albite and oligoclase-andesine ($An_{22} - An_{27}$) in the Munsiri rocks; to andesine (An_{35}) in the Joshimath and the Pandukeshwar psammopelites and to labradorite ($An_{55} - An_{65}$) in the Pindari calc-gneisses. Thus it is apparent that the composition of plagioclase depends mainly on the bulk-rock chemistry. A characteristic feature of the Pindari plagioclases is their 'spotted extinction' (figure 6b), a feature generally shown by high-temperature feldspars (Mackenzie 1966) and by amphibolitic plagioclases (Canon 1966). The plagioclases form perthitic intergrowth (mainly patch-, vein- and string-perthite) with the alkali-feldspars in the Pandukeshwar and the Pindari rocks. The patch-perthite suggests prevalence of high temperature (Agrawal *et al* 1972).

Potash-feldspar is generally absent in the assemblages of the Joshimath, though occurs sporadically in the Pandukeshwar and is quite common in the migmatites and calc-gneisses of the Pindari and Budhi Schist. In the gneisses of the Munsiri, microcline occurs as elongated and stretched lenticular augen porphyroblasts. Whereas in the migmatites and granite-gneisses within the Pindari Fm., it shows variation in microclinal twinning pattern. In the higher-grade rocks the feldspar becomes subperthitic to micropertthitic, approaching the character of orthoclase characterized by small to moderate optic axial angle ($2V_\alpha = 50-60^\circ$).

5. Retrograde Metamorphism

The high-grade minerals have suffered marked retrogressive changes, the degree of diaphoresis being more severe in the proximity of the Main Central Thrust, as evident in the augen gneisses of the Munsiri. In the thrust zone, marked extensive

sericitization of feldspars and muscovite, chloritization of biotite and garnet, epidotization of garnet and hornblende, and obliteration of the earlier fabric are discernible. The cataclastic fabric of the Munsiri rocks such as streaky (flaser) banding exhibited by augen gneisses (figure 6c), flattening of quartz and feldspar-porphyroblasts and the formation of chlorite, biotite and epidote in the fractures of garnet (figure 5d), bear indisputable testimony to dislocation metamorphism.

The alteration of diopside, hornblende and of garnet to epidote (+ clinozoisite) in the calc-gneisses, the pinnitization of biotite, the extensive sericitization of alkali feldspars and deformation of twin-lamellae of plagioclase of the Pindari gneisses indicate retrogressive changes either along so far unrecognized intraformational thrust planes in the higher level of the Vaikrita succession or perhaps are induced by hydrothermal solutions derived from post-tectonic granites. The recrystallization in these sheared rocks probably outlasted the deformation. It can, therefore, be concluded that the progressive regional metamorphism was followed by intense shearing and attendant cataclasis, and retrogression of the augen gneisses to mylonites and phyllonitization of schists when the Joshimath gneisses rode over the Munsiri rocks along the Main Central Thrust.

6. P-T Conditions

The presence of kyanite and almandine garnet in the pelitic composition of the Vaikrita Group rocks suggest that minimum pressure was of the order of 4.5 kb, when the triple-point positions of Al-silicate polymorphs (Holdaway 1971; Richardson *et al* 1969) and the experimental results of Hirschberg and Winkler (1968) are taken into consideration. The upper pressure limit cannot, however, be lowered because no phase such as cordierite could have formed in these rocks due to compositional constraints. As will be shown later, the rocks have higher Fe/Mg ratio which prohibited the development of cordierite in them. The temperature limits seem to be fixed in view of the experimental results on the mineral equilibria pertinent to the rocks of the area.

The widespread zone of muscovite-sillimanite with K-feldspar in the Vaikrita Group rocks further suggests that temperatures were close to the muscovite + quartz = sillimanite + K-feldspar reaction, for which equilibrium data have been provided by various workers, for example Althaus *et al* (1970) and Chatterjee and Johannes (1974). The occurrence of migmatitic rocks and the widespread development of quartz-K-feldspar-plagioclase in the psammitic gneisses suggest that the temperature had been above the minimum melting of granite (Tuttle and Bowen 1958). From this it is concluded that the temperatures were in the range 600–670° C at regional pressures of 5.7 kb (when stratigraphic thickness is taken into consideration). A similar temperature value is also obtained when the development of diopside (Winkler 1976) and scapolite (Goldsmith and Newton 1977) is taken into consideration.

The equilibrium paragenesis, diopside-tremolite-calcite-quartz, observed in calcareous rocks lies within a narrow temperature range for considerable variation in X_{CO_2} and possibly indicates a temperature between 540° C and 655° C and pressure 1 and 5 kb. Further, the occurrence of scapolite with hornblende, biotite, garnet, K-feldspar (perthite) and quartz probably indicates that the conditions of metamorphism locally reached upper amphibolite or lower granulite facies. The presence of biotite and hornblende indicates high PH_2O and correspondingly low PCO_2 , which is quite likely as the emplacement of granitic melt was synchronous with regional metamorphism (*cf* Powar 1972).

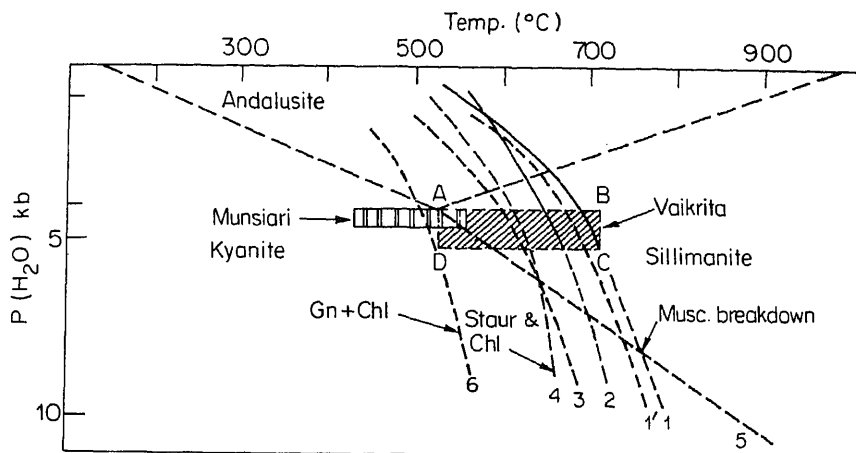


Figure 7. Approximate P-T conditions for metamorphic regimes of the Vaikrita Group (ABCD field with slanting lines) and the Munsiri Fm. (vertical lines). Curves in figure refer to: (1) Breakdown of muscovite in the presence of Quartz, (Evans 1965). (1) Reaction of Mus + Ab = Sill + Ksp (Evans and Guidotti 1966); (2) Breakdown of staurolite (Hoschek 1969); (3) Breakdown of paragonite in the presence of Quartz, based upon natural assemblages given in Guidotti (1974); (4) Reaction of St + Chl = Sill + Bio, Equation 3 of Guidotti (1975); (5) Al-Silicates, (Newton 1969); (6) Reaction of Gn + Chl = Sill + Bio (Guidotti 1974).

At these temperatures the Badrinath Granite pluton could not produce any thermal aureole, for the temperature between the regionally metamorphosed rocks and the post-tectonic intrusions did not differ significantly. However, when the rocks were quenched and unloaded, the thrusting phenomenon including cataclasis occurred whereby earlier progressive minerals were changed into lower-grade association, as in the Joshimath (basal Vaikrita). The Munsiri Formation has characteristically a green-schist facies assemblage and the gneissic or granitic basement of this formation was subjected to a more intense deformation as a consequence of thrusting. The metamorphism of the Munsiri Fm is typically in the temperature range of 450–500°C, in view of the presence of chloritoid and its association in equilibrium with garnet-chlorite-muscovite (Thompson 1976) (see figure 7). However, no feldspathization is discernible in these rocks since the Badrinath Granite intruded only at high levels whereby the Pandukeshwar and Pindari formations were injected profusely by the quartzo-feldspathic material and even locally resulted in partial melting.

7. Chemical Petrology

The rocks representing different mineral assemblages from the Vaikrita Group (13 samples) and the Munsiri Formation (9 samples) were spectrochemically analysed for their major elements by the methods suggested by Shapiro and Brannock (1962) with slight modification for the kyanite- and sillimanite-bearing rocks. The chemical analyses of the rocks with characteristic mineral assemblage and their oxidation ratios are shown in tables 2 and 3. The analyses are plotted in the ACF and AKF diagrams (figures 8 and 9) along with the average analyses of pelites (Shaw 1956), Pierre Shale, Littleton Shale and carbonate-free shales (all from Orville 1969), and greywackes (Pettijohn 1963, 1969) with the field of greywackes after Winkler (1967, fig. 10) for

Table 2. Chemical analyses of the Vaikrita rocks.

Sample No.	D-24	D-26	D-35	D-36	D-38	D-39	D-42	D-46	D-48	D-49	D-50	D-56	D-57
SiO ₂	55.63	69.02	58.52	52.92	58.77	61.22	53.45	61.45	61.22	68.00	51.37	68.99	71.38
TiO ₂	1.00	0.25	0.62	0.53	1.22	0.25	0.37	1.12	0.98	0.56	1.05	0.18	0.38
Al ₂ O ₃	12.17	14.70	15.12	27.86	17.73	15.45	13.38	17.08	15.70	12.56	28.28	15.92	13.85
Fe ₂ O ₃	1.13	0.43	3.47	1.12	1.42	0.93	0.69	1.03	1.33	0.99	1.30	0.69	1.56
FeO	3.08	1.40	8.88	3.72	6.20	2.20	2.80	5.48	3.96	3.20	4.20	1.08	3.20
MgO	8.12	3.42	5.13	6.84	5.34	4.71	5.13	5.56	5.98	6.41	5.13	2.14	3.42
CaO	8.32	0.95	0.71	0.42	0.42	1.31	11.38	0.83	0.60	1.02	1.42	0.56	0.24
Na ₂ O	1.16	2.59	1.38	1.42	1.27	3.50	1.50	0.89	3.70	1.81	1.38	3.65	0.89
K ₂ O	3.33	4.27	3.70	3.95	3.70	7.30	3.40	3.80	3.55	2.67	3.15	4.00	2.78
P ₂ O ₅	Nil	0.56	0.09	0.14	0.11	0.48	Tr	Tr	0.32	Tr	0.14	0.45	0.11
CO ₂	3.74	—	—	—	—	—	6.20	—	—	—	—	—	—
Total	97.68	97.59	97.62	98.97	96.18	96.79	98.30	97.24	97.34	97.22	97.42	97.66	97.81
Oxidation Ratio	24.91	21.77	26.03	21.31	17.13	27.50	18.14	14.40	23.18	21.83	21.75	36.44	30.62
MgO/MgO + FeO	0.826	0.815	0.501	0.768	0.608	0.794	0.768	0.646	0.731	0.783	0.688	0.781	0.658

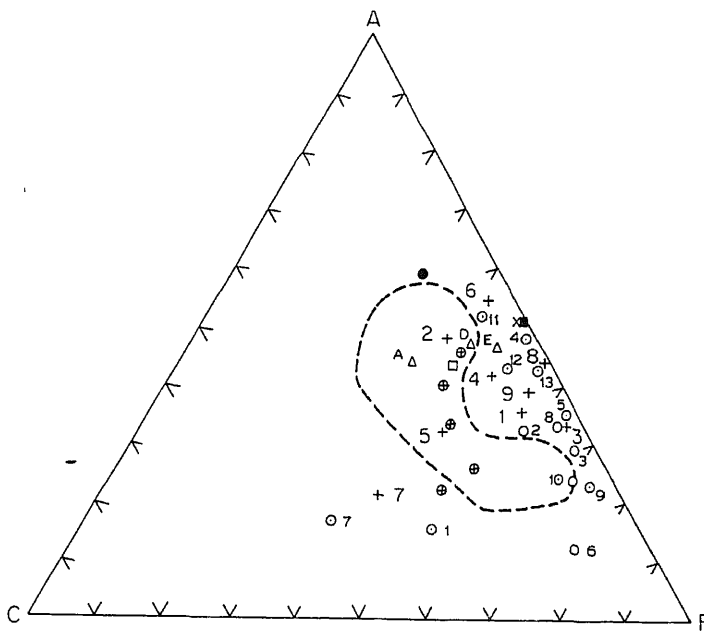


Figure 8. ACF diagram for rocks of the Vaikrita Group. (1-13) and the Munsiri Fm. + 1-9 compared with the average pelite, (Shaw, 1956) ($\square$), average Pierre Shale ($\bullet$), carbonate-free shale ($\blacksquare$) and averages of shale Δ (A Clark), Slate ($\blacktriangle$ D Eckel), Precambrian Slates Δ (Manz) and averages of greywackes ($\odot$), after Pettijohn, (1963, 1969), Average Littleton Shale ($\times$). Field of greywackes after Winkler (1967, figure 10).

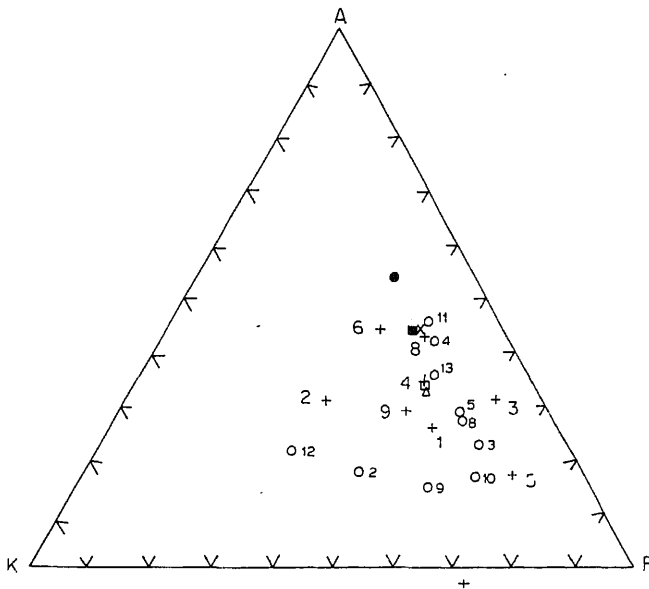


Figure 9. AKF diagram for the Vaikrita rocks and the Munsiri Fm., compared with average pelite, shales, Symbols same as in figure 8. $A = (Al_2O_3 + Fe_2O_3) - (Na_2O + K_2O + CaO)$, $K = K_2O$ and $F = FeO + MgO$. ACF, $A = (Al_2O_3 + Fe_2O_3) - (Na_2O + K_2O)$, $C = CaO$, $F = FeO + MgO$.

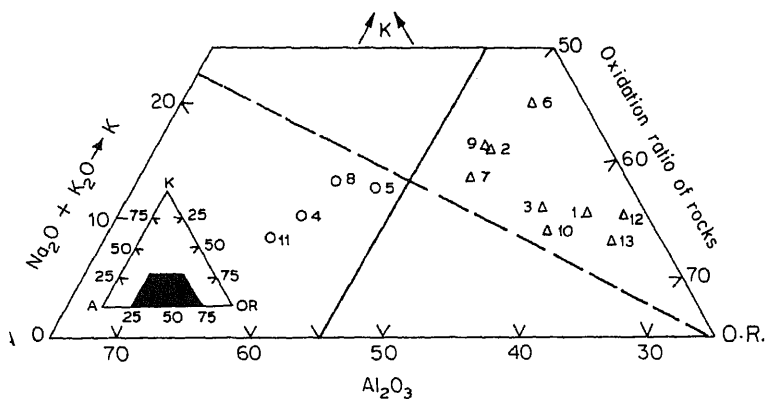


Figure 10. 'KA, OR Diagram' $K = (K_2O + Na_2O)$, $A = Al_2O_3$, O.R. = Oxidation Ratio. For rocks of the Vaikrita Group showing their distribution in sillimanite-bearing (O) and sillimanite-free (Δ) fields, after Goel and Chaudhari (1979).

Table 3. Chemical analyses of the rocks of the Munsiri Formation.

Oxide/samples	D-12	KV-12	KV-35	A-63	A-57	A-31	A-25	KV-228	KV-231
SiO ₂	69.97	69.76	46.62	70.98	68.26	63.12	65.00	53.68	68.21
TiO ₂	0.27	0.00	2.49	0.50	0.70	0.50	0.70	0.54	0.70
Al ₂ O ₃	14.54	16.44	22.42	14.80	15.54	20.37	15.44	24.25	16.04
Fe ₂ O ₃ + FeO	3.31	2.46	15.70	4.66	4.68	5.89	6.21	4.83	4.01
MgO	3.42	0.75	5.13	1.30	1.02	1.31	1.23	6.16	2.56
CaO	0.95	1.01	0.71	1.01	2.24	0.45	5.40	0.24	0.67
Na ₂ O	2.02	2.80	0.61	2.00	5.00	2.80	4.40	0.43	1.50
K ₂ O	4.07	4.90	3.60	3.00	1.20	4.30	2.50	5.25	5.00
P ₂ O ₅	0.38	0.03	0.41	0.18	0.23	0.04	0.14	0.09	0.27
Total	98.93	98.15	97.69	98.43	98.87	98.78	99.02	95.47	98.96

D-12 Mylonitized augen gneiss, Duk Gad, Chirkala, Eastern Dhauli valley.

KV-12 Mylonitized Granite, North of Gangoligaon, Eastern Dhauli valley.

KV-35 Garnetiferous-muscovite-chlorite gneiss, Munsiri, Gori valley.

A-63 Biotite schist, Jhuni, Saryu valley.

A-57 Mylonitized gneiss, Tarspil, Sundardhunga River, Pindar valley.

A-31 Garnetiferous mica schist, between Jaloli and Bhadang, Pindar valley.

A-25 Mylonitized augen gneiss, near Khati, Pindar valley.

KV-228 Chlorite-sericite schist, Tapovan, Western Dhauli valley.

KV-231 Sericite-chlorite schist Shelang, Alaknanda valley.

The diagrams show that the majority of the rocks of the Vaikrita Group (circle with dot) lie in the field of pelites; one lies within the boundary of greywackes, while three are far removed. Two of these three samples (Nos. D-24, D-42) are rich in carbonates (and thus confined within the field of calcareous shales), and the third is granite-gneiss in composition and rich in tourmaline and biotite. These features represent compositional variations of the parental sediments. The lithological subdivision as given in table 1 is thus confirmed by the chemographic relation as well.

The chemical analyses of metamorphics of the Munsiri Fm (+) also fall within the field of pelites and at the boundary of the greywacke field (figure 9). Thus the two groups of rocks are chemically not much different, although have entirely different mineral assemblages resulting from their contrasting grades of metamorphism, as revealed by their mineralogy (see earlier section).

7.1 Bulk rock composition and formation of sillimanite and kyanite

The analyses of rocks of the Vaikrita Group were plotted on KA-OR diagram (figure 10) following the method suggested by Goel and Chaudhari (1979). The rocks fall in two groups — one in the sillimanite-bearing field (SB) and the other in the sillimanite-free (SF). It is obvious that the bulk composition of the majority of rocks of the lower Vaikrita Group was not suitable for the formation of sillimanite, although they had attained the high grade of metamorphism up to upper amphibolite facies, as indicated by various mineral assemblages. It seems that higher oxidation of rocks and their higher alkali content favoured stabilization of muscovite and inhibiting the reaction $\text{muscovite} + \text{quartz} = \text{sillimanite} + \text{K-feldspar} + \text{H}_2\text{O}$, to take place on a wider zone.

The $\text{MgO}/(\text{MgO} + \text{FeO})$ mole ratio of the Vaikrita rocks shows that the $M/(M + F)$ values are more than 0.54 for the bulk of the kyanite-bearing rocks. These values are in conformity with the observation of Naggar and Atherton (1970) that kyanite in the medium-grade metapelites would not form if these values were lower than 0.54. Later, Atherton and Brotherton (1972) suggested that kyanite can form even in rocks with lower $M/(M + F)$ values resulting from higher modal percentage of garnet and iron ores. Similar trend is discernible in the rocks of the Pandukeshwar Fm. (as noticed 1 km south of Urthing in the valley of East Dhauli) having $M/(M + F)$ values of 0.65 and 0.73 respectively, the former containing kyanite with garnet and the latter free of garnet. This clearly suggests the effect of bulk-rock chemistry on the development of kyanite and sillimanite in the rocks. It thus appears that the rocks of the Vaikrita Group actually represent a medium to high grade of metamorphism and the absence of kyanite and/or sillimanite in a number of rocks may be attributed to their bulk-rock compositions.

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References

- Agrawal L, Pandey, Atma Ram and Powar K B 1972 *Him. Geol.* 2 145
 Althaus E, Karotke E, Nitsch K H and Winkler H G F 1970 *Neues. J. B. Miner. Mh.* 7 325
 Atherton M P and Brotherton M S 1972 *Scotland J. Geol.* 8 203

- Auden J B 1937 *Rec. Geol. Surv. India* **71** 407
- Berthelsen A 1951 *Saer. Meddl. Dansk. Geol. Forening* **12** 102
- Bodenhausen J W A, De Booy T, Egler C G and Nitjhuis H J 1964 *Report 22nd Int. Geol. Cong. Delhi*, Sec. 11, 101
- Bordet P 1961 *Centre Nat. Recher. Sci. Paris* p. 275
- Bordet P 1961 *Recherches géologiques dans l'Himalaya du Nepal Region du Makalu*, (Paris: CNRS), p. 275
- Bordet P 1971 *Recherches géologiques dans l'Himalaya du Nepal, region de la Thakkhola* Chap. VII, 1, (Paris: CNRS), (eds) M Colchen, D Krummanecker, P Le Fort, R Mouterde and M Remy
- Bordet P, Colchen M and Le Fort P 1972 *Him. Geol.* **2** 537
- Cannon R T 1966 *Am. Jour. Sci.* **264** 526-542
- Chatterjee N D and Johannes W 1974 *Contrib. Miner. Petrology* **48** 89
- Das A K, Bakliwal P C and Dhoundiyal D P 1975 *Geol. Surv. India Musc. Pub. Calcutta* **24** 115
- Dave V K S and Rawat B S 1968 *Res. J. Roorkee Univ.* **10** 97
- Evans B W 1965 *Am. J. Sci.* **263** 647
- Fuchs G R and Frank W 1970 *Jahrb. Geol. Bundes* **18** 1
- Gansser A 1964 *Geology of the Himalayas* (New York: Interscience) p. 289
- Goel O P and Chaudhari M W 1979 *Lithos.* **12** 153
- Goldsmith J R and Newton R C 1977 *Am. Miner.* **62** 1063
- Greisbach C L 1891 *Mem. Geol. Surv. India* **23** 1
- Greisbach C L 1893 *Rec. Geol. Surv. India* **26** 19
- Guidotti C V 1970 *J. Petrol.* **11** 277
- Gupta L N 1978 *Bull. India Geol. Assoc.* **11** 63
- Hagen T 1958 *Jarbh. St. Gallischen Naturf. Gessel* **76** 3
- Hashimoto H, Ohta Y and Akiba G 1973 *Geology of the Nepal Himalayas* (Japan: Himalayan Committees of Hakkaido Univ. Sapporo)
- Heim A and Gansser A 1939 *Denkschr. Schweiz Naturf. Gessel* **73** 1
- Hirschberg A and Winkler H G F 1968 *Contrib. Miner. Petrol.* **18** 17
- Holdaway M J 1971 *Am. J. Sci.* **271** 97
- Hoschek G 1969 *Contrib. Miner. Petrol.* **22** 208
- Jangpangi B S, Dhoundiyal D P, Kumar G and Dhaundiyal J N 1978 *Him. Geol.* **8(II)** 837
- Jaros J and Kalvoda J 1976 *Him. Geol.* **6** 176
- Kretz R 1966 *J. Petrol.* **7** 68
- Kumar G, Mehdi S H and Prakash G 1970 *J. Palaeontol. Soc. India* **15** 86
- Lombard A 1958 *Denkschr. Schweiz. Naturf. Gassel* **82** 1
- Le Fort P 1971 *Recherches géologiques dans l'Himalaya du Nepal region de la Thakkhola*, Chap. III (ed) (Paris: CNRS)
- Liou J G 1973 *J. Petrol.* **14** 381
- Mackenzie W S 1954 *Miner. Mag.* **30** 354
- Mehdi S H, Kumar G and Prakash 1972 *Him. Geol.* **2** 481
- Misra R C and Bhattacharya A R 1977 *Him. Geol.* **7** 133
- Miyashiro A 1973 *Metamorphism and Metamorphic Belts*, (London: George Allen), p. 492
- Naggar M H and Atherton M P 1970 *J. Petrol.* **11** 549
- Nautiyal S P, Jangpangi B S, Singh P, Guhasarkar T K, Bhate V D, Raghavan M R and Sahai T N 1964 *Report 22nd Int. Geol. Congr.* **11** 1
- Orville P M 1969 *Am. J. Sci.* **267** 64
- Pande I C and Saxena M N 1968 *Pub. Centre Adv. Stud. Geol. Punjab Univ. Chandigarh* **4** 1
- Pecher A 1977 in *Himalaya, Sciences de la Terre* (Pub. No. 268), (Paris: CNRS), p. 301
- Pettijohn F J 1963 *Data of Geochemistry*, 6th ed., US Geol. Surv. Prof. Paper 440-S, Si-S21
- Pettijohn F J 1969 *Sedimentary Rocks*, 2nd Indian reprint (Calcutta, New Delhi, Bombay: IBH Pub. Co.) p. 718
- Powar K B 1972 *Him. Geol.* **2** 34
- Powell C M A and Conaghan P J 1973 *Earth Planet. Sci. Lett.* **20** 1
- Richardson S W, Gilbert M C and Bell P M 1969 *Am. J. Sci.* **267** 259
- Saxena M N 1971 *Tectonophysics* **12** 433
- Shapiro L and Brannock W W 1972 US Geol. Surv. Prof. Paper, 1144-A, p. 56
- Sharma R S and Narayan V 1975 *Neues. Jb. Miner. Abh.* **124** 190
- Shaw D M 1956 *Bull. Geol. Soc. Am.* **69** 919

- Srikantia S V and Bhargava ON 1978 in *Tectonic Geology of Himalaya* (ed) P S Saklani (New Delhi: Today & Tomorrow's)
- Thakur V C 1977 in *Himalaya, Sciences de la Terre* (Pub. No. 268), (Paris: CNRS), p. 433
- Thompson A B 1976 *Am. J. Sci.* **276** 401
- Tuttle O F and Bowen N L 1958 *Mem. Geol. Soc. Am.* p. 74
- Valdiya K S 1962 *J. Geol. Soc. India* **2** 27
- Valdiya K S 1973 *Proc. Sem. Geodynamics of Himalayan Region*, NGR1, Hyderabad, pp. 204-205 (Abstract)
- Valdiya K S 1977 in *Himalaya, Sciences de la Terre* (Pub., No. 268), (Paris: CNRS), p. 449
- Valdiya K S 1979 *J. Geol. Soc. India* **20** 145
- Valdiya K S 1980 *Tectonophysics* **66** 323
- Valdiya K S 1981 *Zagros-Hindukush, Himalaya Geodynamic Evolution* (eds) H K Gupta and F M Delany (Washington: Am. Geophysical Union), Vol. **3**, p. 87
- Vernon R H 1968 *Aust. J. Petrol.* **9** 1
- Winkler H G F 1967 *Petrogenesis of Metamorphic Rocks*, 2nd ed. (New York: Springer-Verlag)
- Winkler H G F 1974 *Petrogenesis of Metamorphic Rocks*, 4th ed. (New Delhi: Springer Int. Student Ed.) p. 334
- Winkler H G F 1976 *Petrogenesis of Metamorphic Rocks*, 5th ed. (Berlin: Springer-Verlag)

Very low frequency electromagnetic prospecting method — A few field applications in India

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Abstract. The very low frequency electromagnetic prospecting equipment measures different space and phase components of the electromagnetic field in the frequency range 15–25 kHz emanating from distant naval radio stations. The field application of the equipment designed and constructed for this purpose has demonstrated the usefulness of the investigations in groundwater and geological mapping. The amplitude measurements necessitate correction of the diurnal variations of the electromagnetic field at the observation point.

Keywords. Elliptic polarization; diurnal variation; groundwater investigations; granitic terrain; very low frequency electromagnetic equipment.

1. Introduction

Very low frequency electromagnetic (VLFEM) method (Sedelnikov *et al* 1971; Pateron and Ronka 1971) utilises electromagnetic fields in the range 15 to 25 kHz radiated by powerful naval radio stations. At sufficiently large distances from the transmitting antenna (distance $R \gg$ wavelength λ) the electromagnetic field near the homogeneous earth can be approximated to a plane wave with the magnetic field horizontal at right angles to the radius vector connecting the transmitting station to the observation point (figure 1). Whenever a conductive geological body, whose strike direction is nearly parallel to the propagation path, is present in an otherwise homogeneous earth, eddy currents are induced in the body giving rise to a secondary magnetic field. This secondary magnetic field will have the same frequency as that of the primary field but lesser in amplitude and differs in direction and phase. The phase of the secondary field will be between $\pi/2$ and π depending upon the product $|Kd|$ where K is the wavenumber given by $K = (\epsilon\mu\omega^2 + j\sigma\mu\omega)^{1/2}$ and d is the characteristic linear dimension of the geological body*. At the observation point, the total magnetic field is elliptically polarized in general.

In VLF prospecting it is the general practice to determine either (a) the different space components of the total magnetic field and the tilt θ (the inclination of the major axis of the ellipse of polarization with the horizontal) in degrees or the tangent of the tilt in per cent or (b) the ellipticity ϵ which is the ratio of the major to minor axes and the tilt θ or (c) the magnetic field intensity in the direction perpendicular to the radius vector (R) and the electrical field component in the earth (by means of grounded electrodes separated by some distance) in the direction of the radius vector, such that the apparent resistivity ρ_a can be computed using the relation

* σ is the conductivity (ohm/m); μ is the magnetic permeability (Henry/m); ϵ is the dielectric constant (Farad/m) and ω is the angular frequency (Radians/sec.).

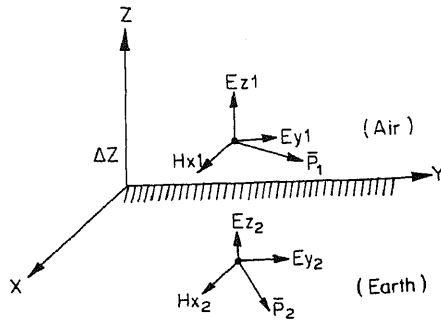


Figure 1. Propagation of VLF EM field.

$$\rho_a = \frac{1}{\omega\mu} \left| \frac{E_Y}{H_X} \right|^2 \quad (1)$$

where E_Y and H_X are the horizontal electric and magnetic field components on the earth's surface respectively.

The VLF method enjoyed popularity all over the world due to the speed of field operation and low overall operation cost. Further, from the existing theory of the EM method and field application in general the following advantages can be attributed to the VLF EM technique.

- The depth of penetration is fairly large (20–50 m for ground resistivities of the order of few hundreds of ohm-m) when compared to those methods which make use of frequencies in the range of 150 kHz to few MHz.
- Owing to the relatively higher frequencies employed when compared to the other EM methods such as SLINGRAM, TURAM, etc., relatively poorer conductors will also contribute to the signal. As a result this technique can be employed in groundwater investigations in hard rock terrain, geological mapping and fresh/salt water interface studies, etc.
- The VLF EM field attenuates slowly with distance from radio stations and are relatively stable in intensities.

However, application of this method in prospecting for very good conductors like polymetallic deposits may pose a problem in view of the fact that one has to identify the required 'signal' from among several anomalies caused by different geological factors in the region.

In India, so far the potential of this technique has not been utilised in groundwater prospecting for rapid coverage of drought prone areas. Instead quite often time consuming electrical resistivity methods are being utilised extensively as the primary methods. Geological mapping is another area where the capabilities of this method are not being fully exploited in India.

This paper is intended to demonstrate the application of the VLF EM equipment designed and constructed in the Department of Geophysics, Osmania University, in geological mapping and groundwater investigations in hard rock terrain by presenting a few field studies in India. A brief theory, equipmental details, field procedure adopted and the detailed discussion of the field results obtained are presented in this paper.

2. Theoretical basis

The EM field of the radio antenna is equivalent to the field produced by a vertical electrical dipole placed on the earth's surface.

Let the transmitting electrical antenna of length Δz carrying an alternating current I with a circular frequency ω be placed at the origin of the spherical coordinate system R, θ, φ . Let the propagation constant for air $K_0 \approx \omega(\epsilon_0 \mu_0)^{1/2}$ and for the homogeneous conductive earth $K = (\epsilon \omega^2 \mu + j \sigma \mu \omega)^{1/2}$. At distances $R \gg \lambda_0$ (λ_0 is the wavelength in air), the electric field may be expressed as (Stratton 1941)

$$E = E_1 + E_2 \quad (2)$$

where

$$E_1 = -(j\omega\mu_0/4\pi) I \Delta z \sin \theta (1 + R_v) [\exp(jKR)/R] a_\theta \quad (3)$$

$$E_2 = (j\omega\mu_0/4\pi) I \Delta z (1 - R_v) F[\exp(jKR)/R] \times \\ \times [a_z + a_r(\sin \theta / Z_{31}^2)(1 + 0.5 \cos^2 \theta)(Z_{31}^2 - \sin^2 \theta)^{1/2}] \quad (4)$$

$$R_v = \frac{Z_{31}^2 \cos \theta - (Z_{31}^2 - \sin^2 \theta)^{1/2}}{Z_{31}^2 \cos \theta + (Z_{31}^2 - \sin^2 \theta)^{1/2}} \quad (5)$$

$$F = [1 + j\sqrt{\pi}W \exp(-W) \operatorname{erfc}(-j\sqrt{W})] \quad (6)$$

$$\operatorname{erfc}(-j\sqrt{W}) = 2/\sqrt{\pi} \int_{-j\sqrt{W}}^{\infty} \exp(-x^2) dx \quad (7)$$

$$W = 0.5 jKR [Z_{31}^2 \cos \theta + (Z_{31}^2 - \sin^2 \theta)^{1/2}]^2 / Z_{31}^4 \quad (8)$$

$$Z_{31}^2 = -\frac{\epsilon}{\epsilon_0} \left(1 + \frac{j\sigma}{\omega\epsilon} \right) \quad (9)$$

a_θ is a unit vector perpendicular to the radius vector R , while a_z and a_r are the unit vectors in a cylindrical coordinate system parallel to the Z and R directions.

It can be shown that, if the conductivity of the earth is finite $R_v \rightarrow -1$ for $\theta \rightarrow \pi/2$ and then $E_1 \rightarrow 0$. In such a case only the E_2 component exists and is termed as the ground wave. Converting into the Descartes coordinate system, it can be shown that only one horizontal magnetic component H_x remains near the earth's surface given by

$$H_x = -\sqrt{\epsilon'/\mu} E_1 \quad (10)$$

where

$$\epsilon' = \epsilon + j\sigma/\omega$$

Thus on measuring the surface impedance $|Z| = |H_X/E_Y|$ and neglecting the influence of the displacement currents the resistivity ρ can be determined.

Further whenever a local conductor is present in the homogeneous earth, the magnetic field is elliptically polarized.

3. Elliptical polarization

Let the linearly polarized primary and secondary magnetic fields be represented by $H_p = H_{p0} \cos \omega t$ and $H_s = H_{s0} \cos(\omega t - \varphi)$ respectively where ϕ is the phase difference between H_p and H_s . Further, let the space angle between H_p and H_s be α . Let the space coordinate system X, Y, Z be oriented such that the primary field H_p is along the OX axis and the direction of propagation is along the Y axis (figure 2) and the Z axis is oriented along the vertical.

Following Svetov (1973) different components of the total magnetic field ($H_T = H_p + H_s$) which is elliptically polarized are given by the horizontal component of the total field

$$H_{TX} = (H_{p0}^2 + H_{s0}^2 \cos^2 \alpha + 2H_{p0} H_{s0} \cos \alpha \cos \varphi)^{1/2} \quad (11)$$

Vertical component of the total field

$$H_{TZ} = H_{s0} \sin \alpha \quad (12)$$

Semi major axis

$$H_a = 0.5[H_{p0}^2 + H_{s0}^2 + 2H_{p0} H_{s0} \cos(\alpha - \varphi)]^{1/2} + [H_{p0}^2 + H_{s0}^2 + 2H_{p0} H_{s0} \cos(\alpha + \varphi)]^{1/2} \quad (13)$$

Semi minor axis

$$H_b = 0.5[H_{p0}^2 + H_{s0}^2 + 2H_{p0} H_{s0} \cos(\alpha - \varphi)]^{1/2} - [H_{p0}^2 + H_{s0}^2 + 2H_{p0} H_{s0} \cos(\alpha + \varphi)]^{1/2} \quad (14)$$

usually $H_s \ll H_p$ and hence

$$H_a/H_p \approx 1 + (H_s/H_p) \cos \varphi \cdot \cos \alpha = 1 + (\text{Re } H_{sx}/H_p) \quad (15)$$

$$H_b/H_p \approx (H_p/H_p) \sin \varphi \cdot \sin \alpha = \text{Im } H_{sz}/H_p \quad (16)$$

tilt of the major axis

$$\theta \approx H_s \cos \varphi \cdot \sin \alpha / H_p = \text{Re } H_{sz}/H_p \quad (17)$$

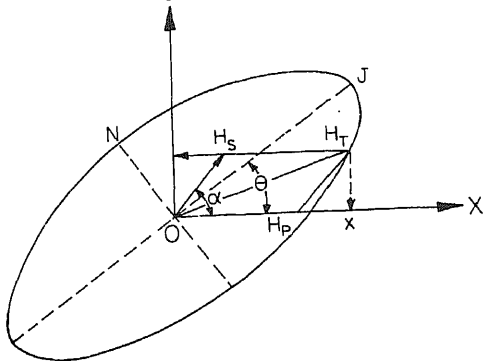


Figure 2. Ellipse of polarization.

From the above relations, it is evident that the profiling curves of the semi major axis are similar to the horizontal component (parallel to the primary field) of the secondary field while its frequency characteristics are similar to the real part of the horizontal component of the secondary field. On the other hand, the profiling curves of the semi minor axis are similar to the vertical component of the secondary field while its frequency characteristics are similar to that of the imaginary part of the vertical component of the secondary field. The tilt angle of the major axis behaves similarly as the real part of the vertical component of the secondary field. Thus by measuring, either the components of the total magnetic field or the polarization ellipse parameters (which are in turn related to the secondary field) it is possible to obtain the geometric and electrical parameters of the local geological objects.

4. Interpretation of VLF EM data

Quantitative interpretation of the VLF EM data is complicated and is not fully developed. However, Fraser (1969) suggested a filter for contouring the VLF tilt angle measurements data which helps in reducing the noise and transforming the true cross-overs to positive peaks. Paterson and Ronka (1971) suggested thumb rules to calculate the depth to the top of the conductor from the distance between the peaks of the real component profiles. For near surface conductors, the depth is over estimated by the thumb rule (Coney 1977). Baker and Myers (1979) conducted modelling investigations on the sheet response due to conductors in a conducting medium and developed interpretation curves. Kaikkonen (1977), Olsson (1980), Sacit Saydam (1981) and Poddar (1981, 1982) presented interpretational schemes for the half plane conductors embedded in a conductive environment. Baker and Myers (1980) and Eberle (1981) presented terrain correction for VLF EM data. Based on the characteristic points, Tarkhov (1961) reported simple relations for determining the depth to vertical conductive vein type bodies and contact between two media differing in electrical parameters.

5. Equipment

The tangent of the tilt of the major axis from the horizontal in per cent (termed as real) and the ellipticity (termed as imaginary or quadrature) in per cent is measured by

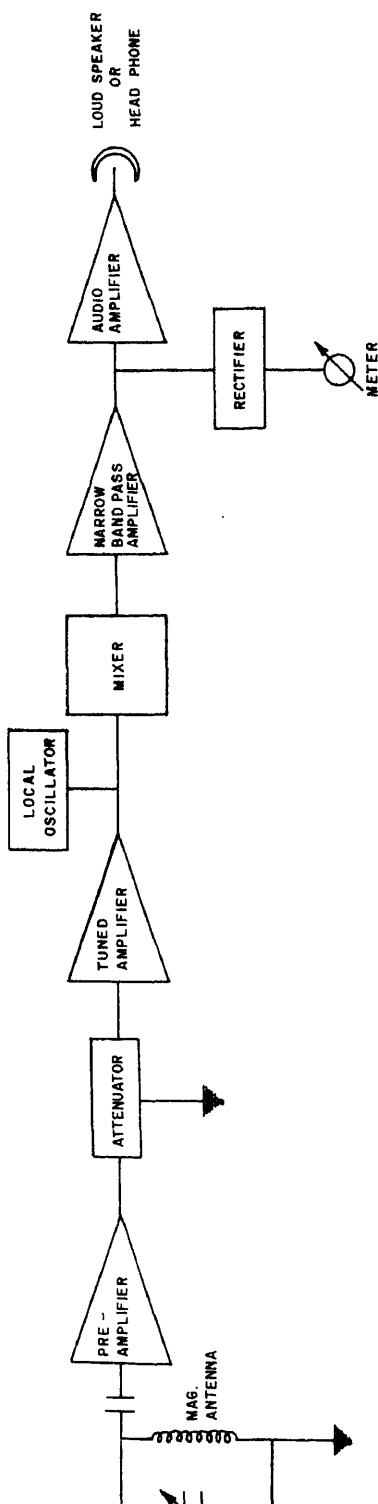


Figure 3. Block diagram of the V.I.F. EM unit.

EM 16 (Geonics) system while EM 16 R (Geonics) measures the impedance of the VLF EM field on the earth's surface. Phoenix system measures the orientation and magnitudes of the major and minor axes of the polarization ellipse. Radem system is employed for measuring the tilt of the major axis in degrees and the ellipticity in per cent. The Scintrex VLF EM equipment measures the amplitudes and geometry parameters of the polarization ellipse. Soviet made VLF EM equipment measures the space components of the total field and the tilt of the major axis of the ellipse of polarization.

The equipment designed by the authors is based on superheterodyne receiver system and is capable of measuring the VLF field emanating from NWC-Australia (22.3 kHz). The block diagram of the equipment is presented in figure 3 and the general view of the entire equipment is presented in figure 4.

The magnetic receiving antenna consists of a ferrite core of 20 cms containing six sections of coils with a sensitivity of 25 Vm/A at 22.3 kHz which can be tuned with the help of a variable condenser provided on the front panel of the equipment. This antenna is rotatable around the vertical and horizontal axes for measuring different space components and the tilt. The signal received by the antenna is fed to a preamplifier stage having a gain of 40 designed on FETBFW-1 I. The output of the amplifier is fed to an attenuator which is incorporated in the emitter circuit of the emitter repeater and provides 30, 100 and 300 μ V full scale ranges for the equipment. The output of the emitter repeater is fed to a tuned amplifier which is designed on IC-741, and has a gain of 50. The local oscillator constructed on LC circuit has a frequency of 26.4 kHz. The

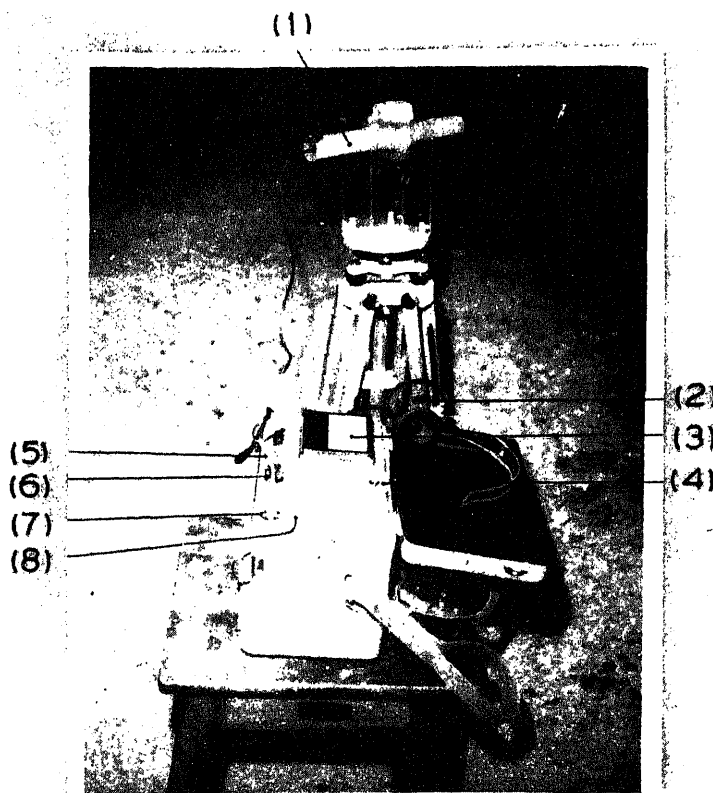


Figure 4. VLF EM equipment.

frequency of the oscillator is stable to 0.1 in 10 for a continuous operation of 3 hrs. The output of the local oscillator and the tuned amplifier are mixed in a mixer stage constructed on a diode whose load is an LC circuit tuned to 4.1 kHz (intermediate frequency). The output of the mixer is fed to an IF tuned amplifier with a gain of 20 having a Q of 3 at IF 4.1 kHz. The signal from the IF amplifier is fed to a fullwave rectifier and then measured by a panel meter. The output of the tuned amplifier is also fed to an audio amplifier, the output of which is given to the head phone.

For measuring the magnetic field components, the magnetic antenna is connected to the input and when the impedance measurements are to be made the grounded electric dipole of length 50 m is connected to the input after measuring the magnetic field component. The selection of the antenna is made by a toggle switch provided on the front panel of the equipment.

6. Field procedure

At every observation point at first, the axis of the magnetic antenna is kept horizontal and rotated around a vertical axis for maximum pick up. While measuring the magnetic field components, the magnetic antenna is tuned with the help of a tuning condenser before taking the observation. The maximum pick up is indicated by a clear loud signal in the earphone and a maximum reading in the panel meter, which is noted down and is termed as $H\rho_{\max}$.

To measure the tilt, the magnetic antenna is rotated around a vertical axis for minimum pick up and then lifted in a vertical plane (rotated around a horizontal axis) for a further minimum signal. The tilt angle is noted from a protractor scale (not shown in figure 4).

For measuring the vertical component H_z the magnetic antenna is kept vertical.

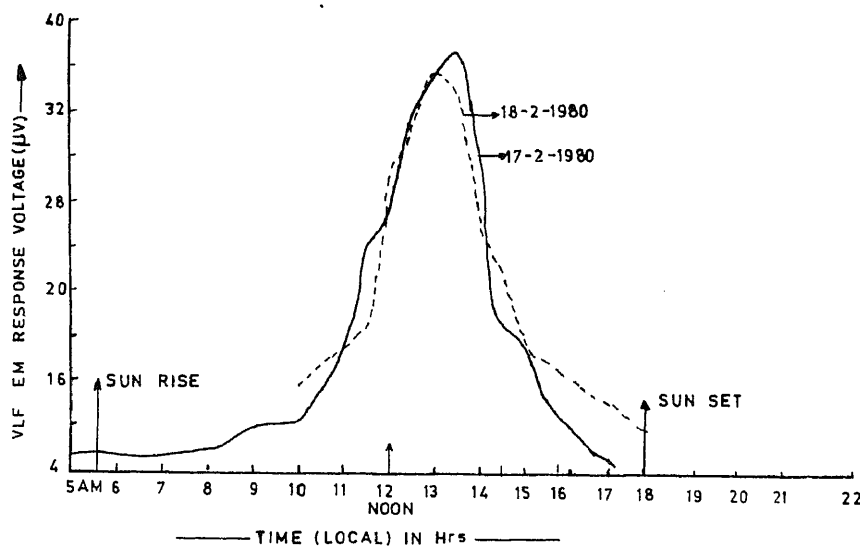


Figure 5. Diurnal variation of VLF EM field from NWC, Australia.

Since we are interested in relative variations of the signal from the base point along a profile, the profiling data in which $H\rho_{\max}$ and H_z are measured can be expressed either in μV or A/m (using a sensitivity value of 25 Vm/A for the magnetic antenna). However, while taking the impedance measurements for computing ρ_a values from (1), the electric component E is expressed in V/m and the magnetic component is expressed in A/m so that the impedance Z has the dimension of ohm.

In practice the intensity measurements have diurnal variations similar to that observed in magnetic methods of prospecting. Figure 5 gives the diurnal variations of $H\rho_{\max}$ observed continuously for two days. It can be noted that the intensity considerably increases from 0930 hrs–1230 hrs local time after which it decreases. For field observations made between 0930 hrs and 1730 hrs local time the diurnal corrections are to be applied by occupying the base station(s) as in the magnetic methods of prospecting and corrected for the variation of intensity with time.

7. Field applications in India

The apparent resistivity ρ_a values computed from (1) from the surface impedance measurements are shown in figure 6. During these measurements the electric component was measured using an electric dipole of length 50 m. Symmetrical Schlumberger resistivity data with a current electrode separation of 10 m is also shown in figure 6. Both the results show a lowering in resistivity between 80–140 m stations. However, this zone is more prominent and localized in the ρ_a value computed from the VLF EM data. Investigations by Schlumberger electrical soundings and borewell (for tapping groundwater) placed in the low resistivity zone revealed a water bearing weathered granite (Patangay 1977).

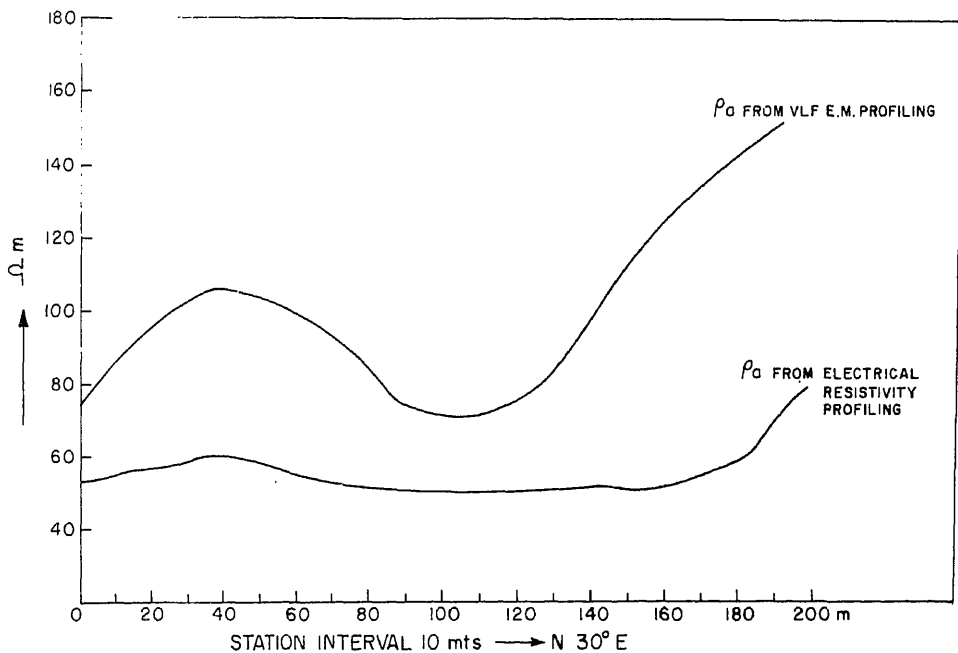


Figure 6. VLFEM and Schlumberger ($AB/2 = 10 \text{ m}$) profiling in groundwater investigations.

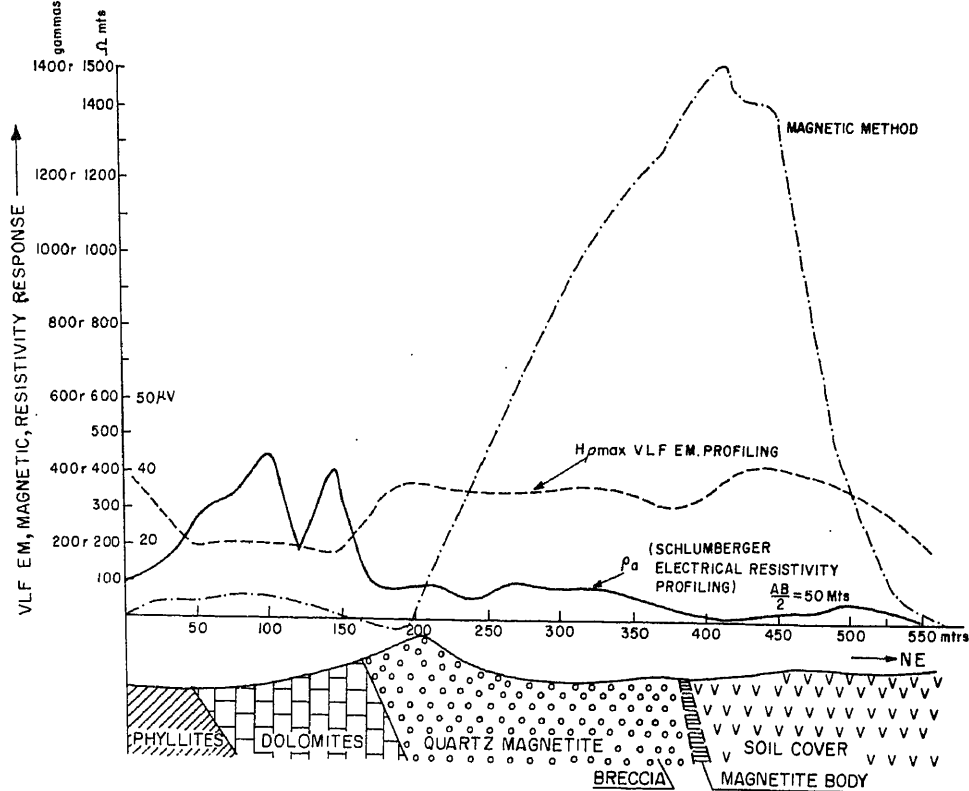


Figure 7. VLF EM Schlumberger resistivity and vertical component magnetic profiles over Shernawala outlier, Khammam, AP.

For estimating the geological mapping capabilities of the VLF equipment the Shernawala outlier, Khammam, AP was selected. In the region metasediments (phyllites, dolomites and quartzites) occurring in a folded pattern have a faulted contact with granites on the western margin and a gradational contact with schists on the eastern side. The presence of faulted breccia and magnetic mineralisation indicated the presence of a nearly NS trending fault within the metasediments (Ramana Rao and Ballurkar 1977; Varaprasada Rao 1977). In figure 7* are shown the results of $H\rho_{\max}$ measurements expressed in μV , Schlumberger resistivity data with a current electrode separation of 100 m and potential electrode separation of 4 m, and vertical component magnetic measurements along a SW-NE profile across the Shernawala outlier. It is interesting to note that resistivity data is reflecting the contacts between the phyllites and dolomites and also the contact between dolomites and quartz magnetites, while the magnetic body and the associated structure are not well demarcated by the resistivity profiling. On the other hand the magnetic measurements show a profound anomaly over the magnetic mineralization, while the quartzite-dolomite contact and the dolomite-phyllite contact are not prominently reflected on the profile. The VLF EM profiling data shows a distinct low over relatively resistive dolomites and a steep gradient over the fault structure containing magnetic mineralization. The expected VLF EM responses over the different geological settings are schematically represented in figure 8 (Khmelovsky 1971).

* The magnetic and resistivity data shown in figure 7 was obtained during a field training programme for an

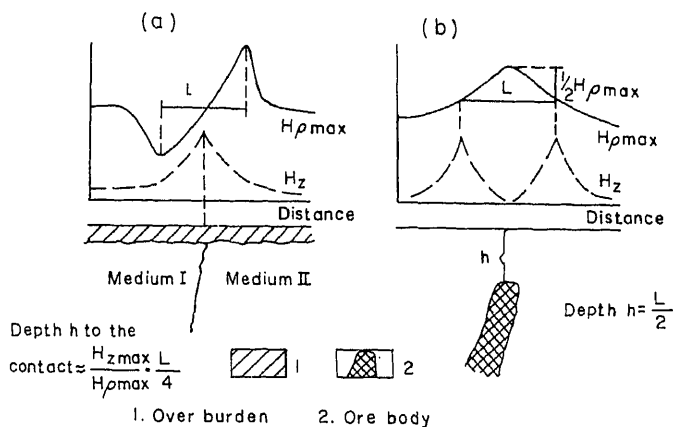


Figure 8. Schematic representation of VLF EM intensity measurements (after Khmelovsky 1971).

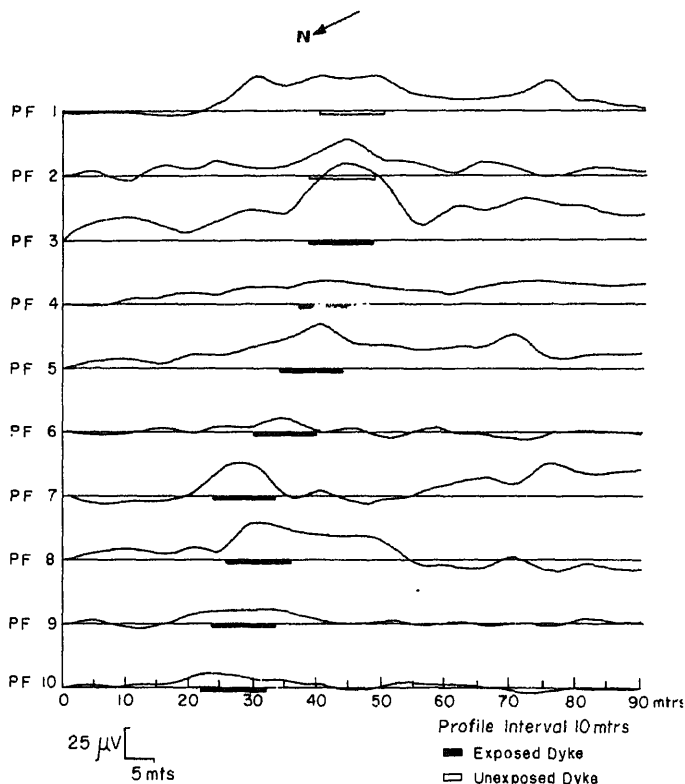


Figure 9. VLF EM results over a dyke near CEG.

The results of VLF EM ($H_{\rho\max}$) profiles across a partly exposed dolerite dyke having a strike of N 60° W in the granitic terrain is shown in figure 9. Ten profiles in N 25° E direction with a profile interval of 10 m were covered with the VLF EM method. A distinct anomaly pattern correlatable along all the 10 profiles are observed over the exposed dyke and also over the unexposed parts.

Usually, when the signal to noise ratio is small or the strike extent is limited to a small number of profiles a visual assessment of the strike direction is not always possible. Under such conditions the cross correlation and stacking techniques (Bhimashankaram *et al* 1973) can be used for effectively enhancing the signal to noise ratio enabling a reliable detection of the strike. Schaub (1963) proposed the use of a modified correlation $K_p^{i,j}$ expressed as a function of the space co-ordinate (Index p) itself, thereby facilitating the exact spatial location of the expected trends. The correlation function $K_p^{i,j}$ can be expressed as

$$K_p^{i,j} = \frac{1}{2n + 1} \sum_{l=-n}^n f_{i,p+l} \cdot f_{j,p-l} \quad (18)$$

By convention the $K_p^{i,j}$ function is referred in space to the p th point of a parallel profile midway between profiles i and j and has certain advantages over other correlation methods such as resolution of multiple trends and is capable of distinguishing between correlation of even and odd shaped anomalies by its positive and negative peaks.

The modified correlation function computed (choosing $n = 1$) for the data presented in figure 9 is shown in figure 10. It is evident from figure 10 that the computed $K_p^{i,j}$ function helps in demarcating the dyke. Such investigations have an application in groundwater exploration in the hard rock terrain.

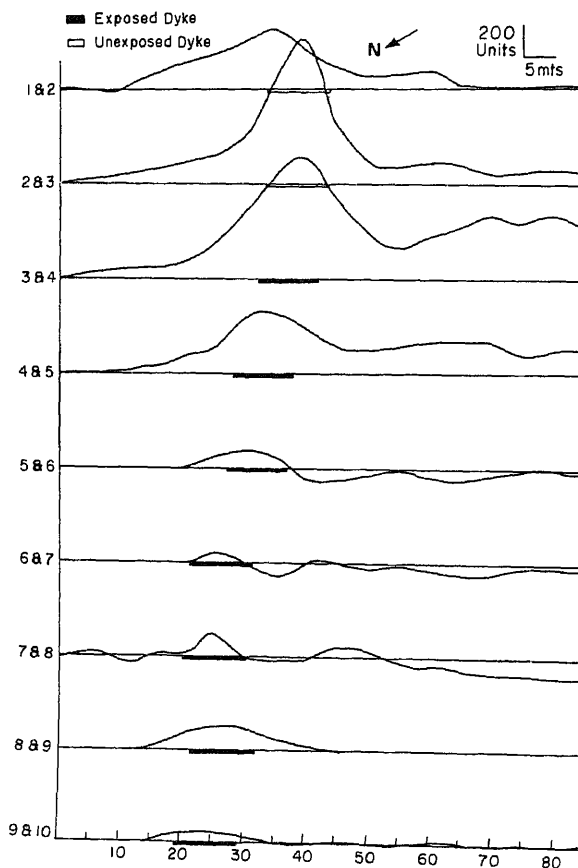


Figure 10. Schaub's modified correlation for VLF data over a dyke near CEG.

- (a) The VLF EM fields emanating from NWC-Australia can be utilised for fast exploration of fairly resistive ground for (i) geological mapping and (ii) groundwater investigations. These are demonstrated by the field experiments in India utilising the equipment designed and developed by the authors.
- (b) Since the diurnal variations are very prominent the intensity measurements in VLF EM prospecting should be properly corrected for the diurnal variations.

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References

- Bhimasankaram V L S, Tarkhov A G, Nikitin A A and Seshagiri Rao S V S¹ 1973 *Geophys. Prospect.* **21** 464-471
- Baker H A and Myers J O 1979 *Geoexploration* **17** 55-63
- Baker H A and Myers J O 1980 *Geoexploration* **18** 135-144
- Coney D P 1977 *Geoexploration* **15** 19-35
- Eberle D 1981 *Geoexploration* **19** 103-114
- Fraser D C 1969 *Geophysics* **34** 958-967
- Kaikkonen P 1977 *J. Geophys.* **43** 179-192
- Khmelovsky V K 1971 *Basic course of electrical prospecting* (in Russian) Part II (Moscow: University Publication) p. 270
- Olsson O 1980 *Geophys. Prospect.* **28** 415-434
- Patangay N S 1977 *Lectures on Geophysics for Geologists and Engineers* (ed.) V L S Bhimasankaram and V K Gaur (Hyderabad: A E G Publication)
- Paterson N R and Ronka V 1971 *Geoexploration* **9** 7-26
- Poddar M 1981 *Pure & Appl. Geophys.* **119** 1024-1036
- Poddar M 1982 *Geophysics* **47** 1059-1067
- Ramana Rao N and Ballurkar A 1977 *Met. Miner. Rev.* **16** 10-13
- Sacit Saydam A 1981 *Geophysics* **46** 1594-1605
- Schaub Yu I 1963 *Bull. Izvestia Acad. Sci. USSR Geophysics series No. 42*
- Sedelnikov E S, Gridchin V M, Gardev S G, Lunin Yu G and Rogachev B V 1971 Method of long wavelength radiokip with SDVR 3 equipment (in Russian) Tchinigri Moscow
- Stratton J A 1941 *Electromagnetic theory* (New York: McGraw Hill) p. 587
- Svetov B S 1973 *Theory, methodology and interpretation of low frequency induction prospecting data* (in Russian) (Moscow: Nedra) p. 247
- Tarkhov A G 1961 *Basis of Geophysical prospecting by radiokip method* (in Russian) Moscow p. 209
- Varaprasada Rao S M 1977 *Integrated Geophysical investigations over Shernavala outlier, Khammam district, A P* Ph.D. Thesis, Osmania University (unpublished)

Interpretation of gravity anomalies due to some two-dimensional structures: A Hilbert transform technique

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Abstract. An interpretation procedure is formulated using the Hilbert transform for analysing the gravity effect of (a) two-dimensional horizontal circular cylinder, (b) semi-infinite thin fault block, and (c) a geologic contact. In all three cases the abscissa of the point of intersection of the gravity anomaly curve or the horizontal derivative curve and its Hilbert transform yields directly the depth of the body. The proposed method is tested on theoretical models. Also, the method is applied to the field data of a geologic contact taken over the Himalayan foothills across the Brahmaputra valley.

Keywords. Hilbert transform; gravity; interpretation; two-dimensional structures.

1. Introduction

In recent years, the Hilbert transform techniques have acquired importance in the quantitative interpretation of gravity and magnetic anomalies (Nabighian 1972; Stanley and Green 1976; Stanley 1977; Mohan *et al* 1982; Sundararajan *et al* 1982). In the present paper an interpretation procedure is developed using Hilbert transforms for evaluating the body parameters of two dimensional horizontal circular cylinder, semi-infinite thin fault block and a geologic contact. An interesting feature observed is that the point of intersection of the curves of the gravity anomaly $g(x)$ of any body, and its Hilbert transform $H(x)$, gives the depth of the body. Further the amplitude of $g(x)$ and $H(x)$, is always maximum over the origin.

2. Hilbert transform

The Fourier transform of the gravity anomaly $g(x)$ of the body is given by

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} g(x) \exp(-i\omega x) \cdot dx, \\ &= \text{Re } F(\omega) + i \text{Im } F(\omega), \end{aligned} \quad (1)$$

where ω is the spatial frequency in radians per unit length and $\text{Re } F(\omega)$ and $\text{Im } F(\omega)$ are the real and imaginary components of the Fourier transform.

The Hilbert transform of $g(x)$ is defined as (Thomas 1969)

$$H(x) = \frac{1}{\pi} \int_0^{\infty} [\text{Im } F(\omega) \cos \omega x - \text{Re } F(\omega) \sin \omega x] d\omega. \quad (2)$$

The Hilbert transform of the field data, which is always discrete is computed by making use of the discrete Fourier transform (DFT) and the discrete Hilbert transform (DHT).

The components of DFT are (Gold and Rader 1969)

$$\operatorname{Re} F(n\omega_0) = \sum_{l=0}^{N-1} g(l \cdot \Delta x) \cos(n\omega_0 l \cdot \Delta x), \quad (3)$$

$$\operatorname{Im} F(n\omega_0) = \sum_{l=0}^{N-1} g(l \cdot \Delta x) \sin(n\omega_0 l \cdot \Delta x). \quad (4)$$

Then the discrete Hilbert transform is defined as:

$$H(l \cdot \Delta x) = \frac{1}{\pi} \left[\sum_{n=0}^{\frac{N}{2}-1} \operatorname{Im} F(n\omega_0) \cdot \cos(n\omega_0 l \cdot \Delta x) - \sum_{n=0}^{\frac{N}{2}-1} \operatorname{Re} F(n\omega_0) \cdot \sin(n\omega_0 l \cdot \Delta x) \right] \quad (5)$$

where Δx = sample interval; N = total number of samples and

$$\omega_0 = 2\pi / (N \cdot \Delta x)$$

2.1 Location of origin

The abscissa where the amplitude curve

$$A(x) = \{[g(x)]^2 + [H(x)]^2\}^{1/2} \quad (6)$$

attains maximum gives the origin (Nabighian 1972).

3. Horizontal circular cylinder

The gravity effect due to a horizontal circular cylinder (figure 1a) is given by

$$g_1(x) = 2\pi^2 GR^2\sigma \frac{D}{D^2 + x^2} \quad (7)$$

where G is the universal gravitational constant, σ is the density contrast, D is the depth of the centre of the cylinder and R is the radius of the cylinder. The real and imaginary components of Fourier transform of $g_1(x)$ are given as:

$$\operatorname{Re} F(\omega) = 2\pi^2 GR^2\sigma \exp(-\omega D), \quad (8)$$

$$\operatorname{Im} F(\omega) = 0. \quad (9)$$

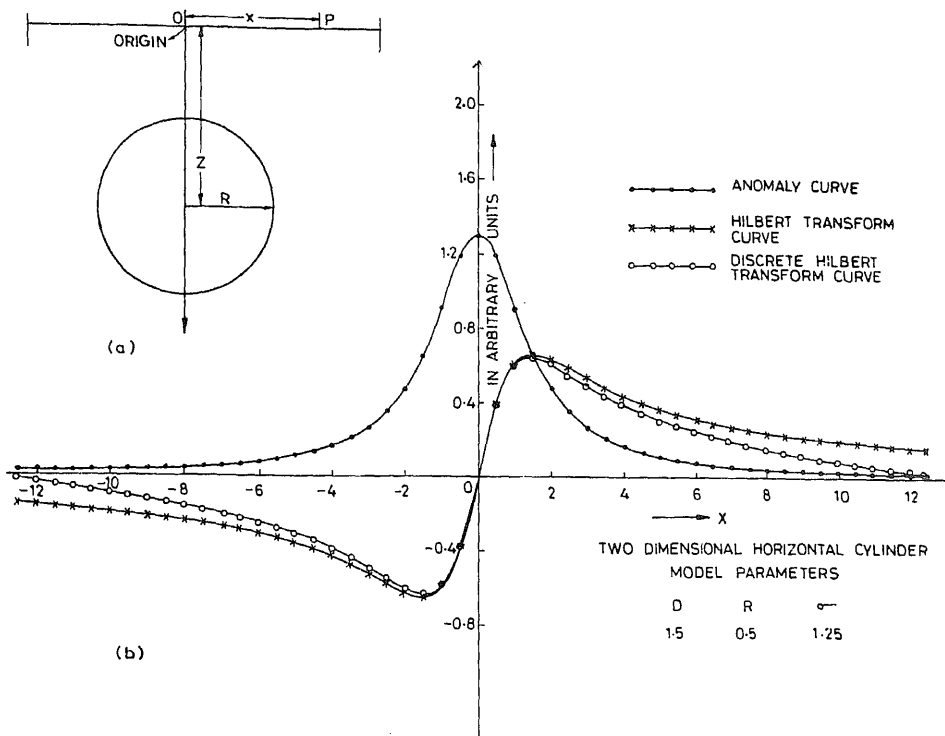


Figure 1. Computed gravity effect the Hilbert transform and the discrete Hilbert transform of the horizontal circular cylinder.

Using (8) and (9) in (2), it is obtained as

$$H(x) = 2\pi^2 GR^2 \sigma \int_0^{\infty} \exp(-\omega D) \sin \omega x \cdot d\omega.$$

On integrating, we get

$$H_1(x) = 2\pi GR^2 \sigma \frac{x}{D^2 + x^2} \quad (10)$$

The gravity effect due to a horizontal circular cylinder $g_1(x)$ and its Hilbert transform $H_1(x)$ are shown in figures 2 and 3. The two curves intersect at a point $X = X_1$, hence

$$g_1(x_1) = H_1(x_1) \text{ yields,} \quad (11)$$

$$D = x_1.$$

Therefore, the depth to the centre of the cylinder is the abscissa of the point of intersection of $g_1(x)$ and $H_1(x)$. It is conspicuous that the magnitude of D remains unaltered even if we consider the Hilbert transform with the minus sign.

Also, squaring and adding $g_1(x)$ and $H_1(x)$ at $x = 0$ we get,

$$R^2 \sigma = \frac{D}{2} \{[g_1(0)]^2 + [H_1(0)]^2\}^{1/2} \quad (12)$$

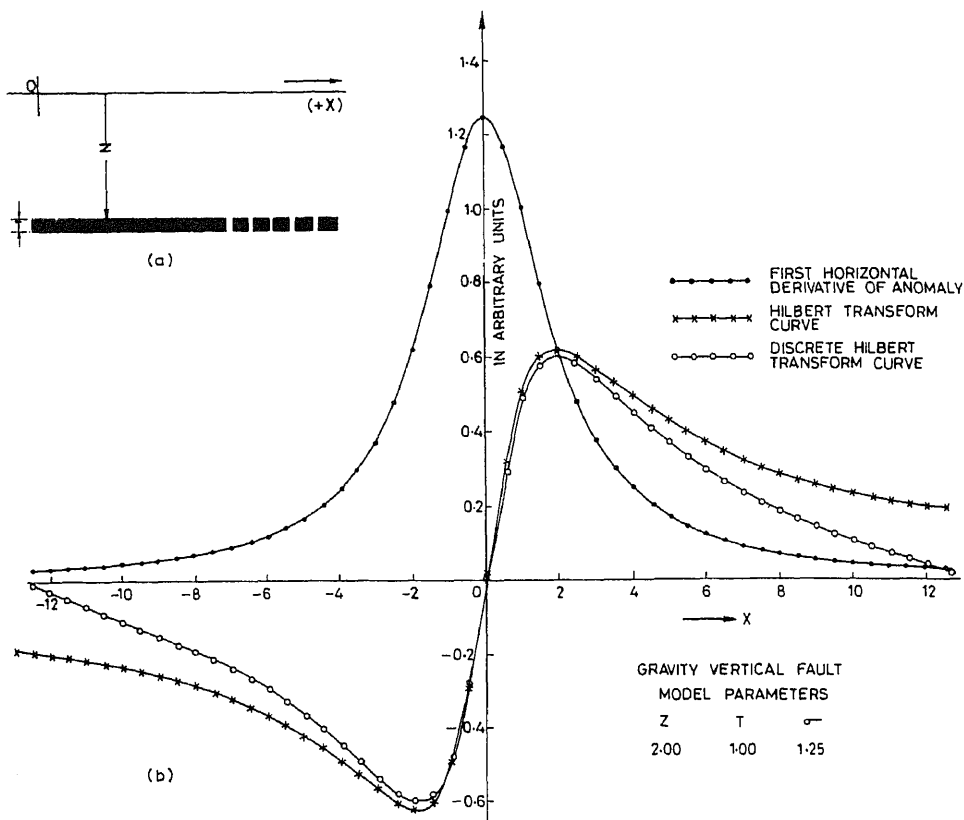


Figure 2. Computed first horizontal derivative of the Hilbert transform and the discrete Hilbert transform of the semi-infinite thin fault block.

Table 1. Theoretical models of horizontal circular cylinder, semi infinite thin fault block and geological contact.

Model	Parameters	Assumed	Evaluated
Cylinder	D	1.5	1.5
	R	0.5	0.5
Semi infinite Thin fault block	Z	2.00	2.05
	T	1.00	1.03
Geologic contact	h	0.5	0.5
	θ	110°.00	110° 30'
	σ	1.00	0.97

Note: Except θ other parameters in arbitrary units.

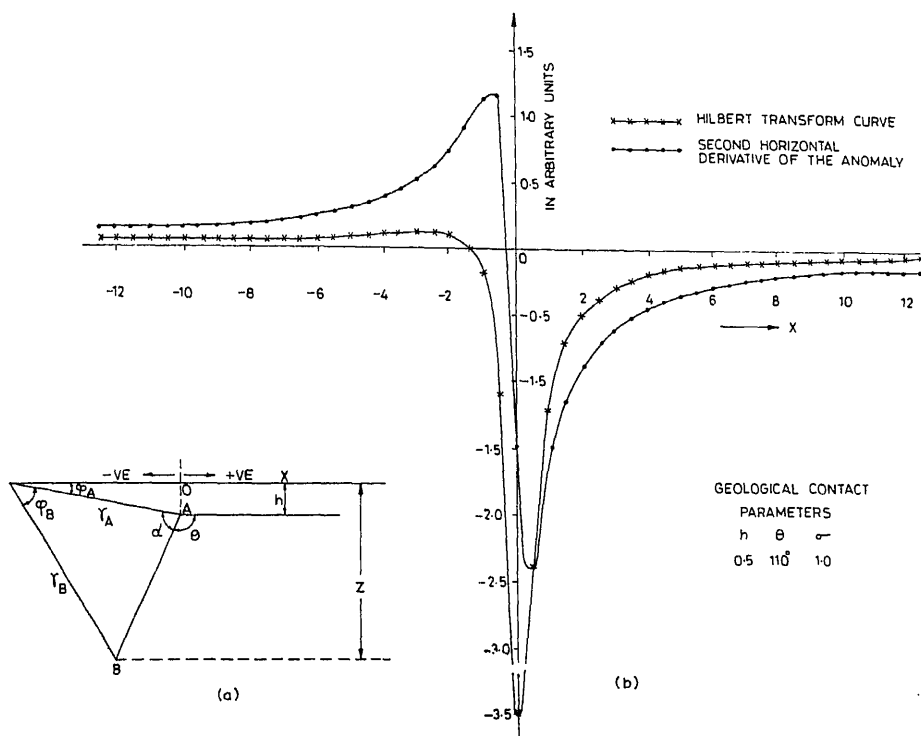


Figure 3. Computed second horizontal derivative of the geologic contact and its Hilbert transform.

From (12), either R or σ is calculated by assuming the other.

Figure 1(b) shows an example (table 1) of the gravity effect $g_1(x)$ of the cylinder and its Hilbert transform $H_1(x)$. It is observed that $g_1(x)$ and $H_1(x)$ intersect at a point, which yields directly the depth to the centre of the cylinder.

In practice, we get the field data in discrete form, and hence the discrete Hilbert transform of $g_1(x)$ is computed by using (5) as shown in figure 1(b). It is observed that the discrete Hilbert transform curve intersects the anomaly curve nearly at the same point as that of the theoretical Hilbert transform and the anomaly curve. Based on the analysis given earlier, the parameters are calculated as shown in table 1.

4. A semi-infinite thin fault block

The gravity effect due to semi-infinite thin fault block (figure 2a) is expressed as

$$g_2(x) = 2G\sigma T(\pi/2 + \tan^{-1}x/Z), \quad (13)$$

where G and σ stand as in $g_1(x)$, Z the depth to the thin plate and T the thickness of the plate.

For the purpose of interpretation it is however convenient to use the first horizontal derivatives.

Differentiating (13) with respect to x we get

$$g'_2(x) = 2G\sigma T \frac{Z}{Z^2 + x^2} \quad (14)$$

$$\operatorname{Re} F(\omega) = 2G\sigma T \cdot \pi \exp(-\omega Z), \quad (15)$$

$$\operatorname{Im} F(\omega) = 0. \quad (16)$$

Substituting (15) and (16) in (2) and then integrating the Hilbert transform is obtained as:

$$H_2(x) = 2G\sigma T \frac{x}{Z^2 + x^2} \quad (17)$$

The first horizontal derivative of the gravity effect due to a semi-infinite thin fault block $g'_2(x)$ and its Hilbert transform $H_2(x)$ are shown in figures 4 and 5. It is observed that these two curves intersect at $x = x_1$, which is the depth to the fault block. Thus

$$g'_2(x_1) = H_2(x_1) \text{ yields } Z = x_1 \quad (18)$$

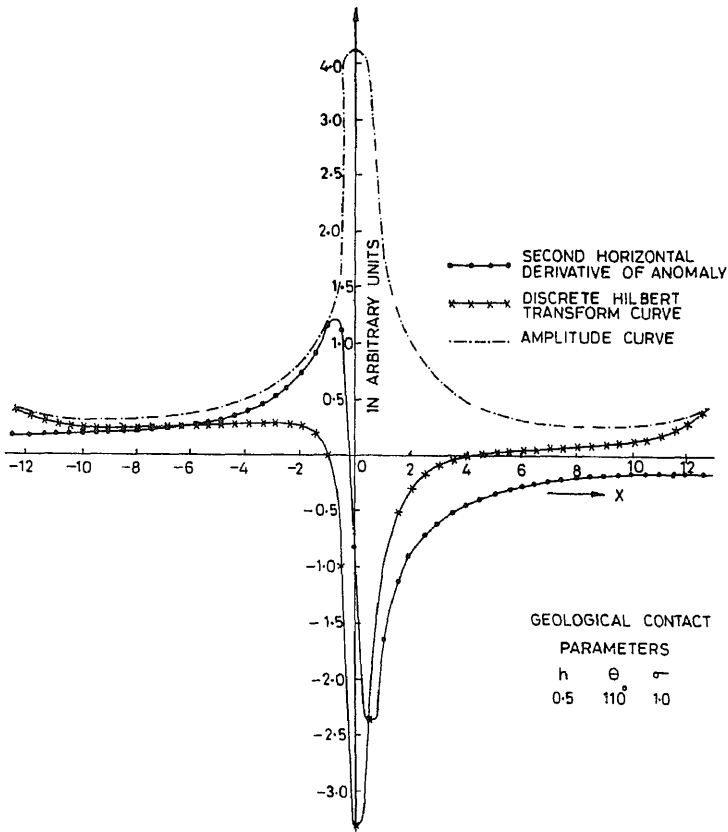


Figure 4. Computed second horizontal derivative of the geologic contact, its discrete Hilbert transform and the amplitude of the second horizontal derivative and its DHT.

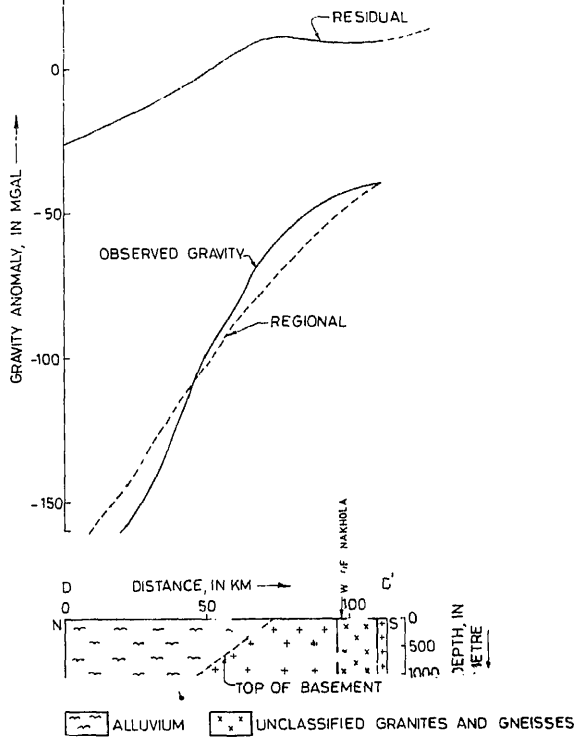


Figure 5. The gravity anomaly due to a geologic contact taken over the Himalayan foothills across the Brahmaputra valley.

Also, squaring and adding $g'_2(x)$ and $H_2(x)$ at $x = 0$, we get

$$T\sigma = Z\{[g'_2(0)]^2 + [H_2(0)]^2\}^{1/2} / 2\pi G. \quad (19)$$

From (19) either σ or T is calculated by assuming the other.

The method is studied by considering a theoretical model (table 1). Using (14) and (17) the horizontal derivative of the gravity effect $g'_2(x)$ of the fault block and its Hilbert transform $H_2(x)$ is computed and shown in figure 2b. Also, $g'_2(x)$ is digitized for generating the discrete data. Using (5), the discrete Hilbert transform of the digitized data is computed and shown in figure 2b. It is observed that the Hilbert transform and the discrete Hilbert transform curves intersect the anomaly curve at the same point which directly gives the depth to the body. The thickness of the body is calculated using (19). The results are presented in table 1.

5. Geologic contact

The geometry of the geologic contact is shown in figure 3a. The gravity effect over the geologic contact is expressed as (Jung 1961)

$$- \cos \theta (\phi_B - \phi_A)] + (Z\phi_B - h\phi_A)\}, \quad (20)$$

where the various parameters are explained in figure 3a.

In this case it is convenient to use the second derivatives in the analysis. The second derivative of $g_3(x)$ is obtained as are obtained by differentiating (20) with respect to x twice,

$$g_3''(x) = 2G\sigma \sin \theta \left(\frac{h \cos \theta - x \sin \theta}{r_A^2} - \frac{(2Z - h) \cos \theta - x \sin \theta}{r_B^2} \right) \quad (21)$$

For the case when $Z \gg h$ and $r_B \gg r_A$ the above equation reduces to,

$$g_3''(x) = \left(\frac{2G\sigma \sin \theta (h \cos \theta - x \sin \theta)}{r_A^2} \right) \quad (22)$$

The Hilbert transform of $g_3''(x)$ is obtained as:

$$h_3(x) = 2G\sigma \sin \theta \left(\frac{x \cos \theta - h \sin \theta}{x^2 + h^2} \right) \quad (23)$$

$g_3''(x)$ and $H_3(x)$ intersect at x_1 given by

$$g_3''(x) = H_3(x) \quad \text{yields} \quad h = x_1. \quad (24)$$

The dip of the geologic contact is obtained as follows:

$$\theta = \tan^{-1} \left(\frac{x g_3''(x) - h H_3(x)}{h g_3''(x) - x H_3(x)} \right) \quad (25)$$

Putting $x = 0$ in (22) and (23) and solving we obtain an expression for σ given by

$$\sigma = \frac{h}{2G \sin \theta} \{ [g_3''(0)]^2 + [H_3(0)]^2 \}^{1/2} \quad (26)$$

The theory is demonstrated by considering an example (table 1). Using (22) and (23) $g_3''(x)$, $H_3(x)$ are computed and shown in figure 3b.

Using (5) and (6) the discrete Hilbert transform and amplitude are computed and shown in figure 4 and the parameters are evaluated and given in table 1.

It may be recalled that the curve of the discrete variable could never totally coincide with that of the continuous variable of the same function. This inherent theoretical concept is responsible for the departure of DHT curves from the theoretical curves in the above examples.

6. Field example

The Hilbert transform technique developed here is demonstrated in field data over a geologic contact (figure 5). This profile runs from the northern edge of the Shillong Plateau to the Himalayan foothills across the Brahmaputra valley (Verma and Mukhopadhyay 1976). The length of the profile considered is 30 km and it is digitized at an interval of 1.25 km. The second horizontal derivative of the anomaly, the discrete Hilbert transform and the amplitude are shown in figure 6. The abscissa of the point of intersection of the second horizontal derivative of the gravity anomaly and its discrete Hilbert transform gives the depth to the upper surface of the contact. The dip and the density contrast are calculated using (25) and (26). The results are given in table 2.

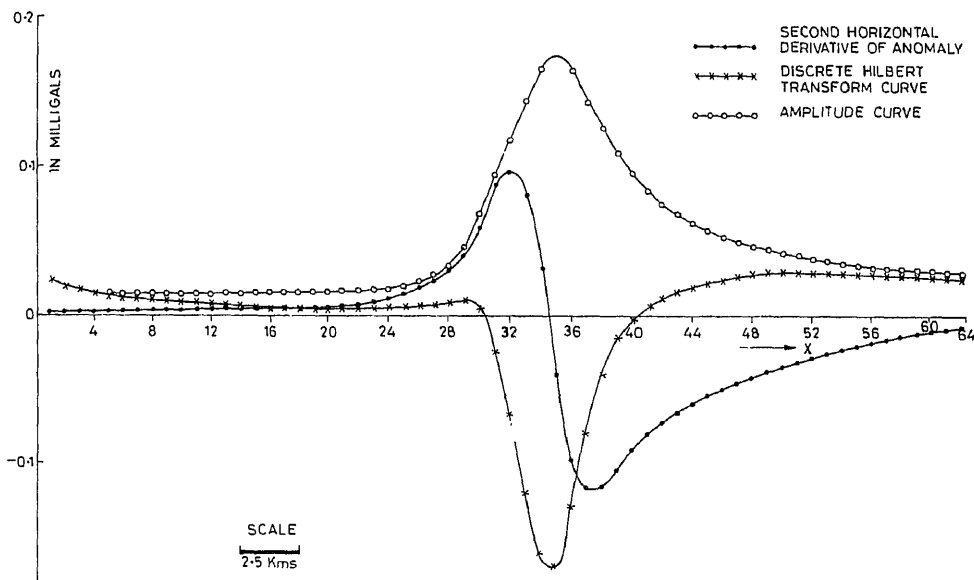


Figure 6. The second horizontal derivative of the anomaly, the discrete Hilbert transform and their amplitude.

Table 2. Evaluated parameters from the gravity data across the Brahmaputra valley.

Parameters	Z (km)	θ (deg.)	σ (g/cc)
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Evaluated

Acknowledgements

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References

- Gold B and Rader C M 1969 *Digital processing of signals* (New York: McGraw Hill)
- Jung K 1961 *Geophysik Geist und Portie*
- Mohan N L, Sundararajan N and Seshagiri Rao S V 1982 *Geophysics* **47** 376
- Nabighian I N 1972 *Geophysics* **37** 507
- Stanley J M 1977 *Geophysics* **42** 1230
- Stanley J M and Green R 1976 *Geophysics* **41** 1276
- Sundararajan N, Mohan N L and Seshagiri Rao S V 1982 *J. Geophys.* (in press)
- Thomas J B 1969 *An introduction to statistical communication theory* (New York: John Wiley) p. 639
- Verma R K and Mukhopadhyay M 1976 *Natl. Geophys. Res. Inst. Bull.* **14** 7

Synthetic P wave seismograms from Longshot, Milrow and Cannikin explosions

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Abstract. The effect of spalling and other transfer functions is considered on the amplitude spectra and waveforms of teleseismic short period P wave seismograms generated due to Longshot, Milrow and Cannikin underground nuclear explosions. Source models of Haskell and Seggern were used for computing P wave seismograms from these events. Step-by-step variations of the amplitude spectra and waveforms of P wave signal as it propagates from source to various seismometer array stations show that the source function, the source crust transfer function, the mantle transfer function and the spalling function are the most important factors that influence shaping of spectra and waveforms of P seismograms. Comparison of computed and observed P wave seismograms at various seismometer array stations for Longshot explosion shows that computed seismograms do reproduce many of the features of observed seismograms. The results also support the conclusion that overburden P wave velocity inferred from observed seismograms is less as compared to its value obtained from on-site measurements.

Keywords. Seismograms; signal; underground nuclear explosions; earthquakes; waveforms; reduced displacement potential; source function; source crust; receiver crust; anelasticity; yield; composite transfer function; source depth.

1. Introduction

Various investigators have reported differences in the amplitude spectra of P waves from underground nuclear explosions and earthquakes having nearly same body wave magnitudes and from same tectonic regions (Wyss *et al* 1971; Bakun and Johnson 1971; Molnar 1971; Hasegawa 1972, 1973). Some investigators have attributed spectral differences from explosions and earthquakes to the differences in source dimensions of explosions and earthquakes (Carpenter and Thirlaway 1966; Liebermann and Pomeroy 1969, 1970), while others attributed these differences to source dimensions as well as the source time functions (Wyss *et al* 1971; Hasegawa 1972, 1973; McEvilly and Peppin 1972). On the other hand, Tsai and Aki (1971) and Helmberger and Harkrider (1972) pointed out that spectral differences between underground nuclear explosions and earthquakes can be explained by focal depth effect itself.

For earthquakes, the source time function is usually considered to be a step (Ben-Menahem 1967; Tsai and Aki 1970) or approximately a step with a finite rise time (Ben-Menahem and Toksoz 1963). However, at present, there is no complete agreement among different investigators about the shape of the reduced displacement potential of underground nuclear explosions (Aki *et al* 1974; Murphy 1977) and many assumptions are made in the literature (Muller 1973). Banghar (1980a, 1981) showed that delta and heaviside types of reduced displacement potentials are not suitable as a source function for underground nuclear explosions, because these functions do not predict the increase in period of P wave seismograms with the increase in yield. Kogeus

explosions, using numerical values of reduced displacement given by Werth and Herbst (1963) and Holzer (1966). Haskell (1967) presented analytic expression for reduced displacement potential and the same was employed by Hasegawa (1971, 1972, 1973) and Douglas *et al* (1972) for computing P and Rayleigh waves seismograms. Seggern and Blandford (1972) presented alternate analytic expression for reduced displacement potential. This expression was used only recently (Burdick and Helberger 1979) for computing teleseismic body waves. However, synthetic seismograms were not compared for Haskell and Seggern source models. Hence, it is desirable to compute and compare short period P seismograms for both of these models.

Several explanations were proposed to explain the long duration of P wave coda in seismograms (because explosion is relatively a short-time phenomenon) from explosive source. One of the possible explanations for long duration of P waves from nuclear explosions is spall-closure (Frasier 1972; Springer 1974), and spalling is observed for many underground explosions. Although several investigators have synthesized P wave seismograms for underground nuclear explosions, a majority of them have not considered the spalling effect (Nair 1977). Hence, it is pertinent to compute P wave seismograms for various shapes of reduced displacement potential for underground explosions, while taking into account the effect of spalling.

2. Theory

We wish to compare the synthesized and recorded short period P wave seismograms from underground nuclear explosions at Eskadalemuir (EKA), Gauribidanur (GBA), Warramunga (WRA) and Yellowknife (YKA) seismometer array stations. Seismogram can be viewed as the final output of a sequence of filters arranged in a cascade, the input to which is the signal generated at the source. Hence, in order to compute seismograms, it is essential to know the transfer functions (impulse responses) of various filters which constitute earth-seismometer system together with the transfer function (impulse response) of source function. The theories of transfer functions of the constituents of the model needed for synthesizing P wave seismograms have been reported earlier by their respective originators (Banghar 1980a). Therefore, only final forms of transfer functions are presented here.

2.1 Source function

Source function for Haskell's formulation of reduced displacement potential is represented as (Banghar 1980a)

$$u(R, \omega) = \frac{\psi(\infty)}{\alpha R} \left[\frac{1 + 24B}{[1 + i(\omega/k)]^4} - \frac{24B}{[1 + i(\omega/k)]^5} \right] \quad (1)$$

where $\psi(\infty)$ is the value of ψ at large times, R is the radial distance, α is the compressional velocity of the medium in which explosion is detonated and ω is the angular frequency. B and k are constants. Numerical values of B and k (Haskell 1967) are available for five kiloton explosions detonated in four geological media, however, source function can be computed for any yield using scaling laws (Banghar 1980a). This expression is subsequently referred to as Haskell model in this paper. Equation (1) shows that far-field displacement spectra behaves as ω^{-4} at high frequencies. This fall-off indicates a high frequency scaling of displacement spectrum inversely proportional to the cube root of yield (Seggern and Lambert 1970). This frequency scaling of displacement spectrum at high frequencies seems to be unrealistic (Aki *et al* 1974).

In order to avoid ω^{-4} dependence of displacement spectra at high frequencies, Seggern and Blandford (1972) presented alternate analytic expression for reduced displacement potential. Using analytic expression for reduced displacement potential of Seggern and Blandford (1972), it can be shown (Banghar 1980a, 1981) that

$$u(R, \omega) = \frac{\psi(\infty)}{\alpha R} \left[\frac{1 + 2B}{[1 + i(\omega/k)]^2} - \frac{2B}{[1 + i(\omega/k)]^3} \right] \quad (2)$$

The meaning of various symbols in (2) are same as in (1); however, the values of B and k in (2) are different from their values in (1). Numerical values of B and k in (2) for five kiloton explosions in four different geological media are given by Banghar (1980a). Source function represented by (2) is referred to as the Seggern model in this paper.

2.2 Source crust transfer function

For an underground explosion, the source crust transfer function is represented by (Fuchs 1966; Banghar 1980a, 1981)

$$T(\omega) = 2i\rho_n\alpha_n^2 \sin i_n^s \cos i_n^s \left[\bar{\Delta}_H - \frac{\bar{\Delta}_V'}{k} \right] \quad (3)$$

where ρ_n , α_n and i_n^s are respectively the density, P wave velocity and angle of incidence of P waves into mantle in source region, k is the horizontal component of wave number, and is related to the apparent velocity c by a relation $\omega = ck$, $\bar{\Delta}_H$ and $\bar{\Delta}_V'$ represent the P wave contributions for the horizontal and vertical point forces.

2.3 Mantle transfer function

The effect of mantle on seismic waves that propagate through it can be accounted for by two factors. One of these factors represents the effect due to anelasticity of the mantle and is described in this section. The second factor representing the variation of the amplitude of seismic waves due to geometrical spreading is related to variation of velocity as a function of depth. This geometrical spreading factor is described separately subsequently.

The mantle is assumed to be homogeneous and anelastic. It is further assumed that absorption mechanism is linear and the quality factor Q is independent of frequency. Following Futterman (1962) and Banghar (1982), the mantle transfer function can be represented as

$$M(\omega) = \exp \omega \left[- \frac{T^*}{2} (1 - \exp(-\omega/\omega_0)) + i \left(T_p - \frac{T^*}{\pi} L_n(\gamma\omega/\omega_0) \right) \right] \quad (4)$$

where ω_0 is low cut-off angular frequency demanded by theory, and $L_n\gamma$ equals 0.5772157. The ω_0 value is not critical, provided it is not zero and is sufficiently lower than the lowest angular frequency taken into account (Kogeus 1968). It is generally taken to be 10^{-3} sec^{-1} . The parameter T^* is defined by $T^* = T_p/Q$, where T_p is the travel time of P waves in the mantle.

2.4 Geometrical spreading factor

This factor (GSF) is represented (Carpenter 1966) by

$$\text{GSF} = \alpha_n (\alpha_n \rho_n / \alpha_n^0 \rho_n^0)^{1/2} G(\Delta), \quad (5)$$

where

$$G(\Delta) = \frac{1}{R_e^2} \left[\frac{1}{\sin \Delta} \frac{dT_p}{d\Delta} \frac{d^2 T_p}{d\Delta^2} \right]^{1/2} \quad (6)$$

and α_n^0, ρ_n^0 are the P wave velocity and density in the mantle (Douglas *et al* 1972) below the station. R_e is the radius of the earth, Δ is the epicentral distance.

2.5 Receiver crust transfer functions

The crustal transfer functions for a plane P wave incident from below on plane parallel layers were presented by Haskell (1953, 1962). In the notation of Banghar (1982), the horizontal (U_H) and the vertical (U_V) transfer functions of the receiver crust are

$$U_H = 2(J_{42} - J_{32}) / D \sin i_n^R, \quad (7)$$

$$U_V = 2(J_{41} - J_{31}) / D \sin i_n^R, \quad (8)$$

where

$$D = (J_{11} - J_{21})(J_{32} - J_{42}) - (J_{12} - J_{22})(J_{31} - J_{41}) \quad (9)$$

and J_{11}, J_{12} , etc. are the elements of crustal matrix J . i_n^R is the angle of incidence of P waves at the base of crust in the region of receiver.

2.6 Seismometer transfer function

We wish to compute seismograms to be recorded by seismometer array stations of the United Kingdom Atomic Energy Authority (UKAEA). The recording systems of these stations are defined by number of zeros (at the origin) and poles (Mowat and Burch 1974). The transfer function $I(\omega)$ for these stations is represented (Banghar 1980a) as

$$I(\omega) = z^n / (z - \gamma_1) \dots (z - \gamma_m), \quad (10)$$

where $z = i\omega$, n is the number of zeros at the origin, and $\gamma_1, \gamma_2, \dots, \gamma_m$ are m poles. The number of poles together with their locations and the number of zeros at origin (for seismometer-array stations of UKAEA) are given by Mowat and Burch (1974).

2.7 Spalling function

Spalling may be defined as the physical parting (Eisler and Chilton 1964) of the near surface layers originally in intimate contact. This parting takes place under the action of stress waves produced by the contained underground explosions (Rinehart 1960) in a particular range of yield and depth of emplacement (Eisler and Chilton 1964). Spalling has been observed for most underground explosions (King *et al* 1974). Hence, it is pertinent to consider the effect of spalling, while computing the seismograms. King *et al* (1974) proposed a model for spalling, according to which, spalling function $SP(\omega)$ can be represented (Banghar 1980a) as

$$SP(\omega) = [1 + A' \exp(-i\omega\tau')],$$

(11)

where A' is the amplitude ratio of P waves generated by slapdown to P waves generated by underground nuclear explosions and τ' is the time interval between the phases mentioned above.

2.8 Composite transfer function

The composite transfer function $F(\omega)$ of vertical component of P wave signal can be obtained by multiplication of various transfer functions given in previous sections. This function is given below

$$F(\omega) = u(R, \omega) * T(\omega) * M(\omega) * U_V * I(\omega) * SP(\omega) * GSF, \quad (12)$$

P wave seismogram can be obtained by taking inverse Fourier transform of (12).

3. Results and discussions

Amplitude spectra and waveforms of teleseismic short period P waves generated by underground nuclear explosions were computed as to be recorded by various seismometer array stations for source models of Haskell and Seggern. Influences of various parameters such as focal depth, yield variation, crustal structure variation at source, on the amplitude spectra and waveforms of P waves were investigated. The results obtained from this study indicate that transfer functions of the source, the source crust, the mantle and spalling are the most predominant factors that influence the shaping of P seismograms. Since the comparison between computed and recorded P seismograms will provide information about the manner in which observed seismograms differ from computed ones, it is desirable to compute teleseismic P seismograms for some known underground nuclear explosions. For this study, Longshot, Milrow and Cannikin underground nuclear explosions were chosen because of the following reasons: (i) crustal structure in the source region and detonation media are known, (ii) focal depths of these events are known precisely, (iii) their yields cover a wide range, (iv) slapdown parameters for these events are available, (v) easy availability of array data from UKAEA. Pertinent data relating to these events are given in table 1. Epicentral distances of these events are about 36.12° , 77.61° , 81.18° and 85.66° from YKA, EKA, WRA and GBA seismic array stations respectively. Crustal model in source region together with the crustal models at these seismometer array stations are given in table 2. Crustal model in source region is that of Orphal *et al* (1970) and Engdahl and Tarr (1970), whereas crustal models at EKA, GBA, WRA and YKA respectively are those of Parks (1967), Arora (1977), Barr (1971) and Cleary (1974).

The earth model used for the synthesis is that of Fuchs (1966). Explosive source is located within a layered crust whereas the receiver is at the free surface of layered crust. All the layers both in source and receiver regions are assumed to be horizontal. The angles of incidence used for computing the source and receiver crusts transfer functions are obtained from Banghar (1970). The crustal models used in these computations assume that the mantle is a uniform half space, whereas in actual calculations, the P wave through the mantle is assumed to follow the standard travel time curve. Seggern and Lambert (1972) reported that the Amchitka underground nuclear explosions follow the 'hard rock' magnitude scaling at Nevada test site. Hence, granite shot medium was used for computing source functions for these events.

Table 1. Underground nuclear explosion locations and other pertinent data.

	Longshot	Milrow	Cannikin
Location	51.42° N 179.18° E	51.44° N 179.18° E	51.5° N 179.1° E
Date	29 Oct. 1965	2 Oct. 1969	6 Nov. 1971
Time	21 00 00.08	22 06 00.04	22 00 00.62
Yield (kiloton)	≈ 85	≈ 1000	≤ 5000
Depth (km)	0.702	1.218	1.791
Shot medium	Andesite/ Basalt	Pillow Lava	Pillow Lava
Magnitude M_s/m_b	3.9/6.1	4.9/6.5	5.7/6.8
Time interval between slapdown and P (sec)	0.87 ± 0.03	1.35 ± 0.6	1.85 ± 0.08
Slapdown amplitude relative to P amplitude	0.3 ± 0.1	0.5 ± 0.1	0.4 ± 0.1

For computing various transfer functions (impulse responses) needed for synthesizing P seismograms, a computer program was developed for BESM-6 computer at the Bhabha Atomic Research Centre, Bombay. For purposes of modelling and comparison with observed data, a time-domain description of wave propagation is more advantageous than frequency domain. Therefore, step-by-step variations of the amplitude spectra and waveforms of P waves are shown. For each of these source models, shaping of the amplitude spectra and waveforms of P waves (vertical component), as signal propagates from source to receiver at GBA are presented in figures 1 to 3 for Longshot, in figures 4 to 6 for Milrow and in figures 7 to 9 for Cannikin underground nuclear explosions. The discussion of the variations in the amplitude spectra and waveforms of signal is arranged as a function of yield.

It is relevant to discuss some of the transfer functions. For Longshot explosion, maxima and minima in the amplitude (trace 2 in figure 1) are due to the constructive and destructive interference between the direct, reflected and converted phases. Low frequency riding over high frequency amplitude is due to constructive and destructive interference between P , pP and higher order reflected and converted phases. This figure (trace 2) shows that amplitude falls with decreasing frequency from say 1 cps, the first minimum being in the vicinity of 0 cps. The second theoretical minimum in the amplitude spectrum occurs at about 3 cps and is due to destructive interference between P and pP phases. This result shows that the minimum in the amplitude spectra of P , and PcP phases of Longshot underground nuclear explosion at about 1 cps obtained by Buchbinder (1968) is probably not due to the effect of focal depth (Hasegawa 1972). Impulse response of this function shows a distinct pP phase (trace 2' in figure 1). Comparison of trace 2 in figures 1, 4 and 7 shows that the fundamental frequency of the amplitude spectrum decreases with increase in focal depth. It may also be seen from these figures (traces 2') that pP phase is more distinct for Cannikin explosion as compared to Longshot explosion. The impulse response of spalling function is composed of two pulses (traces 6' in figures 1, 4 and 7). This result indicates

Table 2. Crustal models used in computations of seismograms.

Layer	Thickness (km)	P-wave velocity (km/sec)	S-wave velocity (km/sec)	Density (g/cm ³)
(a) <i>Structure at source for Longshot, Milrow and Cannikin explosions</i>				
1.	0.30	4.06	1.93	2.35
2.	0.03	3.34	1.62	2.26
3.	0.16	4.06	1.93	2.35
4.	0.01	3.33	1.62	2.26
5.	0.26	4.06	1.93	2.35
6.	0.04	4.61	2.23	2.47
7.	0.03	4.06	1.93	2.35
8.	0.14	4.61	2.23	2.47
9.	0.13	4.06	1.93	2.35
10.	0.18	4.83	2.28	2.13
11.	0.32	4.87	2.32	2.35
12.	10	5.5	3.20	2.70
13.	30	6.6	3.80	3.00
Mantle		8.1	4.70	3.30
(b) <i>Structure at Eskdalemuir (Scotland)</i>				
1.	5.3	6.14	3.55*	2.80
2.	19.7	7.28	4.20*	3.20
Mantle		8.09	4.67	3.38
(c) <i>Structure at Gauribidanur (India)</i>				
1.	8.7	5.62	3.28	2.70
2.	10.5	6.07	3.51	2.80
3.	17.2	6.50	3.95	3.00
Mantle	∞	7.98	4.62	3.33
(d) <i>Structure at Warramunga (Australia)</i>				
1.	20.00	6.20	3.58*	2.70
2.	19.5	6.50	3.75*	2.90
Mantle	∞	8.10	4.70*	3.30
(e) <i>Structure at Yellowknife (Canada)</i>				
1.	1.5	5.64	3.25	2.67
2.	32.5	6.04	3.69	2.71
Mantle	∞	8.15	4.71	3.32

* In those regions where S-wave velocities are not reported, these are determined from the relation $\alpha^2 = 3\beta^2$ for numerical computations.

that spalling can be considered as a secondary source similar to explosive source (Springer 1974). Comparison of traces 1 in figures 2 and 3 shows that the maximum amplitude of the source function occurs at a higher frequency for Haskell's model as compared to Seggern's model. The duration of impulse response (traces 1' in figures 2 and 3) is longer for the source model of Seggern as compared to that of Haskell. Traces *a* (figures 2 and 3) show that beyond about 2.7 cps the amplitude is nearly negligible for

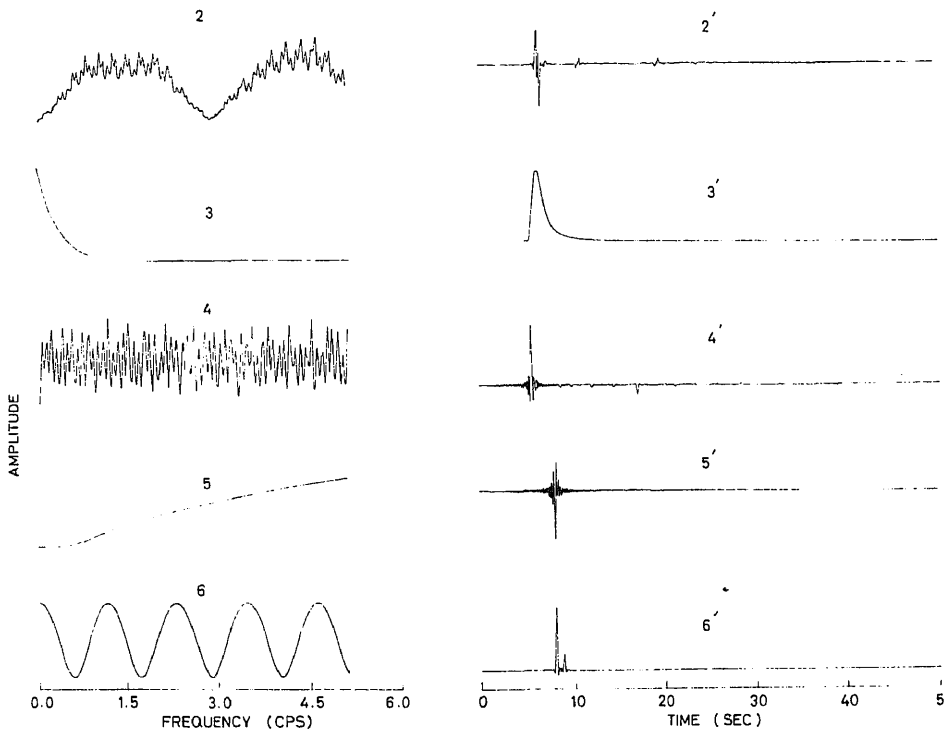


Figure 1. shows the amplitude spectra and impulse responses (arbitrary units) of the transfer functions that remain unchanged while computing variations in theoretical amplitude spectra and waveforms for Longshot as P wave signal propagates from source (Amchitka Island) to Gauribidanur (GBA), India. Procedure for numbering traces in this figure is as follows: Traces 2 and 2' represent amplitude spectrum and impulse response of the source crust transfer function. Corresponding quantities for the mantle transfer function, the receiver crust transfer function, the recording system transfer function, and the spalling function are given by traces 3 and 3', 4 and 4', 5 and 5', and 6 and 6' respectively.

both the source models. Moreover, the second minimum in the amplitude spectrum of signal occurs at a frequency at which it occurs in the amplitude spectrum of source crust transfer function (traces 2 and a in figures 1, 2 and 3). However, the first maximum in the amplitude spectrum of signal occurs at about 1.2 cps (traces a in figures 2 and 3), whereas the first maximum in the amplitude spectrum of source crust transfer function occurs at about 1.4 cps (traces 2 in figure 1). This observation suggests that the fall-off in the amplitude spectra of underground nuclear explosions at low frequencies can be attributed both to the source and source crust transfer functions. For both these models, amplitude spectra are very similar. Examination of traces a' in these figures shows that pP (this phase is quite distinct in the impulse response of the source crust transfer function) and P phases are completely merged. Attenuation in the mantle is incorporated by assuming T^* equals unity. This assumption seems to be reasonable (Carpenter and Flinn 1965) in the case of teleseismic short

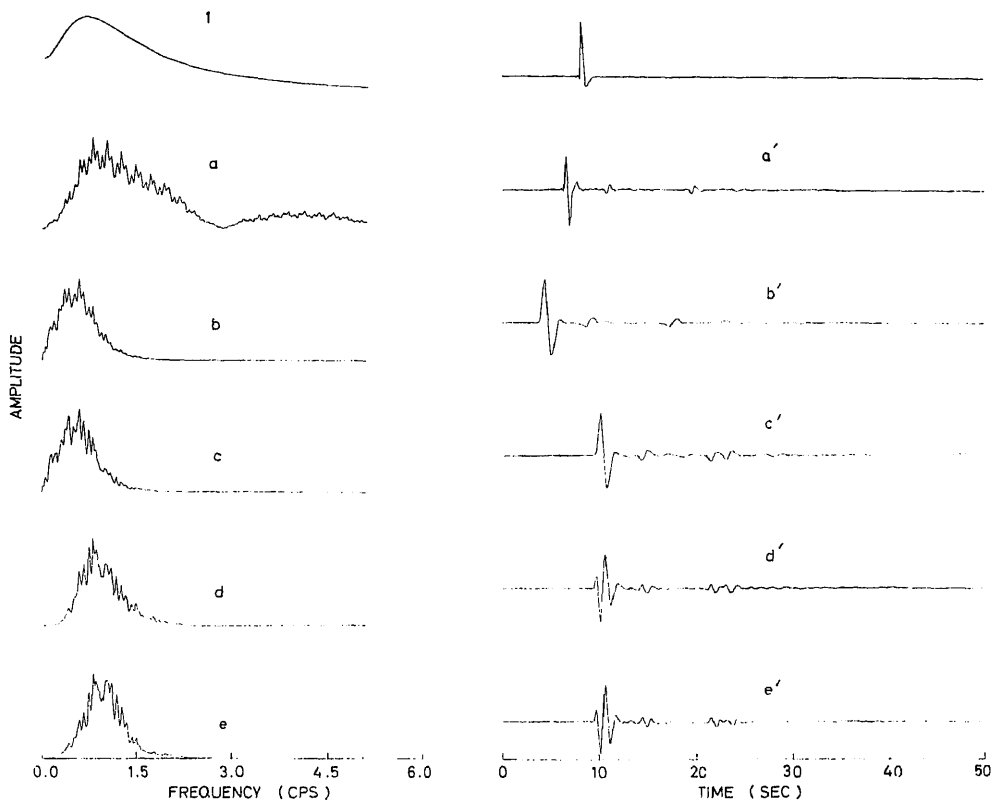


Figure 2. shows the step-by-step variations in the amplitude spectrum and waveform of signal as it propagates from source to GBA. The source model is that of Haskell. Traces 1 and 1' show the amplitude spectrum and impulse response of the source function. The amplitude spectra and waveforms of the signal at the base of crust in source and receiver regions are shown respectively by traces a, a' and b and b'. The amplitude spectra and waveform at the top of crust in receiver region are given by traces c and c' respectively. Traces d and d' and e and e' show respectively the amplitude spectra and waveforms of computed signal without and with the effect of spalling.

period P waves. This value of T^* corresponds to the effective Q value of the order of about 760. It is interesting that the amplitude spectrum is nearly negligible for frequencies greater than about 1.6 cps (traces b in figures 2 and 3), whereas the waveform duration is longer after passing through mantle as compared to its duration prior to its passage through it. The shapes of the amplitude spectra and waveforms of signal for Haskell and Seggern source models are nearly similar. However, the duration of signal (traces b' in figures 2 and 3) for the Seggern model is slightly more than its duration for Haskell model. Comparison of traces b' and c' in these figures shows that the effect of crustal structure in receiver region on P wave seismogram is nearly negligible. Due to the effect of short period recording system on the signal, the

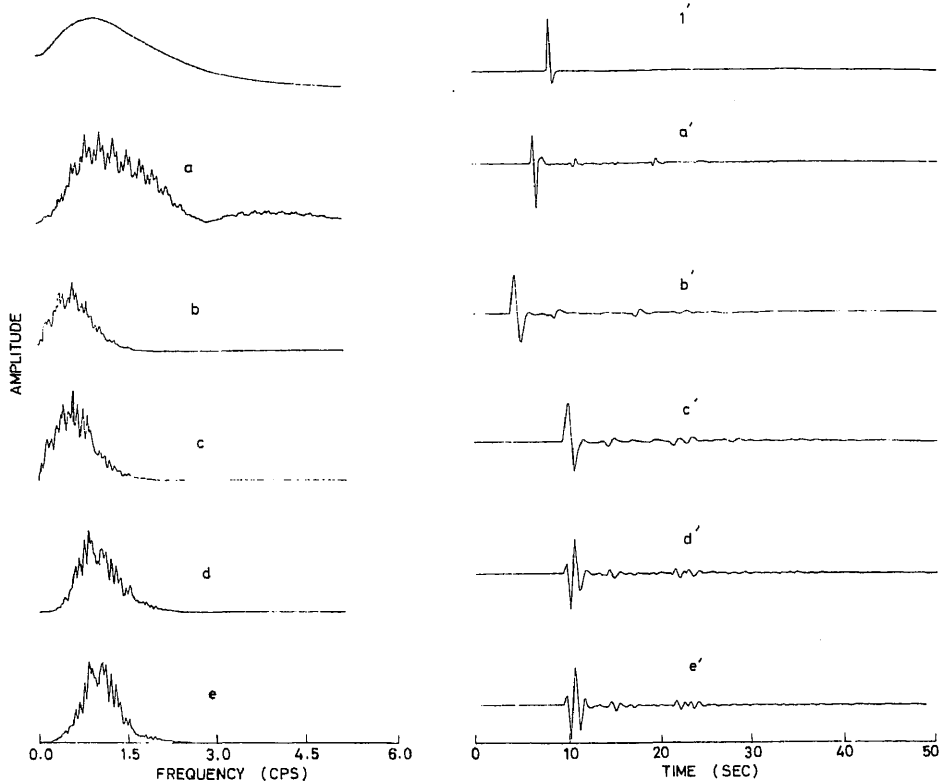


Figure 3. Amplitude spectrum and waveform as signal propagates from source to GBA. The source model is that of Seggern. Symbols same as in figure 2.

peak in the amplitude spectrum is shifted to higher frequency (compare traces *c* and *d*). Another important feature of the amplitude spectra (traces *d* in figures 2 and 3) is their fall-off with decreasing frequency from, say about 0.8 cps. The amplitude spectra of *P* waves from Longshot at YKA also indicated the fall-off in the amplitude spectrum with decreasing frequency for frequencies less than the frequency mentioned above, which was attributed to the combined effect of source function and focal depth by Hasegawa (1972). In order to show the effect of spalling distinctly, its influence was incorporated at this stage. The composite amplitude spectra and seismograms of *P* waves are shown by traces *e* and *e'* in these figures. It may be noted that the amplitude between 0.9 and 1.2 cps is slightly amplified (compare traces *d* and *e*) and the duration of seismogram is elongated due to the effect of spalling (compare traces *d'* and *e'*).

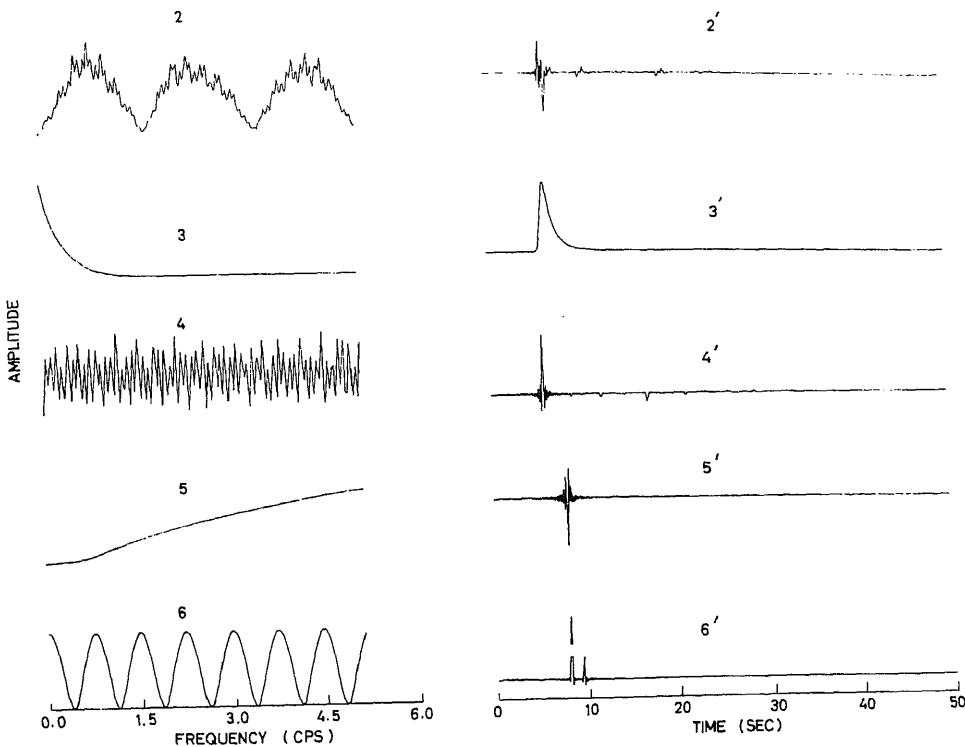


Figure 4. shows the amplitude spectra and impulse responses of the transfer functions that were held constant while computing the variations in the amplitude spectrum and waveform for Milrow as signal propagates from source to GBA. Symbols same as in figure 1.

Figures 5 and 6 show the shaping of the amplitude spectra and waveforms of signal for Milrow underground explosion. It can be seen (traces 1 and 1' in figures 2, 3, 5, 6, 8 and 9 that for a given source model, the amplitude peak shifts to lower frequency and the duration of impulse response increases, with increase in yield. It is interesting to note that for both models, the maximum in the amplitude spectrum of signal at the base of crust (traces *a* in figures 5 and 6) occurs nearly at the same frequency and this frequency lies between the frequencies at which the maximum amplitude in the source function and the first maximum amplitude in the source crust transfer function occurs. This result again shows that the frequency at which the maximum occurs in the amplitude spectrum of signal at the base of crust in the source region, is primarily governed both by source and source crust transfer functions. Comparison of traces *d'* and *e'* shows that the duration of signal is increased due to spalling. It may be noted that the amplitudes of *P* seismogram for first three half cycles are not affected much due to spalling, whereas the amplitudes of fourth and fifth half cycles are more for seismograms which are computed while considering the effect of slapdown as compared to seismograms computed without taking account of slapdown.

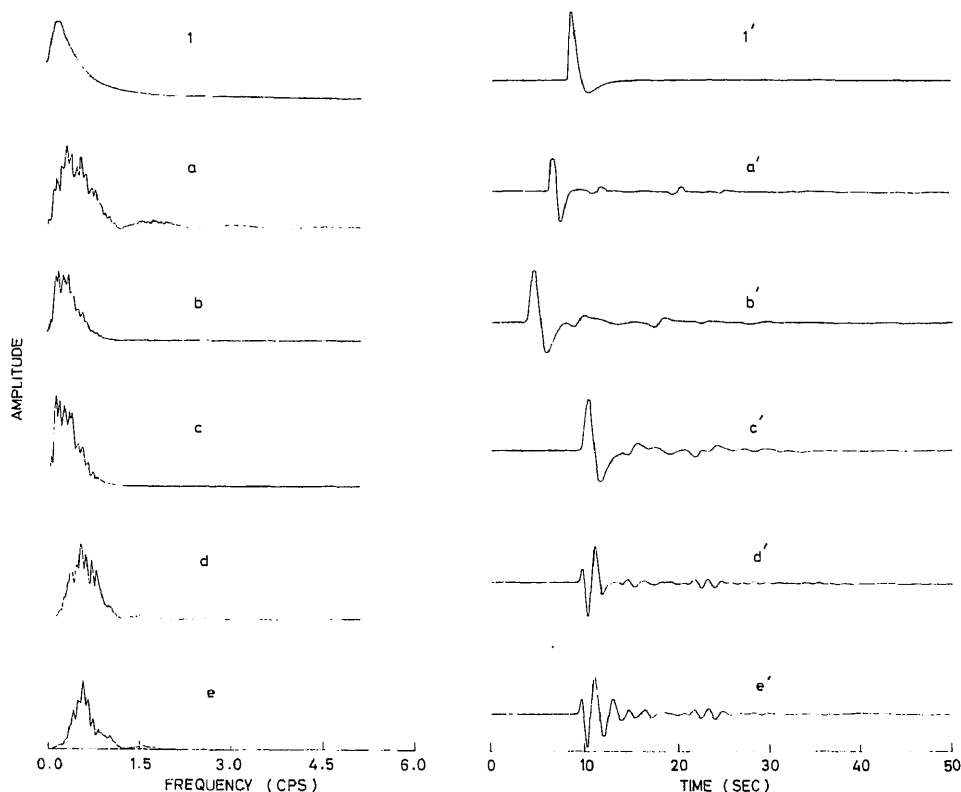


Figure 5. Amplitude spectrum and waveform for Milrow as signal propagates from source to GBA. Source model is that of Haskell. Symbols same as in figure 2.

The amplitude spectrum of signal for Cannikin explosion at the base of crust in source region (traces *a* in figures 8 and 9) for these two source models shows some interesting features. One of these is that the second and third minima are recognizable in the amplitude spectrum for Seggern's model, whereas this is not the case in Haskell's model. Hence for large yields, high frequencies do get transmitted through source crust for Seggern's source model. In this case also, the duration of *P* seismograms (compare traces *d'* and *e'* in figures 8 and 9) is increased due to the spalling effect.

For a given source model, comparison of composite *P* seismograms for Longshot, Milrow and Cannikin underground nuclear explosions, shows that peak-to-peak amplitude and period of *P* waves increase with increase in yield (Banghar 1980b). It is seen that peak frequency in the amplitude spectra is governed by the source function, the source crust transfer function and anelasticity of the mantle. It is also noticed that for a given explosion, pulse shapes of synthesized *P* wave seismograms are almost equivalent for all practical purposes both for source models of Haskell and Seggern. Similarity of composite spectra for these models is mainly due to the dominant effect of the anelasticity of the mantle.

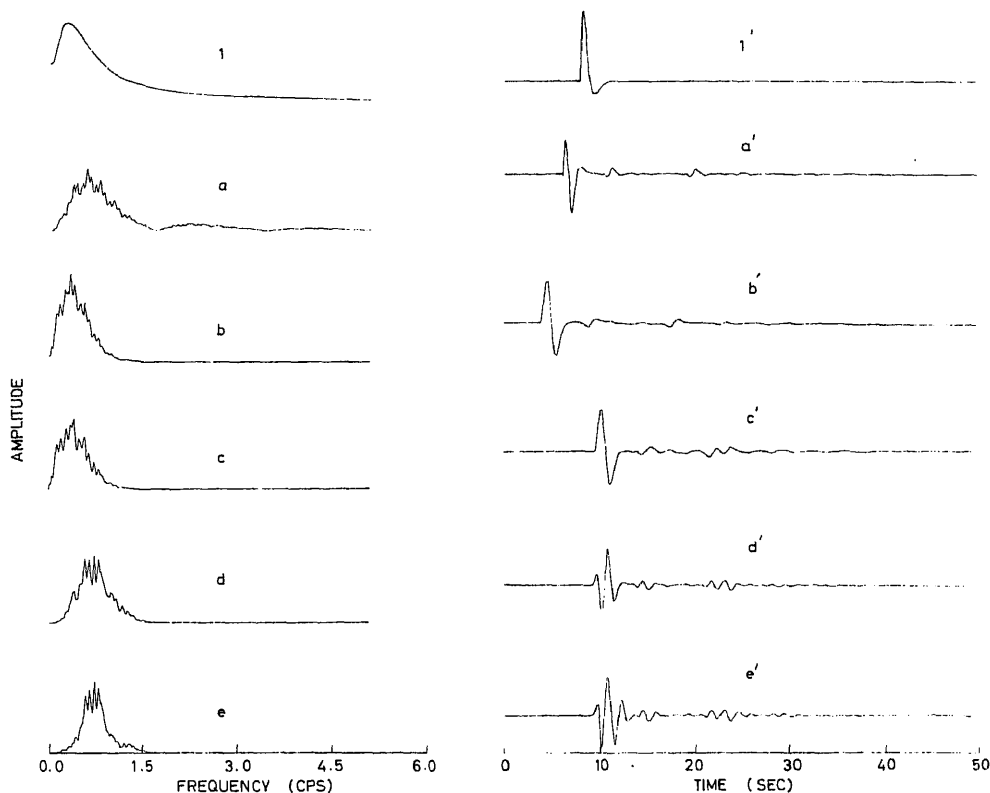


Figure 6. Amplitude spectrum and waveform for Milrow as signal propagates from source to GBA. Source model is that of Seggern. Symbols same as in figure 2.

Theoretical amplitude spectra and waveforms of signal as it propagates from source to receivers at YKA, WRA and YKA were also computed for these events for both the source models. It is noticed that shaping of the amplitude spectra and waveforms at these seismic array stations is almost similar to that at GBA. It is desirable to compare the synthesized and recorded *P* wave seismograms at various seismometer array stations. Computed and observed *P* seismograms are compared at EKA, GBA, WRA and YKA in figure 10 for Longshot explosion only because *P* wave seismograms for Milrow and Cannikin explosions were saturated at these stations. It may be noted (traces a_1 and b_1 in figure 10) that the duration of seismogram is slightly elongated due to spalling. Comparison of traces b_1 and c_1 in figure 10 indicates that computed seismograms reproduce many of the features of observed seismograms. However, the agreement between the coda of computed and observed *P* wave seismograms is much less satisfactory which may be due to the effect of scattering in the source region, inhomogeneities in the upper mantle and multipathing.

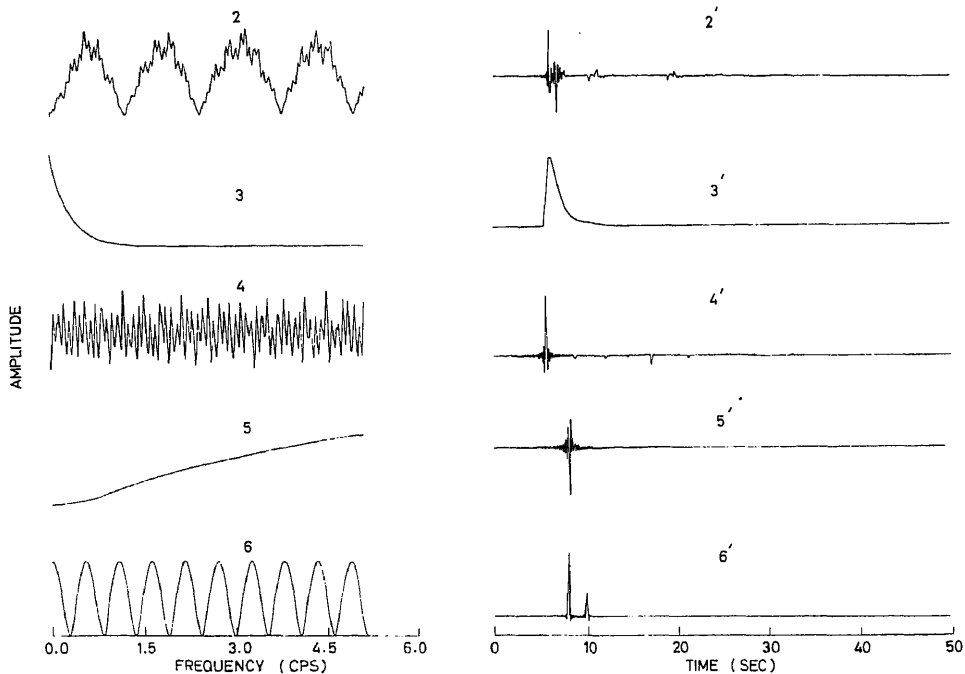


Figure 7. shows the amplitude spectra and impulse responses of transfer functions that were held constant while computing the variations in the amplitude spectrum and waveforms for Cannikin as signal propagates from source to GBA. Symbols same as in figure 1.

Traces c_1 in figure 10 show that the amplitude of the second trough of observed seismograms is more as compared to first trough, whereas the reverse is the case in synthesized seismograms. While studying the spectral amplitude ratios of P waves generated by these explosions, it was reported by Frasier (1972), King *et al* (1972, 1974) that in source region the inferred overburden P wave velocity is less as compared to its value obtained from on-site measurements. The crustal model in source region (table 2a) was modified in such a fashion that the overburden P wave velocity is that inferred by King *et al* (1972) and P wave seismograms were recomputed at GBA, EKA, WRA and YKA seismic array stations for Longshot explosion. Like observed P seismograms, recomputed seismograms show that the amplitude of the second trough is more as compared to the first trough. This result shows that the depth determined from P wave seismograms will be different than actual depth. This result is very significant.

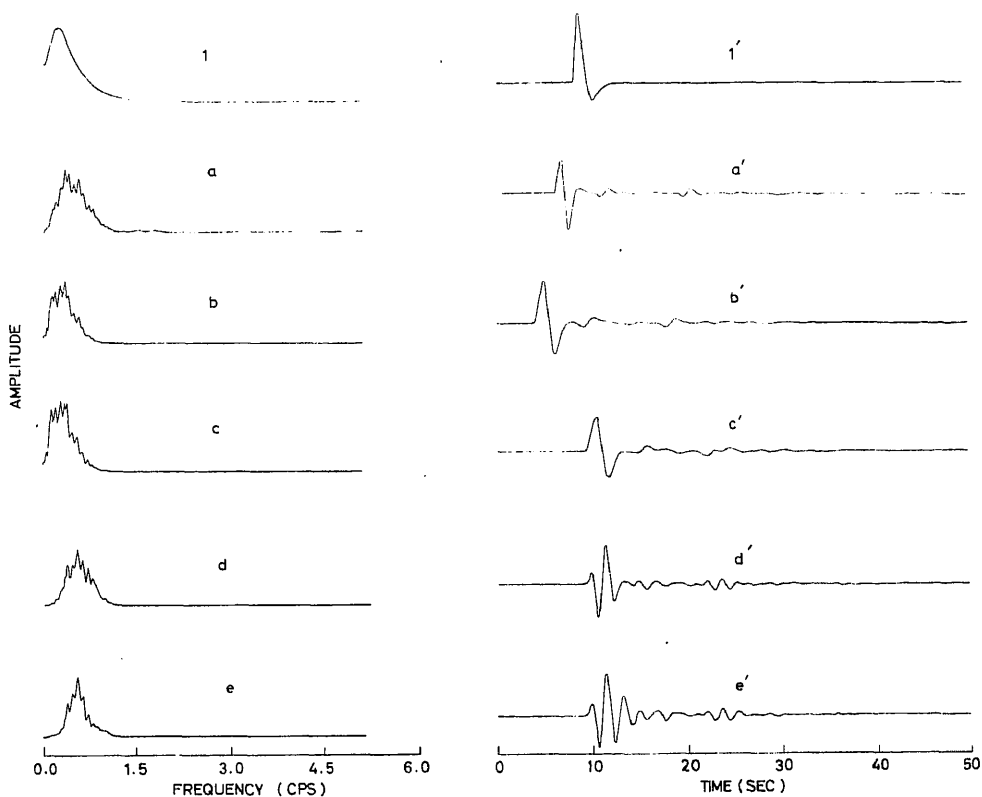


Figure 8. Amplitude spectrum and waveform for Cannikin as signal propagates from source to GBA. Source model is that of Haskell. Symbols same as in figure 2.

4. Conclusions

Step-by-step variations in the amplitude spectra and waveforms of *P* wave signal as it propagates from source region at Amchitka to receiver at GBA, are presented for Longshot, Milrow and Cannikin underground explosions. It is shown that pulse shapes of *P* waves predicted both by Haskell and Seggern source models are almost equivalent for all practical purposes. The period as well as amplitude of computed seismograms increase with increase in yield. Computed and observed *P* wave seismograms for Longshot at various seismic array stations of UKAEA were compared. It is noticed that many features of observed seismograms are reproduced in computed seismograms. Seismograms were also computed using the value of overburden *P* wave velocity given by King *et al* (1974). It is interesting that the recomputed seismograms

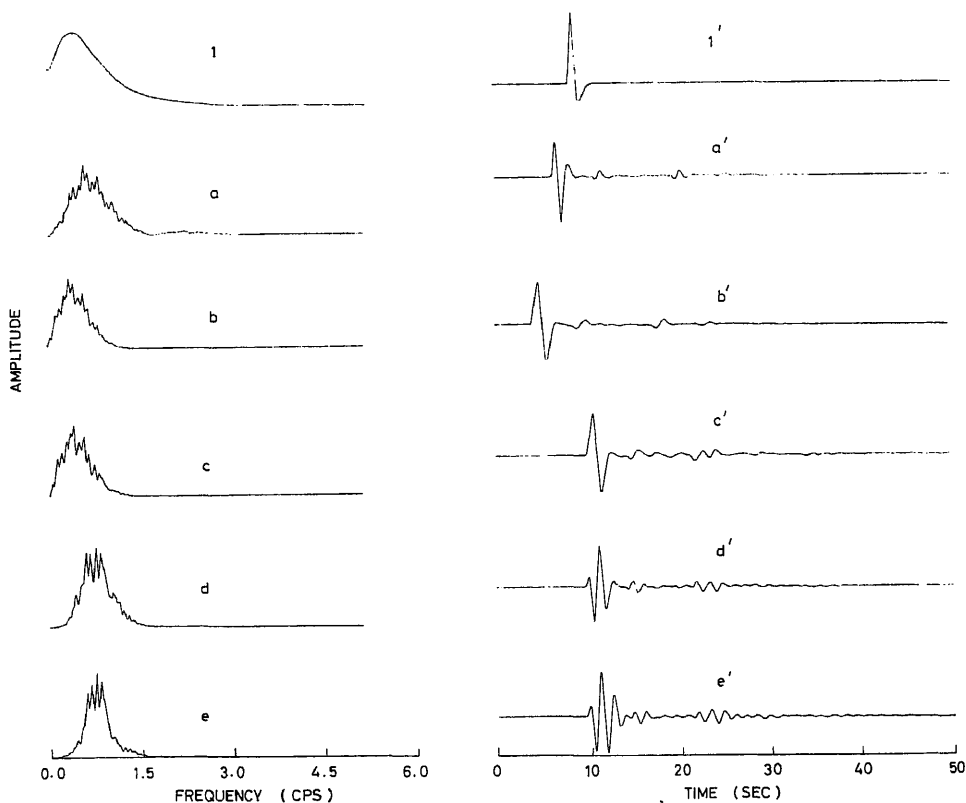


Figure 9. Amplitude spectrum and waveform for Cannikin as signal propagates from source to GBA. Source model is that of Seggern. Symbols same as in figure 2.

compare better with observed seismograms. This result supports the conclusion that overburden *P* wave velocity inferred from seismograms is less as compared to its value obtained from on-site measurements. The results obtained show that the source function, the source crust transfer function, the mantle transfer function and the spalling function are the most important factors that influence shaping of spectra and seismograms. This study is also useful in distinguishing between underground explosions and earthquakes.

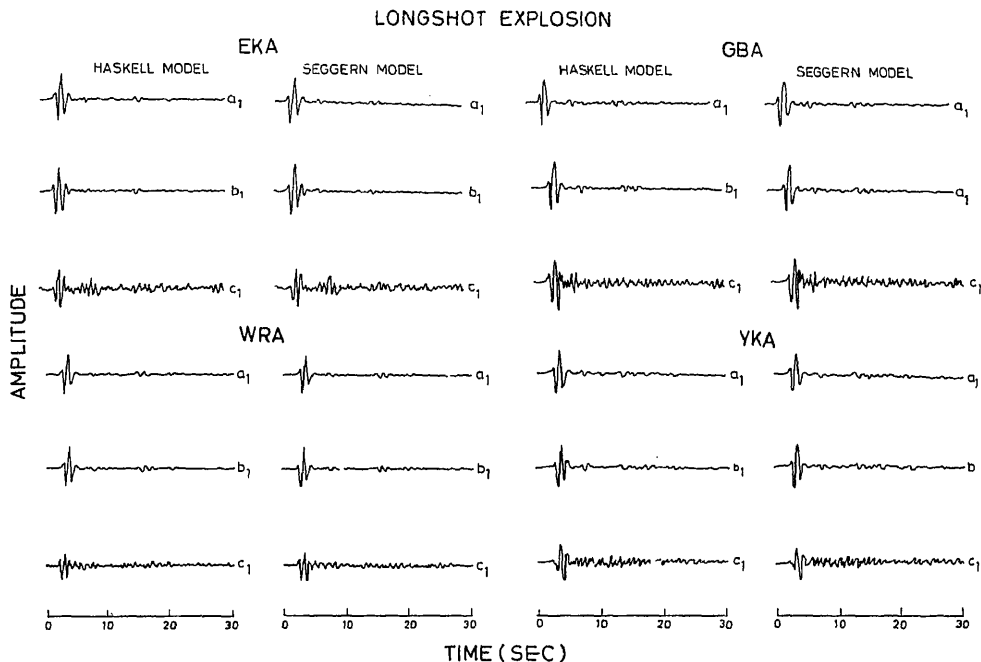


Figure 10. Comparison of synthesized and observed P wave seismograms at GBA, EKA, WRA and YKA for source models of Haskell and Seggern. Computed P wave seismograms without taking into account the effect of spalling are shown by traces a_1 . Composite synthesized seismograms are shown by traces b_1 whereas traces c_1 show observed seismograms.

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References

- Aki K, Bouchon M and Reasenber P 1974 *Bull. Seismol. Soc. Am.* **64** 131
- Arora S K 1977 *Crustal structure near Gauribidanur in Southern India from Gauribidanur data of local earthquakes, rock-bursts and explosions*, Ph.D. thesis, University of Bombay
- Bakun W H and Johnson L R 1971 *Geophys. J. R. Astron. Soc.* **22** 139
- Banghar A R 1970 *Earthquake Notes* **41** 45
- Banghar A R 1980a Synthetic seismograms-I. Transfer functions for synthesis of P wave seismograms from explosive sources and their parametric study, BARC Report I-617
- Banghar A R 1980b Synthetic seismograms -II. Synthesis of amplitude spectra and seismograms from underground nuclear explosions, BARC Report 1085
- Banghar A R 1981 *Mechanisms of earthquakes and synthesis of P wave seismograms from explosions and earthquakes*, Ph.D. thesis, University of Bombay

- Banghar A R 1982 *Phys. Earth Planet. Interior*. (submitted)
- Barr K G 1971 *J. Geophys. Res.* **76** 1929
- Ben-Menahem A 1967 in Current status and future prognosis for understanding the source mechanism of shallow seismic events in the 3-5 magnitude range. *Proc. Vesiac Conf., Willow Run Lab., University of Michigan, USA* p. 85
- Ben-Menahem A and Toksoz M N 1962 *J. Geophys. Res.* **68** 5207
- Buchbinder G G R 1968 *Bull. Seismol. Soc. Am.* **58** 1797
- Burdick L J and Helmberger D V 1979 *Bull. Seismol. Soc. Am.* **69** 957
- Carpenter E W 1966 *Proc. R. Soc. (London)* **A290** 396
- Carpenter E W and Flinn F A 1965 *Nature (London)* **207** 745
- Carpenter E W and Thirlaway H I S 1966 Seismic signal anomalies, travel times, amplitude and pulse shapes. *Proc. Vesiac Special Study Conf., Report 4410-99-X, Willow Run Lab., University of Michigan, USA*
- Cleary J 1974 *Tectonophysics* **20** 241
- Douglas A, Hudson J A and Blamey C 1972 *Geophys. J. R. Astron. Soc.* **28** 385
- Eisler J D and Chilton F 1964 *J. Geophys. Res.* **69** 5285
- Engdahl E R and Tarr A C 1970 Aleutian seismicity: Milrow seismic effects, CGS-746-102. US Coast and Geodetic Survey, Environ. Sci. Serv. Admin., Boulder, Colo. USA
- Frasier C W 1972 *Geophys. J. R. Astron. Soc.* **31** 99
- Fuchs K 1966 *Bull. Seismol. Soc. Am.* **56** 75
- Futterman W I 1962 *J. Geophys. Res.* **67** 5279
- Hasegawa H S 1971 *Geophys. J. R. Astron. Soc.* **24** 365
- Hasegawa H S 1972 *J. Geophys. Res.* **77** 3081
- Hasegawa H S 1973 *Bull. Seismol. Soc. Am.* **63** 1201
- Haskell N A 1953 *Bull. Seismol. Soc. Am.* **43** 120
- Haskell N A 1962 *J. Geophys. Res.* **67** 4751
- Haskell N A 1967 *J. Geophys. Res.* **72** 2583
- Helmberger D V and Harkrider D G 1972 *Geophys. J. R. Astron. Soc.* **31** 45
- Holzer F 1966 *Proc. R. Soc. (London)* **A290** 408
- King C, Bakun W H and Murdock J N 1972 *J. R. Astron. Soc.* **31** 27
- King C, Abe Zena A M and Murdock J N 1974 *J. Geophys. Res.* **79** 712
- Kogeus K 1968 *Bull. Seismol. Soc. Am.* **58** 663
- Liebermann R C and Pomeroy P W 1969 *J. Geophys. Res.* **74** 1575
- Liebermann R C and Pomeroy P W 1970 *Bull. Seismol. Soc. Am.* **60** 879
- McEvelly T V and Peppin W A 1972 *Geophys. J. R. Astron. Soc.* **31** 67
- Molnar P 1971 *Geophys. J. R. Astron. Soc.* **23** 273
- Mowat W M H and Burch R F 1974 Handbook of stations which provide seismograms to the Blacknest Seismology Centre, AWRF, Aldermaston, Berks, UK
- Muller G 1973 *Bull. Seismol. Soc. Am.* **63** 847
- Murphy J R 1977 *Bull. Seismol. Soc. Am.* **67** 135
- Nair G J 1977 Some seismic results of the Rajasthan explosion of 18 May 1974, AG-224, AWRE, Aldermaston, Berks, UK
- Orphal D I, Spiker C T, West I. R and Wronski M D 1970 Analysis of seismic data: Milrow Event, NVO 1163-209, *Environ. Res. Corp.*, Alexandria, Va
- Parks R 1967 *Seismic refraction network in the study of earth's crust*, Ph.D. thesis, University of Edinburgh
- Rinehart R A 1960 *Engg. Mining J.* **161** 98
- Seggern V D and Lambert D G 1970 *J. Geophys. Res.* **75** 7382
- Seggern V D and Lambert D G 1972 Analysis of teleseismic data from nuclear explosion Milrow, Seismic Data Lab. Report 258, AD 743 072, Teledyne Geotech
- Seggern V D and Blandford R 1972 *Geophys. J. R. Astron.* **31** 83
- Springer D I. 1974 *Bull. Seismol. Soc. Am.* **64** 581
- Tsai Y and Aki K 1970 *J. Geophys. Res.* **75** 5729
- Tsai Y and Aki K 1971 *J. Geophys. Res.* **76** 3940
- Werth G C and Hurst R F 1963 *J. Geophys. Res.* **68** 1463
- Wyss M, Hanks T C and Liebermann R C 1971 *J. Geophys. Res.* **76** 2716

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Abstract. Constants occurring in Seggern and Blandford's analytic expression of reduced displacement potential are computed for shot media of alluvium, salt and tuff. The amplitude spectra as well as impulse response of the source function for models of Haskell and Seggern are studied as a function of yield. It is shown that the maximum in amplitude spectrum shifts to lower frequency with increase in yield for a given model. The duration of impulse response is longer for Seggern's model as compared to Haskell's model for a given yield.

Keywords. Source time function; spectra; underground explosions; displacement potential.

1. Introduction

Numerical values of reduced displacement potential are available for a few small underground nuclear explosions detonated in different geological media (Werth and Herbst 1963; Holzer 1966; Rogers 1966; Sisemore *et al* 1969; Healy *et al* 1971). Using cube root scaling laws, values of reduced displacement potential were determined for higher yields to synthesize *P* wave seismograms from underground nuclear explosions (Kogeus 1967, 1968). Haskell (1967) presented analytic expression (involving two arbitrary constants) that fits the numerical values of reduced displacements potential given by Werth and Herbst (1963). This analytic expression made it easy to determine the values of reduced displacement potential at desired intervals of time for various yields. The far-field displacement spectrum derived from analytic expression of Haskell indicates ω^{-4} frequency dependence at high frequencies and led to unrealistic scaling effects, such as a greater high frequency energy from a small underground explosion than the larger ones (Seggern and Lambert 1970). A model of reduced displacement potential of Mueller and Murphy (1971) gives rise to a far-field displacement spectra that indicates ω^{-2} dependence at high frequencies and satisfy data in the short-period band recorded teleseismically. Seggern and Blandford (1972) modified Haskell's analytical expression for reduced displacement potential. Seggern and Blandford's analytic expression like Haskell's expression for reduced displacement potential also involves two arbitrary constants. The essential difference in the models of reduced displacement potential of Haskell and Seggern and Blandford is basically in the fall-off in the far-field displacement spectra at high frequencies. Seggern and Blandford (1972) computed the values of two constants by fitting the computed and observed values of reduced displacement potential for granite shot medium. It is equally essential to know the values of these constants for other shot media. We have determined the values of arbitrary constants of Seggern and Blandford analytic

expression for shot media of salt, tuff and alluvium. It is thus possible to use Seggern and Blandford analytic expression of reduced displacement potential for shot media of salt, tuff and alluvium while synthesizing seismograms.

2. Source function

For a spherically symmetric explosive source in an infinite homogeneous medium, the displacement has a radial component, which can be represented in terms of $\psi(\tau)$ as

$$u(R, t) = - \frac{\partial}{\partial R} (\psi(\tau)/R). \quad (1)$$

where $\psi(\tau)$ = reduced displacement potential, $\tau = t - R/\alpha$ = retarded time, t = time, R = radial distance from the centre of explosion, α = compressional wave velocity of the medium in which explosion is detonated.

Since ψ is only a function of τ in the elastic region, its value is independent of R . Equation (1) shows that ψ has dimensions of volume and normally, it is measured in (metre)³.

Using (1), it can be shown that

$$u(R, t) = \psi(\tau)/R^2 + \frac{1}{\alpha R} \frac{\partial \psi}{\partial \tau} \quad (2)$$

For large values of t , (2) becomes

$$R^2 u(R, \infty) = \psi(\infty). \quad (3)$$

Equation (3) shows that $\psi(\infty)$ is proportional to permanent displacement.

We are interested in far-field values of the displacement. Under this approximation, (2) becomes

$$u(R, t) = \frac{1}{\alpha R} \frac{\partial}{\partial \tau} (\psi(\tau)) \quad (4)$$

The values of reduced displacement potential $\psi(\tau)$ were calculated by suitable integration of the near-field measurements of velocity and acceleration for a few small underground nuclear explosions in different geological media. The values of reduced displacement potential at desired yields can be computed by using scaling laws (Banghar 1980, 1981).

It was desirable to determine an analytic expression that fits the calculated numerical values of reduced displacement potential. Because, such a relation will be extremely useful for calculating $\psi(\tau)$, at desired intervals of time for explosions of various yields. Haskell (1967) proposed a relation for $\psi(\tau)$ for τ greater than zero.

$$\psi(\tau) = \psi(\infty) [1 - f(k\tau) \exp(-k\tau)], \quad (5)$$

where k is a constant to be chosen to fit the time scale of the observed values of $\psi(\tau)$ and $f(k\tau)$ is a polynomial in $k\tau$ to be chosen to match the maximum value of the observed

$\psi(\tau)$. The minimum permissible degree of $f(k\tau)$ was determined by imposing the conditions that displacement, velocity and acceleration calculated from (5) shall be continuous at $\tau = 0$. With these conditions, it can be shown (Haskell 1967), that

$$f(k\tau) = 1 + k\tau + \frac{(k\tau)^2}{12} + \frac{(k\tau)^3}{13} - B(k\tau)^4, \quad (6)$$

where B is a dimensionless constant. Values of B and k were determined by fitting the observed and calculated values of $\psi(\tau)$ for underground nuclear explosions of five kilotons yield detonated in granite, salt, tuff and alluvium. Table 1 shows the values of B , k and $\psi(\infty)$ for 5 kilotons explosions in granite, salt, tuff and alluvium. It may be mentioned here that k being inversely proportional to time depends on yield. It scales as inversely proportional to cube root of yield. It is often convenient to discuss the displacement spectrum. Fourier transform of $u(R, t)$ is defined as

$$u(R, \omega) = \int_{-\infty}^{\infty} u(R, t) \exp(-i\omega t) dt \quad (7)$$

Using (4), (5) and (6), it can be shown (Hasegawa 1972; Banghar 1980) that

$$u(R, \omega) = \frac{\psi(\infty)}{\alpha R} \left[\frac{1 + 24B}{[1 + i(\omega/k)]^4} - \frac{24B}{[1 + i(\omega/k)]^5} \right] \quad (8)$$

where ω is the angular frequency. Subsequently, we shall refer to the relation given by (8) as Haskell's model.

Equation (8) shows that the far-field displacement spectrum indicates ω^{-4} dependence at high frequencies. This fall-off indicates a high frequency scaling of spectrum as inversely proportional to the cube root of yield (*i.e.* large explosions emit less high frequency energy as compared to small ones) and low frequency scaling of displacement proportional to yield (Seggern and Lambert 1970). This frequency scaling of displacement spectrum at high frequencies seems to be unrealistic (Aki *et al* 1974).

The results of comparing the displacement spectral ratios at one station and first motion amplitudes at several stations for Longshot, Milrow and Cannikin explosions (Seggern and Blandford 1972) indicated that the far-field displacement spectra falls as ω^{-2} at high frequencies. Seggern and Blandford (1972) argued that Haskell's requirement of continuous velocity and acceleration at the elastic radius is too restrictive and

Table 1. Parameters for five kilotons explosions for Haskell's model of the reduced displacement potential in various media.

Medium	B	K (sec^{-1})	$\psi(\infty)$ (m^3)	P wave velocity (km/sec)	Density (g/cm^3)
Granite	0.240	31.6	2500	4.80	2.69
Salt	0.171	28.4	4420	4.08	2.13
Tuff	0.050	23.5	5120	2.44	1.84
Alluvium	0.490	17.0	420	1.71	1.87

hence can be abandoned. They proposed a model for $\psi(\tau)$ having discontinuous velocity and acceleration at the elastic boundary of the medium surrounding explosion. This was achieved by removing quadric and quartic terms from the expression of $f(k_1 \tau)$. The new expression for $\psi(\tau)$ proposed by Seggern and Blandford is given as

$$\psi(\tau) = \psi(\infty) [1 - (1 + k_1 \tau - B(k_1 \tau)^2) \exp(-k_1 \tau)] \quad (9)$$

These investigators determined the values of B_1 and k_1 by using the same data as used by Haskell for granite shot medium. We have determined the values of B_1 and k_1 for shot media of salt, tuff and alluvium by using values of reduced displacement potentials that were used by Haskell (1967). Values of B_1 , k_1 and $\psi(\infty)$ are given in table 2. Reduced displacement potentials derived from expressions (5) and (9) together with the observed reduced displacement potential (Werth and Herbst 1963) for explosions in granite, salt, tuff and alluvium are compared in figures 1 to 4 as a function of retarded time. These figures show that analytic and observed displacement potentials agree closely and the agreement is excellent at very small and very large values of retarded time.

Using (4), (7) and (9), it can be shown that (Banghar 1980)

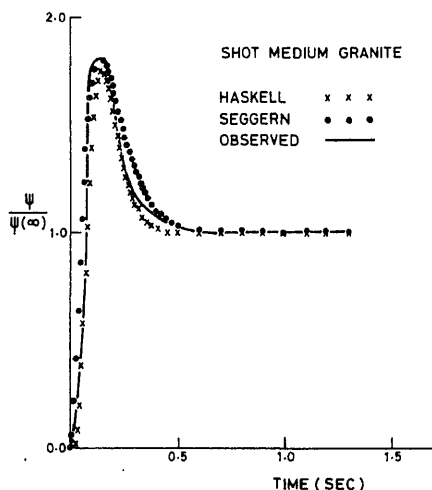


Figure 1. Comparison between observed and analytically derived reduced displacement potentials for 5 kilotons underground nuclear explosions. Analytic expressions used for reduced displacement potential are those of Haskell, Seggern and Blandford.

Table 2. Parameters for 5 kilotons explosions for Seggern and Blandford's model of the reduced displacement potential in various media.

Medium	B_1	K_1 (sec ⁻¹)	$\psi(\infty)$ (m ³)	P wave velocity (km/sec)	Density (g/cm ³)
Granite	2.14	16.80	2500	4.80	2.69
Salt	1.43	15.60	4420	4.08	2.13
Tuff	0.35	17.02	5120	2.44	1.84
Alluvium	4.19	8.76	420	1.71	1.87

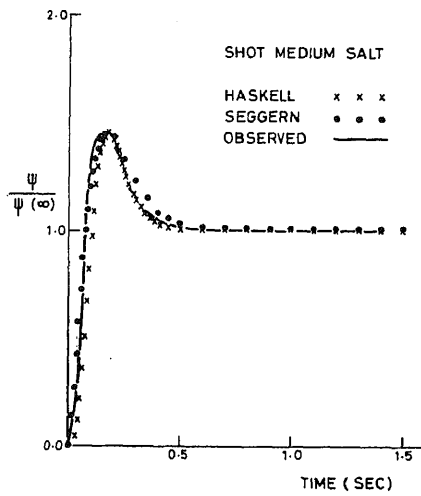


Figure 2. Comparison of observed and analytically derived reduced displacement potentials for 5 kilotons explosions. Models of reduced displacement potential are same as in figure 1.

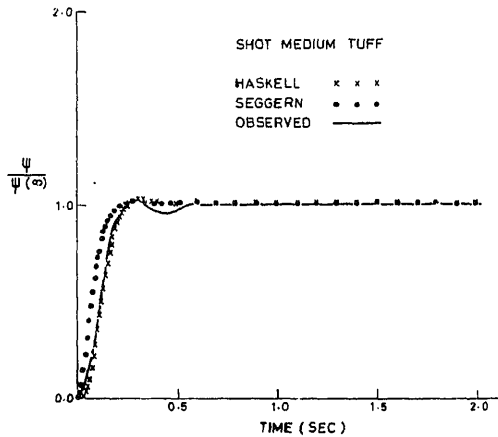


Figure 3. Same as figure 2.

$$u(R, \omega) = \frac{\psi(\infty)}{\alpha R} \left[\frac{1 + 2B_1}{[1 + i(\omega/k_1)]^2} - \frac{2B_1}{[1 + i(\omega/k_1)]^3} \right] \quad (10)$$

Equation (10) shows that the far-field displacement spectrum falls as ω^{-2} at high frequencies. This fall-off indicates that far-field displacement spectrum scales proportional to the cube root of yield at high frequencies. The far-field displacement spectrum given by (10) is referred to as Seggern's model subsequently.

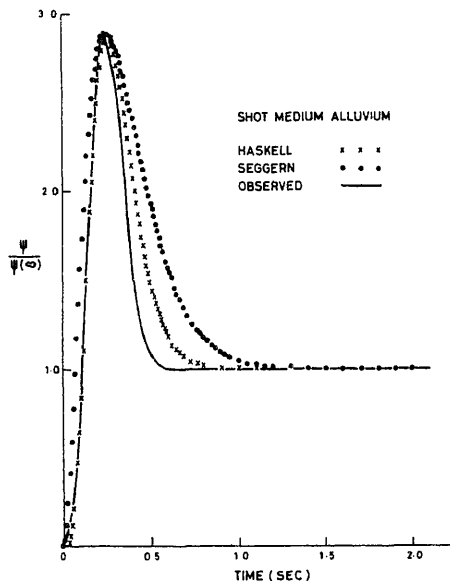


Figure 4. Same as figure 2.

The far-field displacement spectrum (source function) for Haskell and Seggern models given respectively by (8) and (10) is medium as well as yield-dependent. Hence, it is interesting to study the source function as well as its impulse response for both types of these models as a function of yield for various shot media. This study will depict the differences and similarities in the source function and its impulse response due to assumed models. Moreover, an understanding of the behaviour of source function and its impulse response as a function of yield is very useful for synthesizing seismograms from underground nuclear explosions.

Source function for Haskell and Seggern models given respectively by (8) and (10) was computed as a function of frequency for various values of yield for shot media of granite, salt, tuff and alluvium. Figures 5 and 6 show amplitude spectrum (arbitrary units) of source function as a function of frequency for various yields. The assumed source model is indicated at the top in these figures and shot medium is granite. It may be noted that maximum in the amplitude spectrum shifts to a lower frequency with increase in yield for a given model. For example, maximum in amplitude spectrum occurs at 1.8, 0.87 and 0.48 CPS (figure 5) respectively for yields of 10, 100 and 1000 kilotons for Haskell's model. A similar behaviour of source function is observed (figure 6) for Seggern's model also. These figures also show that for a given yield, maximum in the amplitude spectrum occurs at a lower frequency for Seggern's model as compared to Haskell's model.

The impulse response of the source function for these two models was also computed for several yields and the above mentioned shot media. However, impulse response for these two models is depicted (figures 7 and 8) as a function of yield only for granite shot media. Figure 7 shows that the duration of the impulse response of the source function increases with the increase in yield. Comparison of figures 7 and 8 indicates that for a given yield, the duration of impulse response of source function is shorter for Haskell's models as compared to Seggern's model. These results suggest

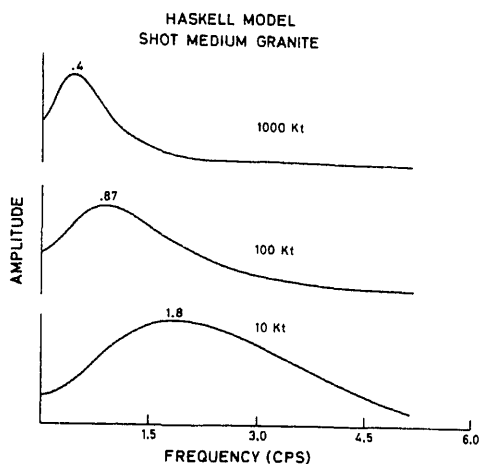


Figure 5. Amplitude spectra of the source function for various values of yield. Source model is that of Haskell and shot medium is granite.

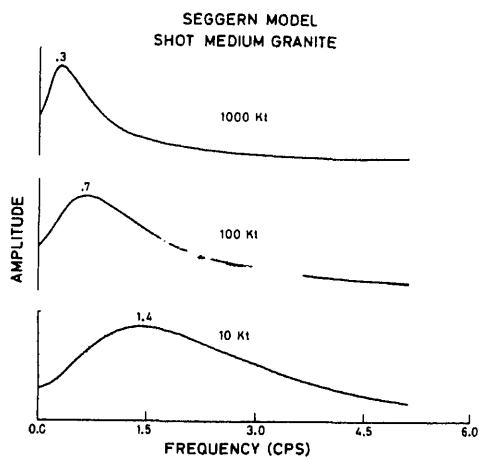


Figure 6. Amplitude spectra of the source function for various values of yield. Source model is that of Seggern and shot medium is granite.

that the period of synthesized *P* seismograms will depend upon the yield of explosion. Hasegawa (1972) and Springer and Hannon (1973) have presented data that agree with this suggestion.

3. Discussion and conclusions

Various constants occurring in analytic expression of reduced displacement potential proposed by Seggern and Blandford were computed for shot media of alluvium, salt and tuff. It is now possible to synthesize *P* seismograms for these media for source model of Seggern. The variation of spectral amplitude is studied for source models of Haskell and Seggern for different yields. It is observed that the maximum in the spectrum shifts to a lower frequency and the duration of impulse response increases with increase in yield for either of the source models. The duration of impulse response is longer for the Seggern model as compared to the Haskell model for a given yield.

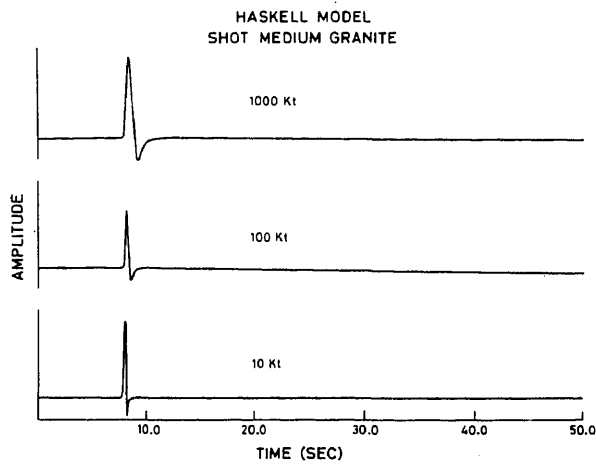


Figure 7. Impulse response of source function for various values of yield. Source model and shot medium are same as in figure 5.

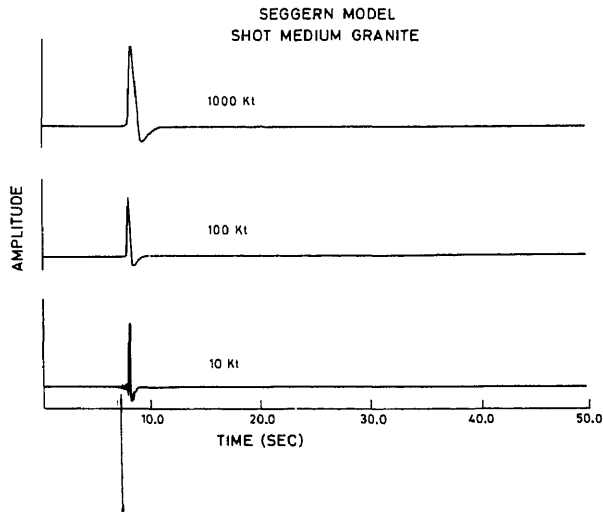


Figure 8. Impulse response of source function for various values of yield. Source model and shot medium are same as in figure 6.

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References

- Aki K, Bouchon M and Reasenber P 1974 *Bull. Seism. Soc. Am.* **64** 131
- Banghar A R 1980 Synthetic Seismograms - I: Transfer functions for synthesis of *P* wave seismograms from explosive sources and their parametric study, BARC Report I-617
- Banghar A R 1981 *Mechanisms of earthquakes and synthesis of P wave seismograms from explosions and earthquakes*, Ph.D. Thesis, Bombay University
- Hasegawa H S 1972 *J. Geophys. Res.* **77** 3081
- Haskell N A 1967 *J. Geophys. Res.* **72** 2583
- Healy J H, King C and O'Neill M E 1971 *J. Geophys. Res.* **76** 3344
- Holzer F 1966 *Proc. R. Soc. (London)* **A290** 409
- Kogeus K 1967 A synthesis of short period *P* wave records from distant explosion sources, Scientific Report No. 54, Seismological Institute, Uppsala University, Uppsala, Sweden
- Kogeus K 1968 *Bull. Seism. Soc. Am.* **58** 663
- Mueller R A and Murphy J R 1971 *Bull. Seism. Soc. Am.* **61** 1675
- Rogers L A 1966 *J. Geophys. Res.* **71** 3415
- Seggern V D and Lambert D G 1970 *J. Geophys. Res.* **75** 7385
- Seggern V D and Blandford R 1972 *Geophys. J.* **31** 83
- Sisemore C J, Rogers L A and Perret W R 1969 *J. Geophys. Res.* **74** 6623
- Springer D L and Hannon W J 1973 *Bull. Seism. Soc. Am.* **63** 477
- Werth G C and Herbst R F 1963 *J. Geophys. Res.* **68** 1463

Characteristics of Pi2 pulsations at an equatorial station and its occurrence association with the phase of the Moon

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Abstract. Diurnal variations of occurrence hours and the period of Pi2 pulsations at Choutuppal (India) for nearly half a solar cycle are presented. Maximum occurrence is noticed to be around local midnight in all the seasons. Shorter periods are observed in the late afternoon hours in E- and J-seasons. Lunar influence on the occurrence of Pi2 pulsations at this station is inferred with the occurrence due to this influence peaking in the vicinity of the lunar phase 4.

Keywords. Pulsations; equatorial station; lunar influence; diurnal variations.

1. Introduction

The irregular pulsations, Pi2, are shortlived fluctuations of the geomagnetic field ranging from 40–150 sec (Jacobs 1970). Pi2 are generally observed, simultaneous with the onset of substorms, over a wide range of latitudes on the ground and in the magnetosphere (Saito 1969; Gupta *et al* 1971; Gupta 1981). The characteristics of Pi2 have been studied earlier, especially in the auroral and high latitudes. However, such studies for low latitudes and equatorial regions are sparse. For a thorough understanding of the wave characteristics responsible for these Pi2s, a comprehensive morphological study at all latitudes will be useful.

In the present paper, the occurrence frequency and the diurnal variations of the period of Pi2s from an equatorial station, Choutuppal (Hyderabad) (Geomag. Lat. 7.5° N), a permanent pulsation recording station in the Indian region, are reported. These morphological characteristics at the equatorial station are compared with those obtained at middle and high latitudes by other investigators. It is shown that the occurrence frequency of Pi2s peaks at more or less the same local hours at different latitudes. Considering that Pi2s observed at all latitudes originate in the geomagnetic tail, which itself extends well beyond the lunar orbit, an attempt is made to investigate lunar influence on this class of pulsations.

2. Data analysis

The basic data for the study are extracted from the magnetic and telluric pulsations records for the period 1 January 1968 to 31 December 1972. These have been published by the National Geophysical Research Institute, Hyderabad. The method for decodin-

the pulsation data is the same as that adopted by Rao (1978) for Pc5 data. By a similar procedure, Pi2 activity reported for each day is decoded for 24 consecutive hours. The hourly interval is considered to have pulsational activity irrespective of the duration of activity. The data are sub-divided into three seasonal groups, summer (J, May to August); winter (D, November to February) and equinoxes (E, March, April, September and October). Diurnal variations of the occurrence of Pi2 as a function of local time (LT) for seasons and for the entire period considered (γ) are shown in figure 1. Also, median, 25th and 75th percentiles of the Pi2 periods for each of the hours are calculated and are shown in figure 2.

3. Results and discussion

3.1 Diurnal variation of occurrence hours

It is apparent from figure 1 that Pi2 activity is predominantly observed during local night at Choutuppal. The maximum occurrence is around local midnight in all the seasons. Similar results have been reported by Sanker Narayan and Sarma (1975) for the same station and by Channon and Orr (1970) for another equatorial station. At middle and high latitudes also, the maximum occurrence of Pi2 is near midnight (Yanagihara 1960; Smith 1973; Gupta 1981). No obvious seasonal variation is observed in the peak occurrence time of Pi2 at Choutuppal which is found to be between 00-01 LT.

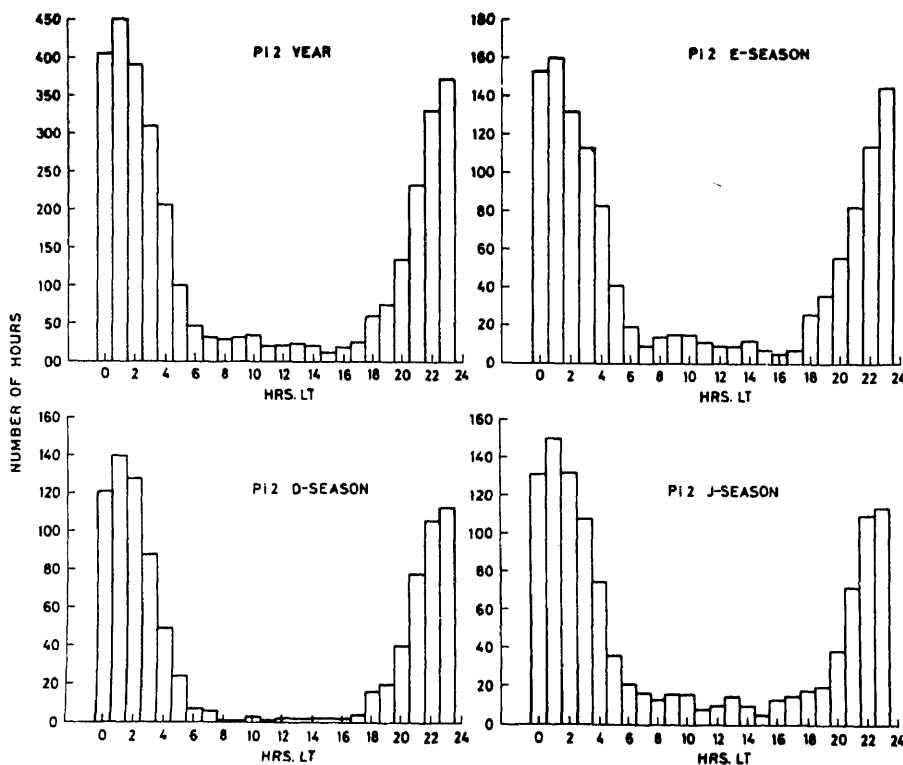


Figure 1. Diurnal variation of occurrence hours of Pi2 pulsations at Choutuppal for J, E and D seasons as well as for the entire period, designated as Y.

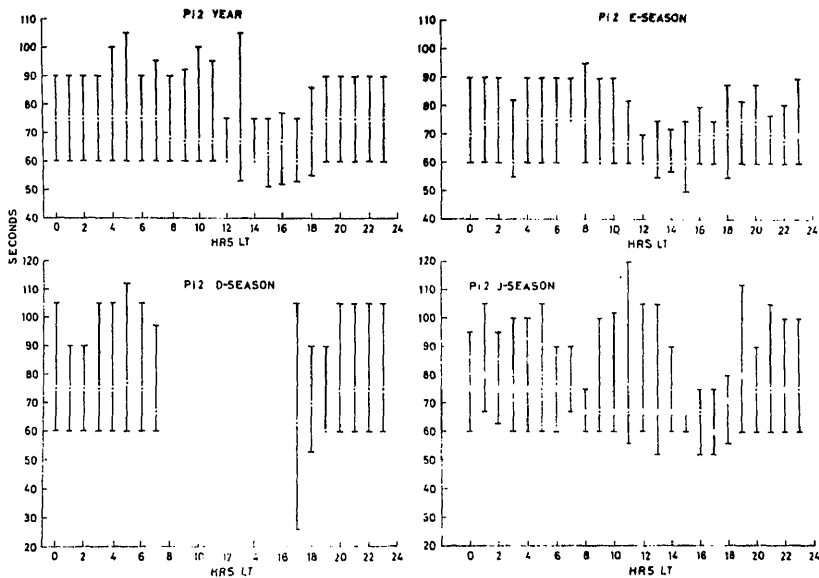


Figure 2. Diurnal variation of median periods of Pi2 at Choutuppal for J, E and D seasons as well as Y. 75th (upper line) and 25th (lower line) percentiles are also shown at each of the median periods.

Gupta (1981) has shown that at mid-latitude station, Ottawa (geomag. Lat. 57° N), the largest number of Pi2s occurred between 00-01 LT in D and E seasons and an hour earlier in J season. At higher latitudes also these pulsations have shown a clear seasonal variation. However, the peak occurrence of Pi2s in the vicinity of local midnight seems to be common at all latitudes.

3.2 Diurnal variation of Pi2 periods

It can be noticed from figure 2 that the Pi2 periods are generally larger during local night hours than during day-light hours. There is no definite progression of periods with the time of the day. However, shorter periods are observed in the late afternoon hours in the E and J seasons (around 14 LT). Gupta (1981) investigated the diurnal variation in the periods of Pi2 pulsations at four stations in the latitude range of 45 to 75° N. He found that there is a significant difference between diurnal variations of periods observed at high latitudes and that observed at the mid-latitude station, Ottawa. At Ottawa, the trend was reasonably well-defined with broad minimum in the early morning hours and a broad maximum in the afternoon.

From a comparison of the results it seems that the minimum of periods shifts from morning at mid-latitude to afternoon in the equatorial regions.

3.3 Lunar influence

Having established in the earlier section that the diurnal variations of occurrence of Pi2s at high and low latitudes are almost similar, it is apparent that the source of the instability can be identified in the same magnetotail region. According to Kikuchi *et al* (1978), the horizontal electric fields, such as those due to magnetospheric convection, are impressed on the high latitude ionosphere. From there they can propagate to the equator within the ionosphere in the guided mode as zero order transverse (TM) waves. Based on the analysis of over 10 years of Pc5 data from Choutuppal and comparing their characteristics with those at high latitude stations, Sarma *et al* (1981) have concluded that amplitudes of TM waves suffer significant attenuation in ionospheric transmission but the signals appearing in the equatorial region would still bear considerable resemblance to those at high latitudes. In the equatorial zone, all types of micropulsations propagate in the same direction (Prince and Bostick 1964; Field and Greifinger 1966); as such the Pi2s at Choutuppal are assumed to have the source in the magnetotail region and the waves propagate to the equatorial region in the same way as those of Pc5.

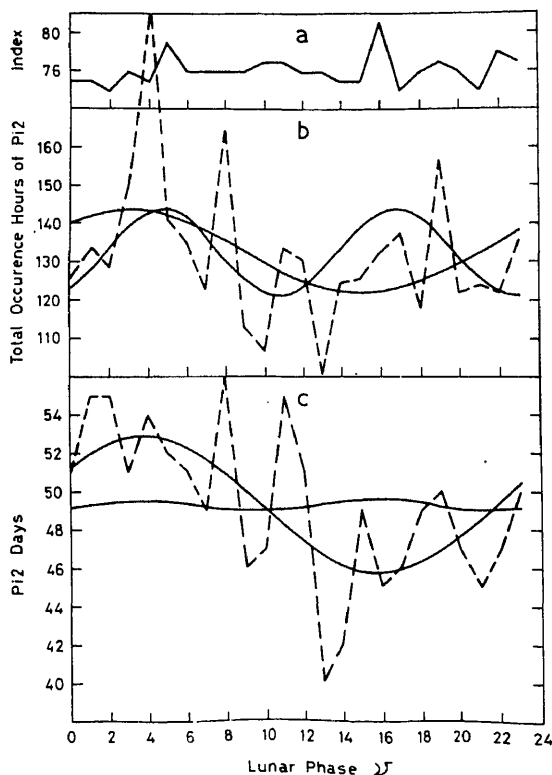


Figure 3. (a) Number of times the same lunar phase is reckoned against each of the lunar phases. (b) Total occurrence hours of Pi2s (broken line) vs the lunar phase numbers. The two smoothed curves are variations from the 1st and 2nd harmonic components. (c) Total number of days of Pi2 occurrences (broken line) vs the lunar phase numbers. The two smoothed curves are variations from the 1st and 2nd harmonic components.

Table 1. Results of harmonic analysis of Pi2 activity grouped according to lunar phases.

Harmonic Number	Total number of hours of Pi2 activity			Number of days of Pi2 activity		
	Amplitude (hr)	Lunar phase of 1st maximum	Percentage accounted in the total distribution	Amplitude (Days)	Lunar phase of 1st maximum	Percentage accounted in the total distribution
1	10.8	3.0	15.4	3.6	3.8	38.9
2	11.4	4.8	17.1	0.2	3.6	0.2
3	4.9	3.4	3.2	2.0	1.9	11.8
4	4.2	4.1	2.3	1.1	5.9	3.6

It is well-known that the average lunar orbit is at 60 earth radii and during the first quarter of lunation, the moon is in the antisolar direction in the magnetotail region. The aim here is to delineate the effect, if it exists, on the observed Pi2 pulsations when the moon, in its orbit, crosses the tail region of the magnetosphere. For this, the occurrence hours per day of Pi2 during 1 January 1968 to 31 December 1972 are grouped according to the lunar phase of that day. In a lunation, the lunar phases vary from 0 at new moon to 24 for the next. During the interval, the number of times the same lunar phase has been reckoned is shown as 'index' in figure 3a and the total occurrences of the Pi2s corresponding to each of the phases are shown in figure 3b. Treating the variations of Pi2 occurrences from phases 0 to 23 as fundamental period, harmonic analysis is carried out to derive the amplitudes and phase angles of the first four harmonics. The individual curves of the 1st and 2nd harmonics are also shown in figure 3b. The number of hours per day of Pi2s is felt to some extent subjective and a little over emphasised index of activity. For this, grouping is redone according to the lunar phase, assigning '1' for the day which had Pi2 activity and '0' otherwise. The total number of days of Pi2 activity and the first two harmonics obtained from them are plotted against each of the lunar phases in figure 3c. Amplitudes, lunar phases of the 1st maximum derived from the phase angles and the percentage accounted by each of the harmonics in the entire distribution for the above two groups are given in table 1.

There seems to be no undue biasing in the frequency of occurrence of the lunar phases in the interval which may have reflected in the pulsational activity. Any lunar phases association, either between the total occurrence hours per day or the number of days on which pulsations occurred, is not readily apparent from figures 3b or 3c. There is, however, a suggestion of lunar semi-monthly (2nd harmonic) association when the total number of hours are considered. The amplitudes of the lunar monthly (1st harmonic) and semi-monthly (2nd harmonic) variations of the total occurrence hours have nearly the same magnitude and they together account for 33% of the total distribution. On the other hand, though the variation of number of days of pulsational activity is not smooth in figure 3c, the lunar monthly harmonic (1st) is the dominant one and accounts for nearly 39% of the distribution. The most interesting aspect to note is that, systematically the variations of the lunar monthly and semi-monthly harmonics in both the cases reach their maximum amplitudes in the vicinity of lunar

phase 4 ± 1 . Thus, it appears that there is a suggestion of lunar influence on the occurrence of Pi2 pulsations at Choutuppal with occurrence due to this influence peaking at the lunar phase around 4.

It suggests that the passage of the moon through the geomagnetic tail may contribute to its dynamics in some way to influence the instability that results in Pi2 activity. The large scatter, which is not accounted for by the harmonics, is evidently due to the stronger influences of the electric and magnetic fields above the ionospheric levels responsible for the generation of Pi2s other than the moon.

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References

- Channon M J and Orr D 1970 *Planet. Space Sci.* **18** 229
Gupta J C, Stening R J and Jansen Van Beek G 1971 *J. Geophys. Res.* **76** 933
Gupta J C 1981 *J. Geophys. Res.* **86** 5665
Field E C and Greifinger C 1965 *J. Geophys. Res.* **70** 4885
Jacobs J A 1970 *Geomagnetic pulsations, physics and chemistry in space* (Berlin: Springer-Verlag)
Kikuchi T, Araki T, Maeda H and Malkawa K 1978 *Nature (London)* **273** 650
Prince C E and Bostick Jr F X 1964 *J. Geophys. Res.* **69** 3213
Rao D R K 1978 *J. Geomag. Geoelectr.* **30** 19
Saito T 1969 *Space Sci. Rev.* **10** 319
Sanker Narayan P V and Sarma S V S 1975 *Proc. Symp. Equatorial geomagnetic phenomenon*, Colaba, Bombay
Sarma Y S, Sastry T S, Sanker Narayan P V and Sarma S V S 1981 *J. Geomag. Geoelectr.* **33** 341
Smith B P 1973 *Planet. Space Sci.* **21** 831
Yanagihara K 1960 *Mem. Kakioka Mag. Obs.* **9** 15

Estimation of the waiting time distributions of earthquakes

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Abstract. Whether the earthquake occurrences follow a Poisson process model is a widely debated issue. The Poisson process model has great conceptual appeal and those who rejected it under pressure of empirical evidence have tried to restore it by trying to identify main events and suppressing foreshocks and aftershocks. The approach here is to estimate the density functions for the waiting times of the future earthquakes. For this purpose, the notion of Gram-Charlier series which is a standard method for the estimation of density functions has been extended based on the orthogonality properties of certain polynomials such as Laguerre and Legendre. It is argued that it is best to estimate density functions in the context of a particular null hypothesis. Using the results of estimation a simple test has been designed to establish that earthquakes do not occur as independent events, thus violating one of the postulates of a Poisson process model. Both methodological and utilitarian aspects are dealt with.

Keywords. Gram-Charlier series; earthquakes; Hermite polynomials; Laguerre polynomials; Poisson process; Polya process.

1. Introduction

The question whether the earthquake occurrences follow a Poisson process model has been widely addressed (Benioff 1951; Aki 1956; Shalanger 1960; Knopoff 1964; Lomnitz 1966; Ferraes 1967; Vere Jones 1970; Schlien and Toksoz 1970; Utsu 1972; Udias and Rice 1975). Under the Poisson process model, the number of earthquakes x per unit time follows a Poisson distribution

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad \lambda > 0, \quad k = 0, 1, 2, \quad (1)$$

where λ is the rate of the process and the inter-arrival time t_1 for earthquakes has a density function

$$p_{T_1}(t_1) = \lambda \exp(-\lambda t_1) u(t_1) \quad (2)$$

where $u(t_1)$ is a unit step function. Most frequently (1) and (2) have been fitted to the empirical data in the form of histograms using least squares criterion and bad fit taken

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as an indication that the Poisson process model is inappropriate. Rarely other density functions such as

$$p_{T_1}(t_1) = \frac{\lambda^p t_1^{p-1}}{(p-1)!} \exp(-\lambda t_1) u(t_1), \quad \lambda > 0, \quad p \text{ positive integer} \quad (3)$$

of which (2) is a special case with $p = 1$, have been fitted in the least squares sense (Udias and Rice 1975) and $p = 2$ was thought to give better fit than $p = 1$, though systematic deviation from the gamma density function of (3) with $p = 2$ has also been reported.

The approach followed in this paper is different. No particular density functions are fitted to the empirical inter-arrival times or waiting times but the density functions are estimated using Gram-Charlier type techniques.

2. Gram-Charlier series

If y is a random variable which has zero mean and unit variance (this can be achieved by standardization), its density function $p(y)$ can be written as (Whalen 1971)

$$p(y) = \varphi(y) + \sum_{n=3}^{\infty} a_n \varphi^{(n)}(y), \quad -\infty < y < \infty, \quad (4)$$

where

$$\varphi(y) = \frac{1}{\sqrt{2\pi}} \exp(-y^2/2), \quad (5)$$

is a standard normal density function, $\varphi^{(n)}(y)$ is its n th derivative given as

$$\varphi^{(n)}(y) = (-1)^n \varphi(y) H_n(y), \quad (6)$$

where $H_n(y)$ is a Hermite polynomial of degree n obtainable by recursive relation

$$H_{n+1}(y) = yH_n(y) - nH_{n-1}(y), \quad H_0(y) = 1, \quad H_1(y) = y. \quad (7)$$

The coefficient a_n in the Gram-Charlier expansion is given as

$$a_n = \frac{(-1)^n}{n!} \int_{-\infty}^{\infty} p(y) H_n(y) dy, \quad (8)$$

wherein the biorthogonality relation

$$\int_{-\infty}^{\infty} H_m(y) \varphi^{(n)}(y) dy = (-1)^n n! \delta_{mn} \quad n, m = 0, 1, \dots \quad (9)$$

is used, where δ_{mn} is the Kronecker delta. Substituting for $H_n(y)$ in (8) from (7) we get

$$a_3 = -\mu_3/3!; \quad a_4 = (\mu_4 - 3)/4!;$$

$$a_5 = -(\mu_5 - 10\mu_3)/5!; \quad a_6 = (\mu_6 - 15\mu_4 + 30)/6!;$$

$$\begin{aligned}
a_7 &= -(\mu_7 - 21\mu_5 + 105\mu_3)/7!; \\
a_8 &= (\mu_8 - 28\mu_6 + 210\mu_4 - 315)/8!; \\
a_9 &= -(\mu_9 - 36\mu_7 + 378\mu_5 - 1260\mu_3)/9!
\end{aligned} \tag{10}$$

where μ_i are standard central moments. These results are well-known (Whalen 1971) and are summarized here just because they are used in this paper as a point of departure. As standard central moments can be estimated from the empirical data, the coefficients a_n can be estimated from (10) and therefore, the density function $p(y)$ can be estimated from (4).

For reasons documented elsewhere (Kenney and Keeping 1963; Fry 1965), for truncating, terms in (4) must be grouped as 0, (3), (4, 6), (5, 7, 9), (8, 10, 12), (11, 13, 15), (14, 16, 18), every group of terms should be included or excluded collectively. The Gram-Charlier series thus truncated is called Edgeworth series.

Using microearthquake data from North-East India, (see § 5) it was found that

$$\begin{aligned}
a_3 &= -0.3137; & a_4 &= 0.1437; & a_5 &= 0.01965; \\
a_6 &= 0.01168; & a_7 &= -0.005139; & a_9 &= -0.001065
\end{aligned} \tag{11}$$

Using data from the Hindukush region (see § 5) it was found that

$$\begin{aligned}
a_3 &= -0.4567; & a_4 &= 0.5661; & a_5 &= -0.7936; \\
a_6 &= 1.1851; & a_7 &= -1.5682; & a_9 &= -1.9469
\end{aligned} \tag{12}$$

Thus particularly for the large earthquakes the coefficients a_n do not decrease rapidly enough as n sweeps the Edgeworth groups. With the zeroth and the third term included the density function has a mode away from the origin. With $n = 0, 3, 4$ and 6 terms included, the density function gives nonphysical values around origin. With $n = 0, 3, 4, 6, 5, 7$ and 9 terms included the density function is monotonically decreasing. Thus the behaviour is not yet stable indicating that $n = (8, 10, 12), (11, 13, 15)$, terms must also be added. This is not very simple. Because in (10) unbiased estimates of the moments must be substituted and obtaining unbiased estimators of central moments using small sample theory (Kenney and Keeping 1963) is a fairly tedious task. Moreover, higher order central moments become less and less reliable when estimated from finite samples. Thus, it is almost certain that the Gram-Charlier series, due to its poor convergence, is not a proper method to estimate density functions in our context.

However, as Gram-Charlier series may be useful in other contexts and as unbiased estimators for central moments are not readily available beyond the fourth moment, they are listed here for easy reference

$$\hat{\mu}_2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - n_1)^2,$$

$$\hat{\mu}_3 = \frac{N}{(N-1)(N-2)} \sum_{i=1}^N (x_i - n_1)^3,$$

$$\hat{\mu}_4 = \frac{N^2}{(N-1)(N^2-3N+3)} \sum_{i=1}^N (x_i - n_1)^4,$$

$$\hat{\mu}_5 = \frac{N^3}{(N-1)(N-2)(N^2-2N+2)} \sum_{i=1}^N (x_i - n_1)^5,$$

$$\hat{\mu}_6 = \frac{N^4}{(N-1)(N^4-5N^3+10N^2-10N+5)} \sum_{i=1}^N (x_i - n_1)^6,$$

$$\hat{\mu}_7 = \frac{N^5}{(N-1)(N-2)(N^4-4N^3+7N^2-6N+3)} \sum_{i=1}^N (x_i - n_1)^7,$$

$$\hat{\mu}_8 = \frac{N^6}{(N-1)(N^6-7N^5+21N^4-35N^3+35N^2-21N+7)} \sum_{i=1}^N (x_i - n_1)^8,$$

$$\hat{\mu}_9 = \frac{N^7}{(N-1)(N-2)(N^6-6N^5+16N^4-24N^3+22N^2-12N+4)} \sum_{i=1}^N (x_i - n_1)^9,$$

(13)

Small sample theory (Kenney and Keeping 1963) is used to derive (13), the terms of the order of N^{-1} are neglected, where N is the number of samples, $x_1, x_2, \dots, x_N$ and n_1 is the sample mean, and the details are reported elsewhere (Malasi 1981).

3. Gram-Charlier series of other types

3.1 The motivation

In Gram-Charlier series of (4), the first term is the normal density function and the other terms measure deviation from it. Thus it is best suited if normal density function is the null hypothesis or prototype. In the context of estimating density function for the inter-arrival times for earthquakes the density function according to the null hypothesis of Poisson process model for earthquake occurrences is a negative exponential density function of (2) (Massey 1971; Udias and Rice 1975). It would therefore be desirable to have a series expansion for a density function in which the leading term is a negative exponential density function so that the remaining terms directly measure deviation from the null hypothesis.

Apart from the derivation using characteristic function, cumulants, etc. (Whalen 1971) the end result of the Gram-Charlier series can be interpreted as a biorthogonal expansion exploiting the biorthogonality relation of (9) between the derivatives $\varphi^{(n)}(y)$ of the normal density function and the Hermite polynomials. The waiting times for future earthquakes can take only non-negative values, whereas the Gram-Charlier

series of (4) was biorthogonal expansion over the range $(-\infty, \infty)$. It would be more appropriate to use orthogonality or biorthogonality properties of some suitably chosen functions over the range $(0, \infty)$.

2. Laguerre type Gram-Charlier series

$L_n(t)$ defined as (Krishnamurthy and Sen 1976)

$$L_n(t) = (-1)^n e^t \frac{d^n}{dt^n} (t^n e^{-t}), \quad 0 \leq t < \infty. \quad (14)$$

and satisfying the recurrence relation

$$\begin{aligned} L_{n+1}(t) &= (2n + 1 - t) L_n(t) - n^2 L_{n-1}(t); \\ L_0(t) &= 1, \quad L_1(t) = 1 - t \end{aligned} \quad (15)$$

called Laguerre polynomial of order n . Let

$$\Psi_n(t) = L_n(t) e^{-t}. \quad (16)$$

then we have a biorthogonality property

$$\frac{(-1)^n}{(n!)^2} \int_0^\infty L_m(t) \Psi_n(t) dt = \delta_{nm}, \quad n, m = 0, 1, 2, \dots \quad (17)$$

using which any square integrable function $f(t)$ defined over the range $[0, \infty]$ can be expressed as

$$f(t) = \sum_{n=0}^{\infty} a_n \Psi_n(t), \quad 0 \leq t < \infty. \quad (18)$$

where

$$a_n = (-1)^n \frac{1}{(n!)^2} \int_0^\infty f(t) L_n(t) dt \quad (19)$$

Using (15), it can be shown that

$$\begin{aligned} a_n &= 1; \quad a_1 = (1 - \nu_1); \quad a_2 = (2 - 4\nu_1 + \nu_2)/(2!)^2; \\ a_3 &= (6 - 18\nu_1 + 9\nu_2 - \nu_3)/(3!)^2; \\ a_4 &= (24 - 96\nu_1 + 72\nu_2 - 16\nu_3 + \nu_4)/(4!)^2; \\ a_5 &= (120 - 600\nu_1 + 600\nu_2 - 200\nu_3 + 25\nu_4 - \nu_5)/(5!)^2; \\ a_6 &= (720 - 4320\nu_1 + 5400\nu_2 - 2400\nu_3 + 450\nu_4 - 36\nu_5 \\ &\quad - \nu_6)/(6!)^2; \end{aligned} \quad (20)$$

where ν_m is the m th moment of the random variable t of which $f(t)$ is the density function.

In addition to the two motivating factors discussed in § 3.1, the Laguerre type Gram-Charlier series has one more advantage. The coefficients of the Gram-Charlier series of (4) depend on the central moments as in (10), so that unbiased estimators for them have to be obtained. On the other hand, the coefficients of the Laguerre type Gram-Charlier series of (18) depend only on the moments of the random variable as in (20) and the sample moments do not have a bias in this case.

The values of the coefficients in the Laguerre type Gram-Charlier series for the inter-arrival times in the Hindukush area were found to be

$$\begin{aligned} a_0 &= 1; & a_1 &= 0.1301 \times 10^{-17}; & a_2 &= 0.5630 \times 10^{-1}; \\ a_3 &= -0.1003 \times 10^{-1}; & a_4 &= 0.3604 \times 10^{-2}; \\ a_5 &= 0.7429 \times 10^{-4}; & a_6 &= -0.8283 \times 10^{-4}. \end{aligned} \quad (21)$$

The density functions estimated by truncating the series after third, fourth, fifth and sixth terms are not very different, thus indicating good convergence of the series. The inter-arrival time data was scaled to have unit mean before fitting the density function.

3.3 Legendre type Gram-Charlier series

The polynomials obtained by the recurrence relation

$$\begin{aligned} (n+1)P_{n+1}(x) &= x(2n+1)P_n(x) - nP_{n-1}(x); \\ P_0(x) &= 1, & P_1(x) &= x, \end{aligned} \quad (22)$$

are called Legendre polynomials and they have a orthogonality property over the range $[-1, 1]$ that

$$\int_{-1}^1 P_m(x) P_n(x) dx = \frac{2}{2m+1} \delta_{mn}. \quad (23)$$

These polynomials cannot be applied directly to our present context, as we require functions orthogonal or biorthogonal over $[0, \infty)$. But this could be achieved by a suitable change of variable (Lee 1960). First let

$$x = 2y - 1 \quad (24)$$

so that

$$2 \int_0^1 P_m(2y-1) P_n(2y-1) dy = \frac{2}{2m+1} \delta_{mn}. \quad (25)$$

Thus functions $P_m(2y-1)$ as functions of y are orthogonal over the range $[0, 1]$. Subsequently, let

$$y = \exp(-pt) \quad (26)$$

so that

$$\int_0^{\infty} p e^{-pt} P_m(2e^{-pt} - 1) P_n(2e^{-pt} - 1) dt = \frac{1}{2m+1} \delta_{mn}, \quad (27)$$

so that the functions $P_m(2e^{-pt} - 1)$ as functions of t are orthogonal in the range $[0, \infty)$ with $p \exp(-pt)$ as the weight function. Therefore, any square integrable function $f(t)$ can be expanded as

$$f(t) = \sum_{n=0}^{\infty} a_n \Psi_n(pt), \quad 0 \leq t < \infty, \quad (28)$$

where

$$\Psi_n(pt) = P_n(2 \exp(-pt) - 1) \exp(-pt), \quad (29)$$

and

$$a_n = (2n+1)p \int_0^{\infty} f(t) P_n(2e^{-pt} - 1) dt, \quad (30)$$

on the basis of (27). From (22) and (29) we have

$$\begin{aligned} \Psi_0(pt) &= \exp(-pt), \\ \Psi_1(pt) &= -\exp(-pt) + 2\exp(-2pt), \\ \Psi_2(pt) &= \exp(-pt) - 6\exp(-2pt) + 6\exp(-3pt), \\ \Psi_3(pt) &= -\exp(-pt) + 12\exp(-2pt) - 30\exp(-3pt) \\ &\quad + 20\exp(-4pt), \text{ etc.} \end{aligned} \quad (31)$$

If $f(t)$ is a density function, the m th moment is

$$\begin{aligned} E(t^m) &= \sum_{n=0}^{\infty} a_n \int_0^{\infty} t^m P_n(2e^{-pt} - 1) \exp(-pt) dt, \\ &= \sum_{n=0}^{\infty} a_n f_{n,m}(p), \end{aligned} \quad (32)$$

where

$$f_{n,m}(p) = \int_0^{\infty} t^m P_n(2e^{-pt} - 1) \exp(-pt) dt. \quad (33)$$

Using terms upto $n = 4$, (22), (33) and replacing m th moment $E(t^m)$, by the m th sample moment $\hat{E}(t^m)$ we get

$$p^2 \hat{E}(t) = a_0 - \frac{1}{2} a_1 + \frac{1}{6} a_2 - \frac{1}{12} a_3 + \frac{1}{20} a_4,$$

$$p^3 \hat{E}(t^2) = a_0 - \frac{3}{4} a_1 + \frac{17}{36} a_2 - \frac{43}{144} a_3 + \frac{247}{1200} a_4,$$

$$p^4 \hat{E}(t^3) = a_0 - \frac{7}{8} a_1 + \frac{151}{216} a_2 - \frac{937}{1728} a_3 + \frac{30689}{72000} a_4,$$

$$p^7 \hat{E}(t^6) = a_0 - \frac{63}{64} a_1 + \frac{44597}{46656} a_2 - \frac{2743363}{2985984} a_3 + \frac{303179}{345600} a_4, \quad (34)$$

where use has been made of

$$\int_0^{\infty} t^m \exp(-qpt) dt = \frac{m!}{(qp)^{m+1}}. \quad (35)$$

These equations yield a polynomial in p

$$S_6 p^7 + S_5 p^6 + S_4 p^5 + S_3 p^4 + S_2 p^3 + S_1 p^2 = 0. \quad (36)$$

Because $p = 0$ is not an acceptable choice, we have

$$S_6 p^5 + S_5 p^4 + S_4 p^3 + S_3 p^2 + S_2 p + S_1 = 0. \quad (37)$$

where

$$\begin{aligned} S_1 &= 0.906854 \hat{E}(t); \quad S_2 = -0.857505 \hat{E}(t^2); \\ S_3 &= 3.0437042 \hat{E}(t^3); \quad S_4 = -4.9862859 \hat{E}(t^4); \\ S_5 &= 3.7042561 \hat{E}(t^5); \quad \text{and } S_6 = -0.9948547 \hat{E}(t^6). \end{aligned} \quad (38)$$

Solving (37) and using a real positive root for p (34) gives a set of five equations (excluding the last) which are linear simultaneous equations in constants a_0, a_1, a_2, a_3, a_4 to be evaluated, which is a standard task. Instead of truncating the expansion in (28) after $n = 4$, terms upto $n = N$ could be retained. Then (34) will have $N + 2$ equations. The polynomial in (36) will become

$$S_{N+1} p^{N+1} + S_{N+2} p^{N+2} + \dots + S_1 p^2.$$

Using the real positive root of this for p , (34), excluding the last equation will have $N + 1$ simultaneous linear equations in $N + 1$ constants $a_0, a_1, \dots, a_N$. Then (28) gives an expansion for the density function.

Using data for interarrival times for earthquakes in Hindukush, we obtain

$$\begin{aligned} p &= 0.12424 \times 10^1; \quad a_0 = -0.43302 \times 10^1; \\ a_1 &= -0.13783 \times 10^2; \quad a_2 = -0.22956 \times 10^2; \\ a_3 &= -0.22951 \times 10^2; \quad a_4 = -0.96675 \times 10^1. \end{aligned} \quad (39)$$

The density function given by these coefficients after substitution in (28) was nonphysical, as suggested by the negative value of a_0 and large values of other coefficients.

3.4 Fast convergent expansions

The idea to be discussed has already been used in § 3.3 but here it is put in a more general context.

If functions $\varphi_n(t)$ are orthonormal, *i.e.* if

$$\int_0^{\infty} \varphi_n(t) \varphi_m^*(t) dt = \delta_{nm}, \quad (40)$$

where the superscript asterisk denotes complex conjugation, it easily follows that the functions $\varphi_n(\beta t)$, where β is an arbitrary positive constant, are also orthogonal, because (40) gives

$$\int_0^{\infty} \varphi_n(\beta y) \varphi_m^*(\beta y) dy = \frac{1}{\beta} \delta_{nm}. \quad (41)$$

Equation (40) implies that any square integrable function over $[0, \infty)$ can be expanded as

$$f(t) = \sum_{n=0}^{\infty} a_n \varphi_n(t). \quad (42)$$

Equation (41) implies that we can also have the expansion

$$f(t) = \sum_{n=0}^{\infty} b_n \varphi_n(\beta t), \quad \beta > 0, \quad (43)$$

where β is arbitrary, but certainly the coefficients b_n will depend on the choice of β . If the expansions in (42) and (43) are not to be truncated, the choice of β is really immaterial. But if the series in (42) and (43) are to be truncated, it would seem advantageous to choose β judiciously so that the expansion has smallest possible number of terms for the same mean error. To illustrate this point, let $f(t)$ be $\varphi_p(\alpha t)$, $\alpha \neq 1$ for a particular value of p . Then in (42) we will need many terms to approximate $f(t)$. But in (43) a single term will suffice if β is chosen to be α .

This problem of choosing the scale factor β will arise for truncated orthonormal expansions over an infinite interval $(-\infty, \infty)$ also. This problem does not seem to have been discussed in literature explicitly. In the Gram-Charlier expansion of (4), standardization of the random variable amounts to choosing the scale factor to be the standard deviation.

As an example of how optimum value of β could be chosen, (19) is written as

$$\frac{(-1)^n}{(n!)^2} \beta \int_0^{\infty} I_m(\beta t) \Psi_n(\beta t) dt = \delta_{nm}. \quad (44)$$

Then instead of (18) we could have the series expansion

$$f(t) = \sum_{n=0}^N a_n \Psi_n(\beta t), \quad (45)$$

where the same symbol is retained for the coefficients but their values would now depend on the choice of β . If $f(t)$ is a density function we would have

$$E(t^m) = \int_0^{\infty} t^m f(t) dt,$$

$$= \sum_{n=0}^N a_n f_{n,m}(\beta). \quad (46)$$

where (16) has been used and

$$f_{n,m}(\beta) = \int_0^{\infty} t^m L_n(\beta t) \exp(-\beta t) dt. \quad (47)$$

The functions $f_{n,m}(\beta)$ can be easily computed. For example, using (15)

$$\begin{aligned} f_{1,2}(\beta) &= \int_0^{\infty} t^2 L_1(\beta t) \exp(-\beta t) dt, \\ &= \int_0^{\infty} t^2 (1 - \beta t) \exp(-\beta t) dt, \\ &= -4/\beta^3. \end{aligned} \quad (48)$$

Thus all the other values of $f_{n,m}(\beta)$ could be calculated. Substituting these in (46), using $N = 4$ as an example and writing (46) for $m = 1, 2, \dots, 6$, we get

$$\begin{aligned} \beta^2 \hat{E}(t) &= a_0 - a_1, \\ \beta^3 \hat{E}(t^2) &= 2(a_0 - 2a_1 + 2a_2), \\ \beta^4 \hat{E}(t^3) &= 6(a_0 - 3a_1 + 6a_2 - 6a_3), \\ \beta^5 \hat{E}(t^4) &= 24(a_0 - 4a_1 + 12a_2 - 24a_3 + 24a_4), \\ \beta^6 \hat{E}(t^5) &= 120(a_0 - 5a_1 + 20a_2 - 60a_3 + 120a_4), \\ \beta^7 \hat{E}(t^6) &= 720(a_0 - 6a_1 + 30a_2 - 120a_3 + 360a_4). \end{aligned} \quad (48)$$

In general $N + 2$ such equations would be obtained. Here population moments $E(t^m)$ are replaced by sample moments $\hat{E}(t^m)$ because of the context of estimating density function from the sample moments. Equation (48) is 6 equations in 6 unknown. On solving these and ruling out the solution $\beta = 0$, we get

$$S_6 \beta^5 + S_5 \beta^4 + S_4 \beta^3 + S_3 \beta^2 + S_2 \beta + S_1 = 0, \quad (49)$$

where

$$\begin{aligned} S_1 &= -\hat{E}(t); \quad S_2 = 5\hat{E}(t^2)/2!; \\ S_3 &= -10\hat{E}(t^3)/3!; \quad S_4 = 10\hat{E}(t^4)/4!; \\ S_5 &= -5\hat{E}(t^5)/5!; \quad S_6 = \hat{E}(t^6)/6! \end{aligned} \quad (50)$$

Solving (49) for a positive real value of β and excluding the last equation in (48), we are left with 5 linear simultaneous equations in 5 unknowns $a_0, a_1, \dots, a_4$, and these can be easily solved.

Once the coefficients a_n and β are evaluated, (45) gives the density function. In general (49) would be a polynomial of degree $N + 1$, yielding $(N + 1)$ possible

choices of the scale factor β . As N increases, many different values of β would be available, making the choice of β less and less important, so that when the expansion is not truncated, any arbitrary value of β would do. This is a fortunate feature because to find out roots of a polynomial of large degree is an increasingly difficult task, whereas for a small value of N , the roots are relatively easily found and the choice of β is also more critical.

Using this method for the Hindukush data for inter-arrival times of earthquakes, we get

$$\begin{aligned}\beta &= 1.1328; & a_0 &= 1.2425; \\ a_1 &= -0.040787; & a_2 &= 1.14667; \\ a_3 &= -0.0095450; & a_4 &= 0.014222.\end{aligned}\quad (51)$$

Note that the value of a_0 has increased in comparison with (21). The other coefficients are still fairly small.

4. A test for independent occurrence of earthquake

It is desirable to check whether the assumption of the independent occurrence of earthquakes, which is made in the derivation of a Poisson process model (Fisz 1963) is valid. A simple test can be set for this purpose. The density function for the waiting time for the next event (*i.e.* the inter-arrival time) has already been estimated using Laguerre type Gram-Charlier series in § 3.2. If the earthquakes occur statistically independently, the density function for the waiting time for the second event would be a convolution of the density function for the waiting time for the next event with itself (Davenport and Root 1958) because the waiting time for the second event is the sum of the waiting time for the next event and the waiting time for the next event again. The density function for the waiting time for the next event is

$$p_{T_1}(t) = \sum_{n=0}^6 a_n L_n(t) e^{-t}, \quad (52)$$

where the coefficients are given by (21). Then

$$p_{T_1}(t) * p_{T_1}(t) = \left\{ \sum_{n=0}^6 a_n L_n(t) e^{-t} \right\} * \left\{ \sum_{n=0}^6 a_n L_n(t) e^{-t} \right\} \quad (53)$$

where $*$ denotes convolution. Substituting for $L_n(t)$ from (15) and using the fact that

$$\begin{aligned}\{t^n e^{-t}\} * \{t^m e^{-t}\} &= \int_0^t u^n e^{-u} (t-u)^m e^{-(t-u)} du, \\ &= e^{-t} \int_0^t u^n (t-u)^m du, \\ &= e^{-t} \frac{(t^{n+m+1}) m! n!}{(n+m+1)!}\end{aligned}\quad (54)$$

we get

$$\begin{aligned}p_{T_1}(t) * p_{T_1}(t) &= e^{-t} \{1.18419t - 0.084268t^2 + 0.12952t^3 + 0.076443t^4 \\ &\quad - 0.013856t^5 - 0.0036101t^6\}\end{aligned}\quad (55)$$

Equation (55) gives the density function for the waiting time for the second earthquake under the assumption that the earthquakes occur independently. The density function for the waiting time for the second earthquake can also be estimated empirically by the method in § 3.2. For the Hindukush data it is (Goel 1982)

$$p_2(t) = e^{-t} [0.27272 + 5.17921t - 1.96555t^2 + 0.56842t^3 + 0.076003t^4 + 0.004638t^5 - 0.000103t^6]. \quad (56)$$

It is obvious that the two sets of coefficients in (55) and (56) do not match. Thus the assumption of the independent occurrence of earthquakes stands refuted.

The density function for the waiting time for the third earthquake has also been empirically estimated for the Hindukush data using the method of § 3.2 and is (Goel 1982)

$$p_3(t) = e^{-t} [-0.006602 + 4.10569t - 3.09795t^2 + 0.88408t^3 + 0.11608t^4 + 0.0069548t^5 - 0.00015204t^6]. \quad (57)$$

Even this could have been used to check the hypothesis of the independent occurrence of earthquakes.

5. Data

The data regarding earthquakes in the Hindukush region are taken from the catalogue of epicentral locations of earthquakes prepared by the Indian Meteorological Department (IMD). These catalogues are prepared by IMD by using the data from USCSS, ISC, ISS and various other agencies. In all, 1535 earthquakes of magnitudes greater than 3.5, from January 1970 to December 1976, of focal depth less than 250 km in an area bounded by 69° E to 72° E longitude and 35° N to 38° N latitude, are used.

Another set of data regarding micro-earthquakes from North-Eastern region of India is also used. These data were collected by the University of Roorkee and GSI under a joint project for a period of 5.5 months from May 1979 to October 1979. There were stations at Raliang (25.47° N, 92.43° E), Borjori (26.40° N, 92.94° E), Burnihat (26.06° N, 91.89° E), and Shillong (25.57° N, 91.88° E). The Shillong station was run by the IMD. Magnitudes were between 2.2 to 5.1. Sources having distances more than about 300 km, i.e. those with S-P times greater than 40 sec were excluded. In all 235 events were used for the analysis.

5.1 Conclusions

- (a) A simple test for independent occurrence of earthquakes was designed. The hypothesis that the earthquakes are independent events stands refuted.
- (b) The Laguerre type Gram-Charlier series, in which the leading term is the negative exponential density function for the interarrival times for earthquakes under the Poisson process model for earthquakes, is the best method for the empirical

estimation of waiting time distribution for earthquake occurrences. Some of the reasons for this are discussed in §§ 3.1 and 3.2.

- (c) The scale factor β should be properly chosen for the fast convergence of orthonormal expansion over infinite and semi-infinite intervals.
- (d) Though one of the assumptions of the Poisson process model for earthquakes is invalid, the estimated density function for the inter-arrival time of earthquakes is not very different from the negative exponential density function as shown by the small values of $a_1, a_2, \dots$ in (21) and (51). The other consequences of the Poisson process model, such as other waiting time distributions, may be incorrect.

5.2 Criticism and scope for future work

(a) Various gamma density functions of (3) have been considered as models for the inter-arrival time of earthquakes (Udias and Rice 1975). The approach used here could be interpreted as a generalization of this notion because the Laguerre type Gram-Charlier series of § 3.2 can be viewed as a linear combination of various gamma density functions. If the gamma density function of (3) is fitted, the parameter p has been interpreted (Udias and Rice 1975) as the reciprocal of the average cluster size. Can this clue be generalized and a better index of clustering defined in terms of the coefficients of the Laguerre type Gram-Charlier series?

(b) The Laguerre type Gram-Charlier series could be interpreted as obtained by considering a linearly independent set of functions of (3), one for every positive integer value of p and obtaining a biorthonormal set of functions from them by Gram-Schmidt procedure (Barrett 1963). Similarly, one could start with a linearly independent set of functions $p \exp(-pt), 2p \exp(-2pt), 3p \exp(-3pt), \dots$ and obtain a biorthonormal set of functions from them by Gram-Schmidt procedure. This could be considered to be the basis of Legendre-type Gram-Charlier series which was derived in § 3.3 from a different starting point. In expanding square integrable functions, orthonormalization or biorthonormalization of a set of linearly independent functions is said to be a proper step. But in obtaining expansion for a density function, it need not be so. For example, one could have written

$$p(t) = \sum_{n=1}^{\infty} \alpha_n \frac{\lambda^n t^{n-1}}{(n-1)!} \exp(-\lambda t) u(t), \quad (58)$$

directly which amounts to writing a density function $p(t)$ as a convex combination of gamma density functions provided

$$\alpha_n \geq 0, \quad \sum \alpha_n = 1. \quad (59)$$

Equation (18) is equivalent to (58) for square integrable bipolar functions, but nothing equivalent to (59) has been imposed therein. As a result there is a no guarantee that (18) will give a function which will satisfy $f(t) \geq 0$, so that $f(t)$ can be a valid density function. This can be clearly seen from (21), (51), (56) and (57). Similarly, one could have written

$$p(t) = \sum_{n=1}^{\infty} \alpha_n n p \exp(-npt) \quad (60)$$

with (59) imposed. On the other hand Legendre type Gram-Charlier series has already yielded a blatantly nonphysical density function as reported in § 3.3. This is the

consequence of using notions of orthonormality, biorthonormality, Gram-Schmidt procedure, etc. applicable for square integrable functions to the so-called class D of functions which are candidates for density functions. Suitable class D procedures should be developed. The Gram-Charlier expansion of § 2, however, cannot be interpreted as a convex combination of a family of density functions. This could be viewed as one more disadvantage of it.

(c) It has been suggested that the Laguerre type Gram-Charlier series or its class D counterpart is a particularly good choice for estimating the waiting time density functions for the future earthquakes in the context of a Poisson process model. This point should be raised to a general level. Suppose the null hypothesis is not a Poisson process model but a Polya process model (Fisz 1963; Sharma 1982; Sharma *et al* 1983). Then the various waiting time density functions are

$$p_{T_p}(t) = \frac{v(v+1)\dots(v+p-1)}{(a+t)^{v+p}} \frac{t^{p-1}}{(p-1)!} \quad t > 0 \quad p = 1, 2, 3, \dots \quad (61)$$

where T_p is the waiting time for the p th earthquake, and a and v are positive parameters. Then to use a Gram-Charlier type approach, we should obtain a set of orthonormal functions over the interval $[0, \infty)$ by Gram-Schmidt orthonormalization procedure and use this set of functions to develop a Polya type Gram-Charlier series for the estimation of waiting time density functions for future earthquakes. Or alternatively we should write

$$p(t) = \sum_{n=1}^{\infty} \alpha_n \frac{v(v+1)\dots(v+n-1)}{(a+t)^{v+n}} \frac{t^{n-1}}{(n-1)!} \quad t > 0 \quad (62)$$

with (59) imposed.

(d) The Poisson process model of earthquakes implies that the inter-arrival times of earthquakes follow a negative exponential distribution. There are conflicting views in the literature whether negative exponential distribution for the inter-arrival times of earthquakes implies a Poisson process model for earthquakes (Massey 1971; Udias and Rice 1975; Vere Jones 1970). But the correct answer to the second question must be negative as explained below. Under the Poisson process model, the density function for the waiting time T_p for the p th earthquake can be shown to be (Goel 1982; Sharma 1982; Sharma *et al* 1983)

$$p_{T_p}(t_p) = \frac{\lambda^p t_p^{p-1}}{(p-1)!} \exp(-\lambda t_p) u(t_p), \quad \lambda > 0, \quad p \text{ positive integer.} \quad (63)$$

It is important to note that (63) lists various implications of a Poisson process model. It can also be seen that (63) cannot be derived from (2) alone. Thus a Poisson process model implies (2), but (2) does not necessarily imply a Poisson process model, because (63) for $p = 2, 3, \dots$ may still be violated. It is, therefore, instructive to find out under what conditions (2) implies (63). It can be shown that (Goel 1982).

$$p_{T_m}(t) * p_{T_n}(t) = p_{T_{m+n}}(t) \quad (64)$$

where $*$ represents convolution. Thus convolution of the waiting time density functions for the n th and m th earthquakes given by (63) gives the waiting time density function for the $(n + m)$ th earthquake according to (63). In particular

$$p_T(t) = p_{T_1}(t) * p_{T_2}(t) * \dots * p_{T_p}(t), \quad (65)$$

where the term on the right side is a p -fold convolution. Convolution of the density functions of two or more random variables gives the density function of their sum provided the random variables are independent (Davenport and Root 1958). Thus (63) follows from (2) only if the events are statistically independent. Equation (2) alone does not imply that the earthquakes are independent. The conclusion in § 5(d) should be viewed in this light.

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References

- Aki K 1956 *Zisin* **8** 205
 Barrett J F 1963 *J. Electron. Control* **15** 567
 Benioff H 1951 *Bull. Geol. Soc. Am.* **62** 331
 Davenport Jr W B and Root W I 1958 *An introduction to the theory of random signals and noise* (New York: McGraw-Hill)
 Ferres S G 1967 *Geophys. Int.* **7** 43
 Fisz M 1963 *Probability theory and mathematical statistics* (New York: John Wiley) p. 52, 164, 276, 298
 Fry T C 1965 *Probability and its engineering uses* (Princeton: D Van Nostrand) p. 257
 Goel S M J R 1982 *Estimation of the waiting time distributions for earthquakes* M. Tech. thesis, University of Roorkee (unpublished)
 Kenney J F and Keeping F S 1963 *Mathematical Statistics* (Princeton: D Van Nostrand) Part II p. 108, 160
 Knopoff I 1964 *Bull. Seism. Soc. Am.* **54** 1871
 Krishnamurthy F V and Sen S K 1976 *Computer based numerical algorithms* (New Delhi: Affiliated East West Press) p. 421
 Lee Y W 1960 *Statistical theory of communication* (New York: John Wiley) p. 459
 Lomnitz C 1966 *Rev. Geophys.* **4** 377
 Malasi S K 1981 *A statistical model for the inter-event arrival times for microearthquake in North East India*, M.Tech. thesis, University of Roorkee (unpublished)
 Massey L D 1971 *Probability and statistics* (New York: McGraw-Hill)
 Schlien S and Toksoz M N 1970 *Bull. Seism. Soc. Am.* **60** 1765
 Shalanger (Ben Menahum) A 1960 *Gerlands Beitr. Geophys.* **69** 68
 Sharma N K 1982 *Generation and testing of hypotheses for earthquake occurrences*, M.Tech. thesis, University of Roorkee (unpublished)
 Sharma N K, Moharir P S and Gaur V K 1983 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* (Communicated)
 Udias A and Rice J 1975 *Bull. Seism. Soc. Am.* **65** 809
 Utsu T 1972 *J. Fac. Sci. Hokkaido University Ser III Geophys.* p. 4
 Vere Jones T 1970 *J. Roy. Statist. Soc.* **332** 1
 Whalen A D 1971 *Detection of signals in noise* (New York: Academic Press) p. 126, 246, 250, 252, 254

Daily variation of the geomagnetic field near the focus of Sq-current system in Indian longitude

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Abstract. The paper presents the first results on the behaviour of solar quiet-day variations of the geomagnetic field components at Gulmarg. Combining the data from Russian stations in the same longitude belt, the annual average daily variations are calculated which show, in the horizontal component (H), a reversal of phase between Gulmarg and Tashkent. Studying the Sq-variations at Gulmarg separately for the three seasons, the daily variation of H during *d*-months is predominantly diurnal in character with the maximum before noon. During *e*-months, and more so in *j*-months, daily variation of the H field is predominantly semi-diurnal in character with minimum around 08-09 hr LT and maximum around 14 hr LT consistently during 1978, 1979 and 1980. These features of the Sq at Gulmarg are suggested to be due to the deformations of the current loops caused by the changing latitude of focus during the course of the day.

Keywords. Solar quiet-day variation; overhead current system; Indian longitude; geomagnetic field.

1. Introduction

India has a long tradition of monitoring the geomagnetic field since early nineteenth century and had a regular magnetic observatory in Colaba for the years 1846 to 1907 which has been transferred to Alibag in 1905 and is still in continuous operation as the primary magnetic observatory of India. During the IGY two new magnetic observatories were established at Trivandrum and Annamalainagar which in conjunction with Kodaikanal have provided valuable data on equatorial electrojet since then. During the geomagnetic meridian project additional observatories were established at Ujjain and Jaipur. The latest addition to this network of 12 magnetic observatories in India is at Gulmarg (Geog. Lat. $34^{\circ} 03' \text{N}$; Long. $74^{\circ} 24' \text{E}$) being also the northernmost observatory in the Indian network. Until this latest addition, Sabhawala (Geog. Lat. $30^{\circ} 22' \text{N}$, Long. $77^{\circ} 48' \text{E}$) used to be the northernmost magnetic observatory in India.

Yacob and Rao (1970) studied the seasonal variations of the diurnal and semi-diurnal components of Sq in horizontal component (H) at low latitude observatories in India. Rastogi and Iyer (1976) studied the Sq(H) variations at Indian observatories from Trivandrum to Alibag. Srivastava and Prasad (1979) studied the solar (Sq) and lunar (L) variations at Hyderabad and Sabhawala and detected large amplitude of the second solar harmonic in H at Sabhawala during *e*- and *j*-months. Arora *et al* (1980) studied the solar and lunar tides in the geomagnetic field at all the magnetic observatories in India and confirmed the predominance of the second harmonic in S at Sabhawala. Rangarajan and Ahmed (1981) studied the geomagnetic H field at Sabhawala for

the period 1964–78 and found that during January and December there is no appreciable variation indicating that during these months the Sq focal latitude is close to the station while the amplitude of second harmonic exceeds that of the first harmonic in summer and equinoxes. With this background information it was felt necessary to examine the geomagnetic data at Gulmarg to assess its position *vis-a-vis* the Sq-focus during different seasons.

2. Data treatment and results

In the present work, use is made of hourly values of geomagnetic field components, *viz* horizontal (H), declination (D) and the vertical (Z) components, from 13 magnetic stations whose data are available in common with Gulmarg. The names and locations of these stations are shown in figure 1. The mean pattern of daily variation in each of the components is obtained by averaging the hourly field values over all the international quiet (IQ) days of the year 1978. The hourly values for each IQ day have been corrected for non-cyclic variation before they are employed in the present analysis. The

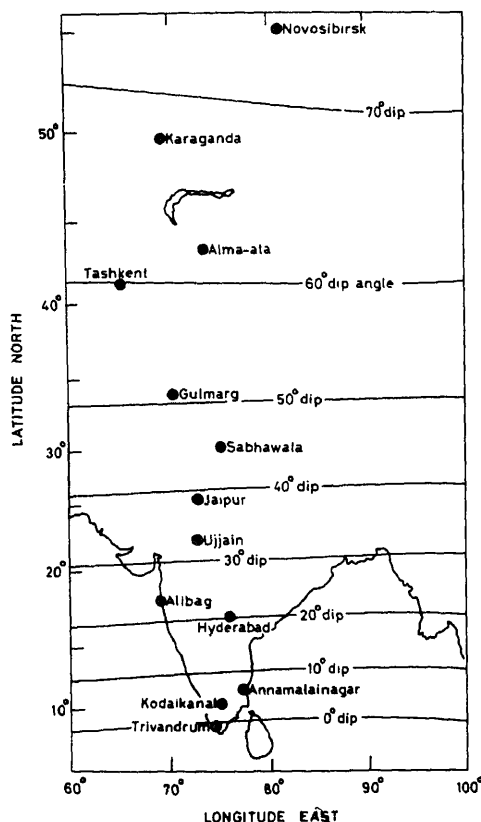


Figure 1. Map showing the locations of magnetic observatories whose data are used in the present analysis.

mean Sq-variation in D, H and Z for each of the stations are plotted as a function of 75° E time and are shown in figure 2. Figure 3 gives the latitudinal profile of the amplitude and phase of the diurnal and semi-diurnal terms obtained by the harmonic analysis of daily variations shown in figure 2. As the primary features of the latitudinal profile in this zone up to Sabhawala have already been discussed elsewhere (Rastogi and Iyer 1976; Arora *et al* 1980), prominent features having special reference to Gulmarg or indicating behaviour typical of the focal latitudes would only be emphasized.

Examination of figures 2 and 3 reveals that in regard to the pattern of DV (daily variation) as well as its latitudinal dependence, the variations at Gulmarg and at the adjoining stations broadly conform to the behaviour as revealed by global studies through spherical harmonic analysis (SHA) (*eg.* see Matsushita 1967). For example D and Z variations in their latitudinal progression attain maximum close to Gulmarg and H-variations are diminished in magnitude. The phase of the diurnal as well as semi-diurnal term in H shows a sharp jump between Gulmarg and Tashkent approximating to a reversal of the DV pattern. All these features lead to the inference that Sq-focus is located close to the latitude of Gulmarg. A noteworthy feature to emerge from figure 2 is that moving northward from Ujjain, the near-noon maximum of H-variation

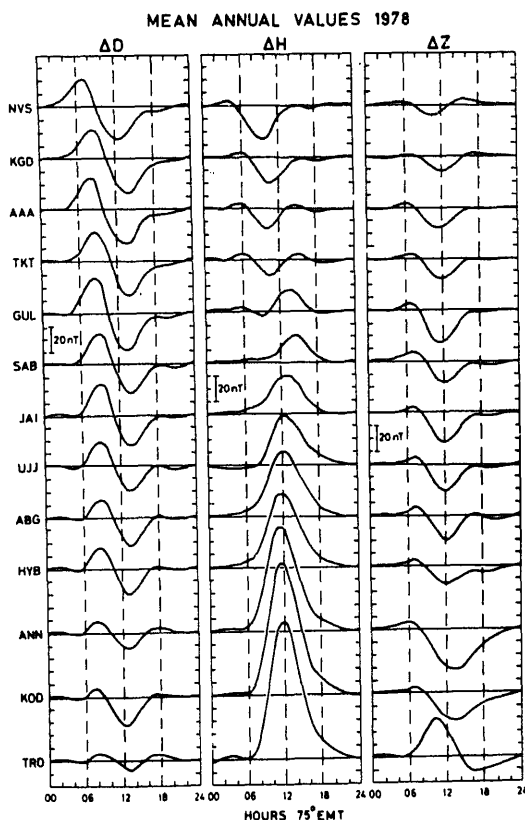


Figure 2. Solar quiet-day variations in geomagnetic field components, D, H and Z at magnetic observatories along Indian longitudinal belt for the year 1978.

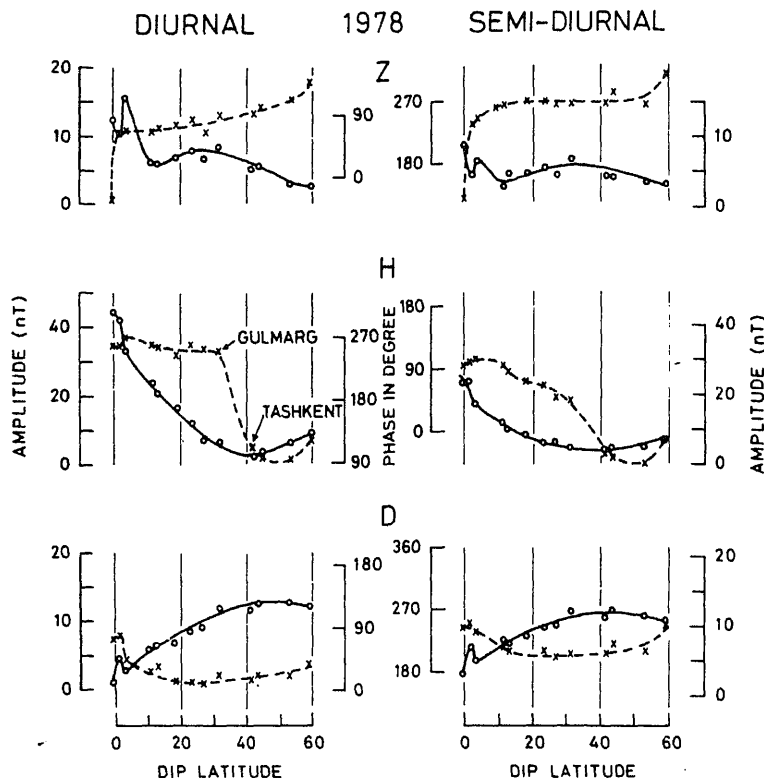


Figure 3. Latitudinal variations of the amplitude (solid line) and phase (broken line) of the diurnal and semi-diurnal terms of daily variations in the three geomagnetic components **D**, **H** and **Z**.

progressively shifts to a later hour until a reversed DV pattern is registered at Tashkent (TKT). However, this reversed H-variation at TKT is not completely out of phase as there exists a considerable time difference between the daily minimum at TKT and maximum at SAB. The pattern at Gulmarg is characterized by mixed-features, showing a feeble minimum close to the time of TKT minimum and a pronounced maximum near the time of SAB maximum. Yet another feature of DV borne out by the harmonic analysis is that at latitudes close to the Sq-focus first and second harmonics of H-variation tend to be comparable in magnitude as against the dominance of the first harmonic at all other latitudes.

In order to have an insight into the stability or otherwise of DV pattern near Sq-focus with seasons, we have evaluated Sq-variations at Sabhawala, Gulmarg and Tashkent for the three seasons by grouping data for *d*-, *e*- and *j*-months of the year 1978. For *d*-months, four consecutive months, November 1977 to February 1978, are employed rather than combining data of January, February, November and December of the same year. The results are shown in figure 4. Apart from the regular seasonal changes in intensity, as implied by the strength of D-variation, the H-pattern depicts strong

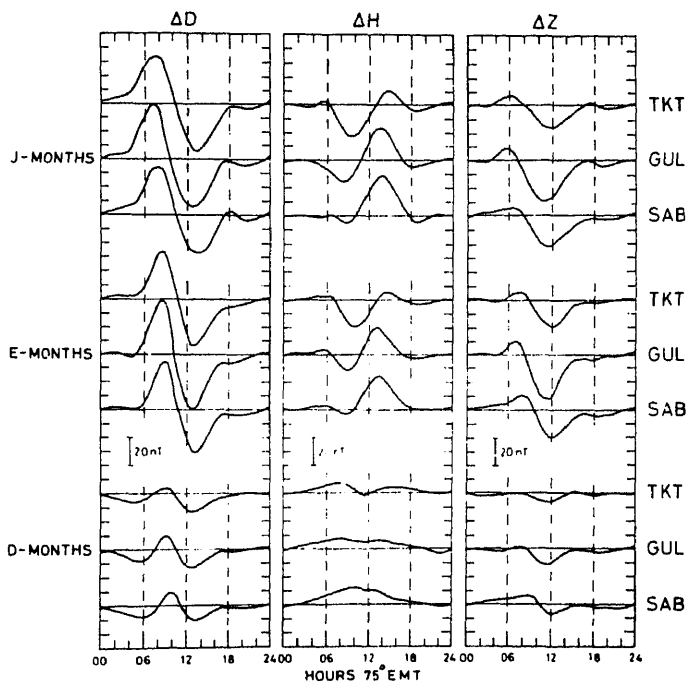


Figure 4. Mean seasonal quiet-day variations in D, H and Z at Sabhawala, Gulmarg and Tashkent for the year 1978.

seasonal variability of form, leading to the conjecture that overhead current system undergoes large seasonal variation both in shape and intensity. Figure 5 illustrates the mean seasonal and annual variation at Gulmarg separately for the years 1978, 1979 and 1980. The similarity of seasonal DV patterns among the three years, further suggests that seasonal progressions are stable and repetitive in nature.

The most striking aspect of the DV during *j*- and *e*-months as noticed from figures 4 and 5, is the absence of normal pattern in H-variation at Gulmarg, *i.e.* absence of a single maximum or a minimum close to the local noon. In contrast, the DV has a pronounced minimum around 9–10 hr LT and a maximum around 13–14 hr LT. Both the maximum and the minimum have equal prominence in DV pattern at Gulmarg. However, the behaviour differs markedly at stations immediately north (TKT) and south (SAB) of Gulmarg. While at Sabhawala the strength of the maximum in the early afternoon tends to be greater than that of the minimum in the forenoon, more or less reversed features are seen in the diurnal variation at Tashkent. In the *d*-months, Gulmarg H-variation is primarily made up of inverted V-type variation with positive (northerly) field during daylight hours suggesting that the Sq-focus is located further north of Gulmarg in this season. An interesting feature of winter H-variation is the presence of a depression, close to the local noon, in the northerly field variation giving the appearance of a secondary minimum included in the main northerly variation. This feature is more prominently registered at TKT than at GUL and SAB (figure 4) and stands out more clearly in the winters of 1978, 1979 than in 1980 (figure 5).

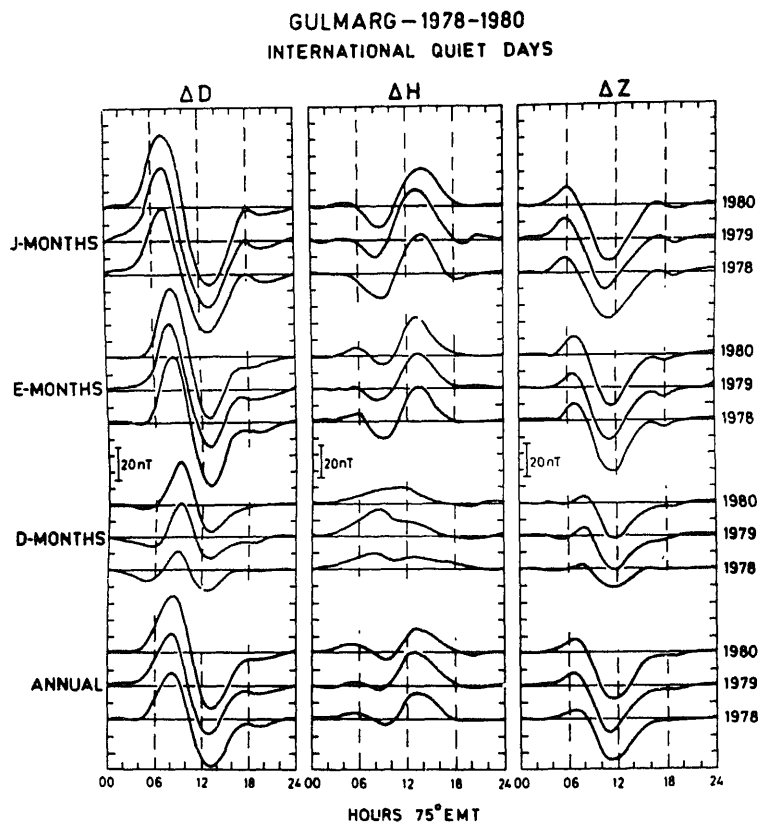


Figure 5. Mean seasonal and annual quiet-day variations in D, H and Z of Gulmarg during the years 1978, 1979 and 1980.

3. Discussion

Mayaud (1967) has visualised some schematic models of Sq-current system which individually or in combination bring out various patterns of DV registered near the focal latitudes. The basic model, termed the rectilinear model, includes an elliptical-shaped current vortex in the overhead current system responsible for DV in geomagnetic component. The other two models, called tilt (T) and deformed (F) models, are obtained by giving some simple deformation to the rectilinear model. In the tilt model, current vortex is deformed in such a manner that the currents are shifted towards either higher (*h*) or lower (*l*) latitudes in the morning (*m*) hours. These two possible forms are named T_h^m or T_l^m models. In other words, in the T model a kind of 'skewing' of the current vortex in the clockwise or anti-clockwise sense with respect to the axis parallel to latitude is implied. At the latitude of the focus or very close to it, the DV in H associated with T_l^m model is made up of a minimum and a maximum, similar to the pattern depicted in the DV pattern at Gulmarg during *j*- and *e*-months. Thus, it may be inferred that during *j*- and *e*-months northern hemispheric current vortex as it passes

over the Indian region tends to have a tilted shape wherein current lines are shifted towards low latitudes in the morning hours. It may also be added that DV near the Sq-focus are very prone to the changes in the latitude of focus, and, therefore, it is likely that H-variation, both with southerly and northerly fields, may result from latitudinal movement of focus during the course of a day.

In the deformation model discussed by Mayaud (1967), the focus is shifted either to higher or lower latitudes such that current contours tend to be closer on the poleward side than on the equatorward side or *vice-versa*, the inherent assumption being that current vortex does not move as a rigid body and the focus appears eccentric with respect to current contours. For such a model where the focus has moved to a lower latitude, called F_1 model, the daily variation in H at latitudes little lower than the focus consists of a secondary minimum included in the broad day-time maximum, a signature well-marked in the DV pattern of H at Gulmarg and Tashkent during *d*-months (figures 4 and 5). The fact that no anomaly is seen in D variation associated with this secondary minimum in H-variation lends further support to the interpretation that secondary minimum has resulted from the concentration of current lines towards the equator side of the focus as such concentration of lines would not produce any discernible influence on the form of D-variation (Mayaud 1967). The secondary effect is more pronounced in the winter months of 1978, 1979 than in the winter of 1980. The relative sunspot numbers during the winter season of these three years were 92.5, 131.2 and 168.5 respectively. This leads to a conjecture that the phenomenon of the deformation of sq-current vortex associated with the secondary effect has a measure of dependence on solar activity.

4. Conclusions

It follows from the above discussion that various perturbations from the normal pattern (as observed at low latitudes) noticed in the daily variation plots at Gulmarg and other neighbouring stations can reasonably be understood in terms of the effects arising from some simple deformations in the overhead currents system. The typical signatures noted suggest that during *j*- and *e*-months, current vortex appears tilted in an anti-clockwise sense with respect to the latitudinal axis whereas during the *d*-months focus is shifted to lower latitudes resulting in a greater concentration of contours towards lower latitudes.

References

- Arora B R, Rao D R K and Sastri N S 1980 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **89** 333
- Matsushita S 1967 in *Physics of geomagnetic phenomena* (ed.) S Matsushita and W H Campbell (New York: Academic Press) p. 301
- Mayaud P N 1967 *IAGA Bull.* **21** 67
- Rangarajan G K and Ahmed K 1981 *Indian J. Rad Space Phys.* **10** 20
- Rastogi R G and Iyer K N 1976 *J. Geomag. Geoelectr.* **28** 461
- Srivastava B J and Prasad S N 1979 *Geophys. J. R. Astr. Soc.* **57** 159
- Yacob A and Rao D R K 1970 *Indian J. Met. Geophys.* **21** 119

Low latitude airglow*

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Abstract. Tropical airglow work during the last few years is reviewed. Airglow instrumentation is becoming more complex. Some of these sophisticated airglow experiments giving important information about the upper atmosphere such as ionospheric F region electron density, height of maximum electron density, dynamics of and irregularities in the F region, mesospheric neutral temperature and its variation, dynamics of mesospheric, etc. are mentioned. At the end some problems which could be tackled in near future with airglow techniques have been suggested.

Keywords. Airglow; earth's atmosphere; F region; irregularities and dynamics; mesospheric temperatures.

1. Introduction

We shall review only a few areas of interest in the studies of the upper atmosphere where airglow techniques are used. The tropical region and the magnetic equator are of particular interest for some special phenomena occurring exclusively in these latitudes and we shall, by and large, confine our attention to this region. An earlier review (Kulkarni 1974) on tropical airglow may be noted. We shall examine the use of airglow emission to study the F region of the ionosphere and the mesosphere.

2. Night time F region of the ionosphere and the airglow

During the last 10 years or so equatorial ionosphere has been extensively studied and the use of airglow techniques for such studies is showing great promise. While the importance of forbidden 6300 Å OI emission and its association with the F region of the ionosphere was recognised in early sixties or even before, the association of the F₂ ionosphere with the airglow due to forbidden 1356 Å line and permitted lines such as 1304 Å, 4368 Å and 7774 Å all due to atomic oxygen is only during the last decade.

Around 1960s it was Daniel Barbier who observed the enhancement of 6300 Å airglow emissions on the equator in the tropical latitudes on the African continent and called these enhancements as the "intertropical arc" and the "western sheet". This was quite different from the Stable Auroral Arcs (SAR) discovered by Barbier (1957) in the mid-latitudes. While the equatorial/tropical enhancements are related to the local ionosphere the SARs are due to the different mechanism. SARs last for several hours and are rather infrequent. Their morphology is described by Roach and Roach (1963). The equatorial enhancements of 6300 Å, however, are quite dynamic.

A semi-empirical relation given by Barbier (1959) to estimate the 6300 Å emission from ionospheric parameters was extensively used for many years and was a good approximation for the physical mechanism emitting 6300 Å radiation in the F region of the ionosphere. The expression for emission was later derived by Peterson *et al*

* Invited Review paper, Commission 21, IAU, Patras, Greece, August, 1982.

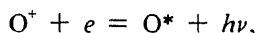
(1966). It is very interesting to note that results calculated with the above formulae and compared with the 6300 Å observation on a typical night at a tropical station show that the observations are in better agreement with Barbier's formula than that with Peterson's.

From Barbier's observations, King (1968) pointed out the similarity in the night time movement of the *F* layer anomaly crests and the 6300 Å airglow emission in the low latitudes. With certain exceptions normally the 6300 Å in the mid-latitudes is quiet, showing a fast decay after the evening twilight, remaining steady during the night for several hours and showing enhancement before the morning twilight. On the other hand, the 6300 Å emission in the tropics is quite lively and its manifestation has been reported in the forms of (i) intertropical arcs (Barbier 1961); (ii) gradients (Steiger *et al* 1966; (iii) fingers (Steiger 1967); (iv) pair of humps (Kulkarni and Rao 1972) showing the above mentioned features in the geomagnetic latitudes $\pm 15^\circ$ and in some cases tracing the movement of such features towards the equator in the early hours of the night.

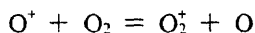
2.1 Satellite studies

With satellite observations it was possible to monitor the 1304 Å and 1356 Å ultraviolet oxygen airglow lines and enhanced emissions were reported around $\pm 15^\circ$ geomagnetic latitude (Hicks and Chubb 1970; Barth and Schaffner 1970). They further predicted that the 7774 Å and 4368 Å oxygen permitted lines also should accompany the above emission. Weill and Joseph (1970) reported preliminary ground-based observations of 7774 Å. Under the Appleton anomaly, in Brazil, Tinsley (1972) observed 7774 Å emission and found that its intensity decreases from end of twilight to well after mid-night (from ~ 100 R to a few R). Later Tinsley *et al* (1973) reported variation of intensity of 7774 Å from 10 to 30 R at Agulhas Negras.

Generally accepting that the 7774 Å emission is due to the radiative recombination reaction



to which ion-ion recombination could be added but shown to be not very significant and the 6300 Å emission due to the dissociative recombination



and $\text{O}_2^+ + e = \text{O} + \text{O}^* (^1\text{D})$ (and other branchings)

it has been shown by Chandra *et al* (1975) that monitoring of these two radiations could be used to understand the tropical *F* region of the ionosphere. Tropical *F* region shows spread *F* activity usually during early evening hours and from the Jicamarca radar extensive *F* region work has been reported. *F* region irregularities have been named as holes, bubbles, bite-outs, plumes to describe various shapes and sizes which are basically due to the depletions in the electron density in the night-time ionosphere mainly at the *F* layer heights. It has been observed that monitoring of airglow radiations even from ground-based station provides a lot of high resolution information for better understanding of the phenomenon.

2.2 Ground-based studies

It was shown by Tinsley and Bittencourt (1975) that height and peak electron density of *F* region of the ionosphere could be determined by calculating column rate emission, J of 1356 Å and 6300 Å from the intensity measurements I_{1356} and I_{6300} .

J_{1356} and J_{6300} being proportional to the square root of the electron density and the electron density respectively, Tinsley *et al* (1975) showed that peak electron density $n_m(e)$ is proportional to $(J_{1356})^{1/2}$ and to a good approximation the ratio $(J_{1356})^{1/2} / J_{6300}$ is a single-valued function of the layer height $h_m F_2$. While the value of $n_m(e)$ thus derived is little affected by the assumed exospheric temperature, $h_m F$ is strongly affected. However, it must be remembered that 1356 Å observation must be taken from a space-borne vehicle due to its strong absorption by the atmospheric ozone. It was further shown (Tinsley and Bittencourt 1975) that 7774 Å is also mainly emitted due to radiative dissociation thus making $(J_{7774})^{1/2}$ proportional to the electron density and $(J_{7774})^{1/2} / J_{6300}$ can be used to estimate $h_m F$. It should be remembered that 7774 Å emission can be monitored by a ground-based instrument. The intensity I_{7774} and I_{6300} has been actually measured and used for the above purpose.

Recently, Sahai *et al* (1981) have reported from a ground-based station airglow observations of 6300 Å and 7774 Å under the southern *F* region anomaly (22.7° S) in Brazil. They find good correlation between the column density $(J_{7774})^{1/2}$ and the *F* layer peak electron density $n_m(e)$. They also find good correlation between $(J_{7774})^{1/2} / J_{6300}$ and the maximum height $h_m F_2$ of the *F* layer. They have concluded that simultaneous measurements of these two emissions would be a very useful technique for remote sensing the ionospheric *F* layer dynamics.

2.3 Spread *F* and neutral temperature

From Doppler width temperatures observed with FP interferometer Rajaraman *et al* (1979) found interesting correlation at Mt Abu (Geographic Latitude 24.6° N, Geomag. Lat. 15°, Long. 72.7° E) between the emission layer temperature of the 6300 Å night airglow emission and the spread *F* phenomenon. It was observed that the *F* region temperature increased by as much as 200° K over its usual night-time value, when spread *F* is present on the magnetic equator and without spread *F* occurring on the magnetic equator there was no increase in the temperature at Mt Abu. *F* layer temperature at Mt Abu increases after 15–30 min following the occurrence of spread *F* at the magnetic equator. The basic heating mechanism is suggested to be the Joule dissipation of the electric currents associated with turbulent electric fields during the equatorial spread *F*.

2.4 Aircraft studies

In 1972 from a jet aircraft simultaneous observations of 6300 Å airglow and *F* region plasma structures with an ionosonde were made by Markham *et al* (1975). The conclusion was that no anomalies in either atmospheric composition or reaction rates are associated with the regions of unusually strong emission (400 R), the entire complicated structure of 6300 Å emission over an extended range of latitude seems entirely to reflect corresponding changes in *F* region plasma structure.

To investigate the F region irregularities experiments on board an aircraft were conducted in 1977 and 1978 (Weber *et al* 1980). The 6300 A imaging photometer, ionosonde and receiver for satellite amplitude scintillation measurements were used. They measured 6300 A depletions, spread F , and scintillation producing irregularities all associated with low density plasma features in early night equatorial ionosphere. With the above instruments they have estimated the electron densities inside and outside the depletion and find that on a typical night during early hours electron density inside a typical depletion is 66% near the magnetic equator as well as near the southern Appleton anomaly. These results agree very well with those obtained from the ionosonde and satellite data.

Continuing their air-borne experiments in 1979 Moore and Weber (1981) found that within 15° magnetic latitude between Africa and South America plasma depletions existed and on 14–15 December 1979 peak electron density varied from $3.3 \times 10^6 \text{ cm}^{-3}$ outside the depletion to $1.1 \times 10^6 \text{ cm}^{-3}$ inside the depletion. The depletion on another night produced even larger variations. These inferences are drawn from airglow observations of 6300 A and 7774 A. They conclude that 6300 A depletions, spread F , and scintillations are associated with low density bubbles in the post-sunset equatorial ionosphere. 6300 A airglow depletions are bottom-side signatures of F layer plumes of very small size (metre size) irregularities as measured by 50 MHz back scatter radar.

3. Suggested studies

Tinsley and Bittencourt (1982) have pointed out the possibility of “imaging” F region bubbles, plumes and depletions at night-time through airglow measurements from ground by taking advantage of the plasma flow from over the equator towards the tropical latitudes. Again 7774 A and 6300 A airglow emissions arising at the peak of the F_2 and at the bottom-side of F_2 layer respectively are proposed to be used to advantage. On the magnetic equator the plasma is lifted to various heights (upto or more than 1000 km altitude) depending on the magnitude and direction of the electric field. This plasma rolls down along the earth's magnetic lines of force and at appropriate height and density manifests itself as airglow emission at various latitudes depending on at what altitude on the equator it starts flowing along the magnetic lines of force. Tinsley and Bittencourt (1982) have shown that from a chosen station say at about 18° dip latitude if airglow observations are taken at zenith angles 60, 70, 80, 85° , etc. towards the magnetic equator, they intercept the airglow emissions layer at various latitudes thus allowing the study of bubbles at various stages of developments. The emission layer being relatively narrow, the scanning or imaging of airglow structures will be without much blurring and a sort of cross-section at that height.

The observations will have maximum accuracy when the line of sight having airglow emission layer intercepts tangentially to the magnetic lines of force. They have also estimated the blurring due to misalignment with respect to the magnetic lines of force. The technique can also be used for studying the morphology of the artificial bubbles or ionospheric holes produced by chemical releases. Problem of background due to OH (9–4) band which must be properly eliminated to get the correct 7774 A intensities must be seriously considered.

High resolution Fabry-Perot spectrometer can be conveniently used in pressure scanning mode (or any other technique) to determine the Doppler broadening and the Doppler shift of the airglow lines convenient for such purpose. The temperature and

the neutral wind velocity at the emission altitude can be estimated from such measurements. Emissions such as 6300 Å from the *F* region of the ionosphere and 5577 Å from about 100 km altitude are quite useful for such purposes. Such measurements on the magnetic equator and at low latitudes will be quite useful.

3.1 *OH studies*

Two ground-based techniques recently used for studying (i) temperature and (ii) dynamics of the mesosphere will be discussed.

3.2 *Mesospheric temperatures*

It is generally accepted that OH emission in the mesosphere is due to hydrogen-ozone reaction. In the 85–90 km region of the upper atmosphere, before radiative de-excitation, OH radical undergoes about 250 collisions and attains the ambient neutral temperature. The OH radical emits the rotational vibrational band system and the intensity of a rotational line is linked to the temperature by the following relation.

$$I_J = C\nu^4 S_J \exp(-F(J)hc/kT),$$

where I_J is the intensity of a rotational line and T the neutral ambient temperature. Spectroscopically it is possible to determine the temperature from the relative intensities of the rotational lines in a given OH band by photographing or by photoelectrically scanning the OH bands. However, by selecting a proper band, say OH (7–2) where rotational lines are well separated and no other emissions overlap it is possible to isolate a single rotational line with a moderately broad filter (~ 10 Å) and monitor its intensity. Majmudar and Kulkarni (1975) used photoelectric photometer and measured intensities of $P_1(3)$ line and ΣR branch of the OH (7–2) band. From the above relation it is possible to associate a unique temperature for one value of ratio of intensities of $P_1(3)$ and ΣR . With usual background elimination when the ratio is taken it eliminates problems of absolute calibration to some extent.

Mesospheric temperatures by the above method have been measured at Mt Abu for a few years. The nocturnal pattern of temperature shows that the mesosphere has variations in temperatures even during night. Atmospheric models show a minimum between 84 and 94 km. Any variation in temperature in this part of the atmosphere is quite important for *D* region chemistry. Its coupling with higher altitude, if any, could be very important.

The study over a number of nights reveals that about a fifth of nights show that the temperature is steady during these nights, while a third of the nights individually show temperature fluctuations during those nights. There are some nights on which the temperature increases by as much as 40° K within a few hours in the night. A few nights also show decrease in temperature. While the constant temperature conditions can be attributed to the static conditions in the mesosphere, fluctuations or changes are difficult to interpret. In his work on internal gravity waves Hines (1960, 1965) has shown that in the 90 km region of the earth's atmosphere reversible adiabatic heating is produced due to the gravity waves and the heating could manifest itself as fluctuations in the temperature and the periods of such fluctuations could be between 10 and 200 min. On many nights fluctuation in temperature was observed by Majmudar and Kulkarni (1980) at Mt Abu showing some order of periodicity as predicted. Very likely these fluctuations in temperature could be attributed to the internal gravity waves.

With other techniques temperatures in this region have been measured (Rai and Fejer 1971; Krassovsky 1972; Shagev 1974; Meriwether 1975; Armstrong 1975). Some of these authors have detected fluctuations. Manson and Meek (1976) from radiometer data have reported the periodicity of IG waves greater than 2 hr in this region.

It is also possible to estimate wave-induced horizontal component, U_x , of the neutral wind from the temperature fluctuations $\Delta T/T$ from the following relation given by Hines (1965);

$$\Delta T/T = \pm i(\gamma - 1)^{1/2} C^{-1} U_x,$$

where i indicates the phase quadrature between the time and place of maximum in T and U_x . Unless additional information on phase is available it is not possible to decide the direction, that is $+$ or $-$ sign in the above equation. In its absence only the dominant periodic component would give the magnitude of U_x . The magnitude of neutral winds from the above relation was estimated at 10 to 25 msec⁻¹ from Mt Abu data.

3.3 Mesospheric wavy structures

Moreels and Herse (1977) in the past few years have used the technique of photographing airglow emission in the very near infrared (7580 to 9850 Å) which is dominated by strong OH emissions. In about 10 to 15 min exposure they take a picture. On a number of photographs they have seen wave-like patterns. By triangulation they could determine the height of the emission layer at 85 km with an accuracy of ± 2 km. The wavelength of such waves is not very constant but varies. The approximate value is estimated at a few tens of kilometers. The wave velocity is of the order of 15 ms⁻¹ and a period of 50 min is estimated from such photographs. This period fits into that derived theoretically for the gravity waves.

The structure shown in the near infrared photographs due to OH emission supports independently the possible presence of gravity waves in the atmosphere in the 80 to 90 km region. These observations are taken at mid-latitudes. A sophisticated experiment proposed by Service d'Aéronomie, CNRS, France and Physical Research Laboratory, Ahmedabad, India, for photographing these structures from Sky-Lab has been accepted by NASA. This would reveal the global structure of OH waves.

It might be very interesting to measure simultaneously temperatures in the F region and the D region of the atmosphere to investigate if there is any coupling between the two.

References

- Armstrong E B 1975 *J. Atmos. Terr. Phys.* **37** 1585
- Barbier D 1957 *C. R. Acad. Sci.* **244** 1809, 2277
- Barbier D 1959 *Ann. Geophys.* **15** 179, 412
- Barbier D 1961 *Ann. Geophys.* **17** 3
- Barth C A and Schaffner S 1970 *J. Geophys. Res.* **75** 4299
- Chandra S, Reed E I, Meier R R, Opal C B and Hicks G T 1975 *J. Geophys. Res.* **80** 2327
- Hicks G T and Chubb T A 1970 *J. Geophys. Res.* **75** 6233
- Hines C O 1960 *Can. J. Phys.* **38** 1441
- Hines C O 1965 *J. Geophys. Res.* **70** 77
- King J W 1968 *J. Atmos. Terr. Phys.* **30** 391
- Krassovsky V I 1972 *Ann. Geophys.* **28** 739

- Kulkarni P V 1974 *Ann. Geophys.* **30** 105
- Kulkarni P V and Rao V R 1972 *Ann. Geophys.* **28** 475
- Rajmudar Neeta H and Kulkarni P V 1975 *Indian J. Radio Space Phys.* **4** 228
- Rajmudar Neeta H and Kulkarni P V 1980 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **89** 109
- Ranson A H and Meek C E 1976 *J. Atmos. Sci.* **33** 1650
- Starkham P, Buchau J, Amettil R E and Noxon J F 1975 *J. Atmos. Terr. Phys.* **37** 65
- Teriweather J W 1975 *Planet. Space Sci.* **23** 1211
- Moore J G and Weber E J 1981 *J. Atmos. Terr. Phys.* **43** 851
- Moreels G and Herse M 1977 *Planet. Space Sci.* **25** 265
- Peterson V L, Van Zandt T E and Norton R B 1966 *J. Geophys. Res.* **71** 2255
- Sai D B and Fejer B G 1971 *Planet Space Sci.* **19** 561
- Rajaraman T N, Desai J N and Degaonkar S S 1979 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **A88** 69
- Roach F E and Roach J R 1963 *Planet Space Sci.* **11** 523
- Shihai Y, Bittencourt J, Teixeira N R and Takahashi H 1981 *J. Geophys. Res.* **86** 3657
- Shagev M V 1974 *J. Atmos. Terr. Phys.* **36** 367
- Seeger W R, Brown W E and Roach F E 1966 *J. Geophys. Res.* **71** 2846
- Seeger W R 1967 *Aurora and airglow* (ed.) B M McCormac (New York: Reinhold) p. 419
- Stensley B A 1972 *Ann. Geophys.* **28** 155
- Stensley B A and Bittencourt J 1975 *J. Geophys. Res.* **80** 16
- Stensley B A, Christensen A B, Bittencourt J, Couveia H, Angreji P D and Takahashi H 1973 *J. Geophys. Res.* **78** 1174
- Stensley B A and Bittencourt J 1975 *J. Geophys. Res.* **80** 2333
- Stensley B A and Bittencourt J 1982 *J. Atmos. Terr. Phys.* **44** 547
- Weber E J, Buchau J and Moore J G 1980 *J. Geophys. Res.* **85** 4631
- Will G and Joseph J 1970 *C. R. Acad. Sci., Paris* **271** 1013

Ice accumulation rate in Changme-Khangpu glacier, Sikkim

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Abstract. Vertical profiles of ^{137}Cs and ^{210}Pb have been determined in a 9 m column of ice from accumulation zone of Changme-Khangpu glacier in north Sikkim valley. ^{137}Cs activity varies from 4 to 22 dpm/L. In many samples ^{210}Pb occurs at a level of 20 to 65 dpm/L which is much higher than the expected fallout value. ^{137}Cs and ^{210}Pb activities correlate well with each other but not with the dust content. Possibility of ^{210}Pb production in the nuclear explosions is discussed. Several peaks appear in the depth profile of ^{137}Cs and ^{210}Pb which can be matched with Chinese atmospheric nuclear explosions with some phase difference if a uniform ice accumulation rate of 0.7 m per year is assumed since 1969.

Keywords. Glaciers; bomb debris; ice accumulation

1. Introduction

The estimation of long term mass balance of a glacier constitutes one of the important parameters in understanding glacier dynamics. In the Himalayas, direct measurements of flow rates have been made on a few glaciers but the accumulation rates have been computed only from indirect methods as winter observations are difficult. These estimates fluctuate significantly from year to year and it has not been possible to obtain long term average accumulation rates from such data. The deposition of nuclear debris in discreet events during the second half of this century has provided definite time markers. Tracer techniques on radioactive-labelled ice have provided dynamic parameters of glaciers averaged for several years in the past and have been used extensively in Alps and polar ice caps (Picciotto *et al* 1964; Merlivat *et al* 1975; Lambert *et al* 1975; Koide and Goldberg 1981). Here we report some radiotracer studies on Himalayan glaciers.

Changme-Khangpu, located at 27°N , $88^\circ 42'\text{E}$ at an altitude of 4800–5500 m, is a small (5.6 km long and 0.88 km wide on the average) glacier situated in Sikkim valley (Bhandari *et al* 1982b). Radiometric studies using ^{32}Si and ^{210}Pb isotopes of this glacier and their comparison with other Indian glaciers (Nehnar and Gara) studied previously are discussed elsewhere (Nijampurkar *et al* 1981, 1982; Bhandari *et al* 1982b). Our recent studies on various samples collected during the 1981 expedition show the presence of bomb produced activities such as ^{95}Zr , ^{95}Nb , ^{137}Cs , etc in snow samples which have been attributed to October 1980 Chinese atmospheric test (Bhandari *et al* 1982a). The nuclear tracers can thus be used to provide a chronological frame for deposition and flow of ice. We have chosen the longlived ^{137}Cs and determined its vertical profile in samples taken from accumulation zone for determining ice accumulation rate.

2. Experimental technique

Samples representing vertical and longitudinal profiles were collected in 1981 expedition with a view to study the natural radioactive tracers *e.g.* ^{210}Pb and ^{32}Si and artificial tracers like ^{137}Cs . A total of thirty-two samples (numbered CK-91 to CK-122) in sections of 30 cm were collected from an ice face in a crevasse in the accumulation zone at an altitude of 5250 m. This profile covers 0–9.6 m except for 420–480 cm where two samples CK-105 and CK-106 were lost in transit. Because of the inclination of the crevasse ($\approx 30^\circ$) the 960 cm depth would correspond to a vertical depth of 831 cm, each section representing a vertical depth of 26 cm. The melt waters were brought to the laboratory and processed as follows. The water samples were acidified with 1–2 ml of HCl and the dust was filtered. The ^{210}Pb activity from the water was scavenged by adding FeCl_3 carrier and precipitating $\text{Fe}(\text{OH})_3$. ^{137}Cs was measured in filtrate as well as the precipitate and dust before it was chemically processed for bismuth. The values given in table 1 represent the total ^{137}Cs activity. The counting system used was a HPGe detector located in 10 cm lead shield. Background in ^{137}Cs peak was 0.11 ± 0.01 cpm and its counting efficiency as determined from ^{137}Cs standard was found to be 3.8%. The procedure for separation and counting of ^{210}Bi — the daughter of ^{210}Pb — has been described in detail earlier (Nijampurkar *et al* 1982). ^{210}Bi was counted on a gas flow GM counter with NaI(Tl) anticoincidence having a background of about 1.4 counts/hr. In most cases, the activity of ^{210}Bi was confirmed by following its decay for a few weeks.

Table 1. Depth profile of ^{210}Pb in ice face samples, Changme-Khangpu glacier.

Sample Number	Depth (cm)	Water processed (L)	^{210}Bi counting rate (cpm) at $t = 0$	^{210}Pb (dpm/L)	^{137}Cs (dpm/L)
CK-91	0–30	0.490	1.05	24.7	15.2
CK-92	30–60	0.800	3.25	33.3	7.3
CK-95	120–150	0.600	1.02	14.8	9.3
CK-96	150–180	0.750	1.73	17.0	8.5
CK-97	180–210	0.730	1.41	18.5	3.7
CK-99	240–270	0.750	2.10	21.5	7.7
CK-100	270–300	0.600	4.21	66.1	20.8
CK-102	330–360	0.700	3.85	40.9	22.1
CK-103	360–390	0.700	2.82	35.7	17.1
CK-110	570–600	0.650	2.06	24.1	7.5
CK-111	600–630	0.780	0.52	5.9	1.9
CK-112	630–660	0.620	1.22	17.1	4.6
CK-114	690–720	0.840	1.37	14.6	4.0
CK-115	720–750	0.260	0.47	13.7	4.9
CK-117	780–810	0.740	4.24	45.9	13.2
CK-121	900–930	0.950	2.25	27.9	11.3
CK-122	930–960	0.935	2.00	15.2	7.5

*Separation of ^{210}Bi from ^{210}Pb .

3. Results and discussion

The ^{137}Cs activity in thirty core sections varies between 4 and 22 dpm/L with four main peaks. ^{210}Pb data measured in 17 of these samples are given in table 1 together with ^{137}Cs activity in the same samples. The histogram of ^{137}Cs activity as a function of sample depth is shown in figure 1. An analysis of the data indicated that the dust content does not correlate either with ^{137}Cs or ^{210}Pb activity. In figure 2 we have plotted ^{210}Pb vs ^{137}Cs activities. In general, the two activities seem to correlate well with each other.

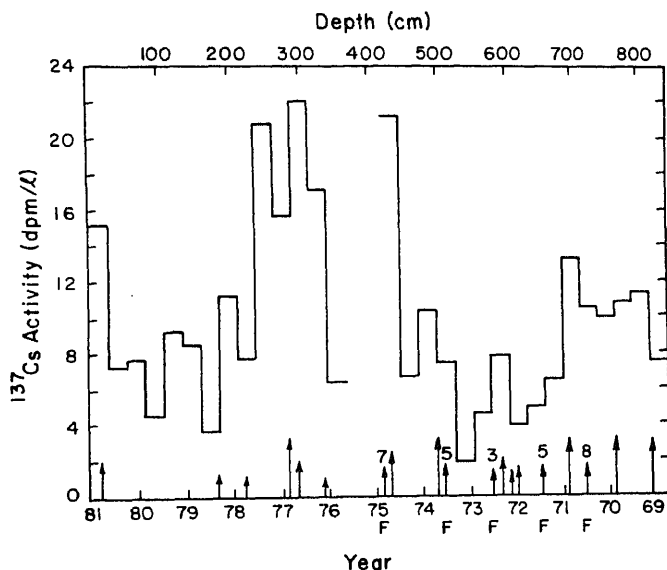


Figure 1. Variation of ^{137}Cs activity with depth of ice. Tests conducted by France are designated by *F* and the total number of explosions in a year are indicated above the arrows.

China and France have conducted several atmospheric nuclear tests since 1964 although since 1975 France has stopped atmospheric detonations. The Indian nuclear explosion of 1974 was carried out underground and was found to be well-contained in the cavity (Bhandari *et al* 1978). Whereas the production of ^{137}Cs in nuclear explosions is well-established, the production of ^{210}Pb is still controversial (Jaworowski *et al* 1979). The values of ^{210}Pb ranging between 6 to 66 dpm/L suggest one of the following possibilities: (i) the fallout of ^{210}Pb at altitudes of 4–5 km is much higher than 8 dpm/L assumed by Bhandari *et al* (1982b) and the data represent annual variations in fallout values. In such a case the observed correlation between ^{210}Pb and ^{137}Cs should not be expected. The correlation shown in figure 2 suggests a common origin of ^{137}Cs and ^{210}Pb ; (ii) ^{210}Pb is indeed produced in the nuclear explosions. The data shown in figure 2 would support such a conclusion but it has been pointed out by Grotowski *et al* (1977) that production of ^{210}Pb in thermonuclear reactions is improbable. Although Crozaz (1967) did not find any evidence of bomb-produced ^{210}Pb in Antarctic, alpine and Greenland glaciers, Jaworowski *et al* (1978) noted that ^{137}Cs and ^{210}Pb activity

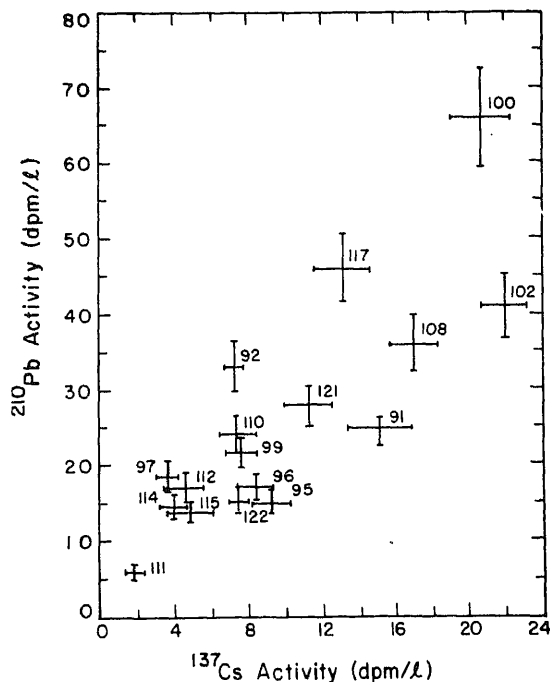


Figure 2. Variation of ^{210}Pb and ^{137}Cs activity in ice samples. Numbers refer to the CK samples as given in table 1.

peaks and troughs occur together for various glaciers including Nepalese Himalaya. However, since ^{210}Pb also correlates with ^{226}Ra and U which are of terrestrial origin, the situation is very complicated. Also for another Himalayan glacier, Nehrur, the horizons containing ^{210}Pb activity above natural levels are also associated with ^{137}Cs activity and a maximum in beta activity (Nijampurkar *et al* 1982). Further, observation of high energy neutron products like ^{54}Mn and ^{88}Y in Chinese nuclear test of 16 October 1980 (Bhandari *et al* 1982a) also seems to support this possibility; (iii) the possibility that the ice concentrates ^{210}Pb and ^{137}Cs identically as water percolates through it cannot be ruled out. Although this phenomenon is not well understood and occurs mostly in ablation zone where melt water is produced profusely (Nijampurkar *et al* 1982), such a process is expected to result in an activity profile without a structure especially towards the deeper depths. The presence of a well-structured profile with several sharp peaks for both the activities does not support this mechanism for concentration of ^{210}Pb . Further, as mentioned earlier, both the activities do not correlate with dust content which should have been the case if they were being concentrated together as a result of percolation and absorption on dust; (iv) lastly there is a contribution of ^{210}Pb from *in situ* decay of uranium series. Again absence of correlation of ^{210}Pb activity with dust content suggests that this contribution is small. Jaworowsky *et al* (1979) have invoked other mechanisms such as atmospheric circulation pattern to explain the correlation of ^{210}Pb and ^{137}Cs , etc. but it is difficult to verify such a mechanism from the present data. Whatever be the origin of ^{210}Pb , from its depth profile one can conclude that the whole core sequence represents a time scale smaller or comparable to the half-life of ^{210}Pb (22 years), which is consistent with accumulation rate of 0.7 m/yr deduced below from ^{137}Cs profile.

The major contribution to the observed ^{137}Cs activity could be from Chinese and early French tests. The cumulative yield of various atmospheric tests carried out by China from December 1968 to 1980 far exceeds the cumulative yields of French atmospheric tests during the same period (Carter and Moghissi 1977; Carter 1979). Most of the French tests were carried out in southern hemisphere at Mururoa and Fangataufa atolls (21°S , 137°W) and hence a major fraction of the radioactive fallout from these tests is expected to be deposited in the southern hemisphere. Only a small fraction will be transported to the northern hemisphere as shown by Rangarajan *et al* (1970) and Gopalakrishnan and Rangarajan (1972) who have continuously monitored different bomb produced activities in Bombay air samples during these periods. Thus only a small contribution of ^{137}Cs activity from French tests is expected in the samples analysed here but because of the geographical proximity between the Chinese tests site (Lop Nor, 40°N , 89°E) and Changme-Khangpu glacier, the major contribution is expected from Chinese tests.

From the list of nuclear explosions carried out by China (G S Murthy, Private Communication, SIPRI Year Book, 1981) we made an attempt to match the peaks in figure 1 with high yield explosions based on an arbitrary but uniform accumulation rate with the uppermost peak attributed to October 1980 Chinese test as discussed elsewhere (Bhandari *et al* 1982a; Sadasivan and Mishra 1982). For Chinese tests the length of the arrow is roughly proportional to the yield, the highest yield being 2–3 MT and lowest being 20 KT or less. Total number of French tests in a particular year is also indicated in figure 1. As can be seen in this figure there is a good correlation between the dates of Chinese explosions and ^{137}Cs peaks if a uniform accumulation rate of 0.7 m/yr is assumed. A phase lag of a few months (2–3 months) is observed for some ^{137}Cs peaks after the explosion which could be partly due to time lapse between the nuclear explosion and precipitation and could partly arise if the accumulation rate is not constant every year.

This estimate of accumulation rate is based on the assumption that the activity once deposited is preserved well. However, in temperate glaciers, such as Changme-Khangpu, this is not strictly true. Melting of snow and refreezing of water in such glaciers can cause some diffusion of the stratigraphy, but the sharpness of various peaks indicate that this process is not important in the accumulation zone.

Changme-Khangpu glacier has been studied by the Geological Survey of India since 1977. By using a network of stakes, the accumulation rates have been computed for brief periods of time. The estimates range between 0.2 mm/day to 6.7 mm/day. The average accumulation rate estimated above falls in this range. The snout of the glacier has also been monitored and is found to retreat at the rate of about 6 m/year for the past five years. Radioactive dating using ^{32}Si and ^{210}Pb indicate that the snout ice is about 100–500 years old. All these parameters together with the topographical information are useful in formulating a model of glacier dynamics which is now in progress.

References

- Bhandari N, Datta P S and Gupta S K 1978 in *Current trends in arid zone hydrology* (eds) S K Gupta and P Sharma (New Delhi: Today and Tomorrow's Printers and Publishers) p. 127
- Bhandari N, Nijampurkar V N, Shukla P N and Puri V M K 1982a *Curr. Sci.* **51** 416
- Bhandari N, Nijampurkar V N and Vohra C P 1982b *Variations of the global water budget*, Proc. Symp., Oxford, (eds) A Street-Perrott, M Beran and R Ratcliffe (D Reidel) p. 207

- Carter M W 1979 *Health Phys.* **36** 432
- Carter M W and Moghissi A A 1977 *Health Phys.* **33** 55
- Crozaz G 1967 in *Radioactive dating and methods of low level counting*, IAEA, Vienna p. 385
- Gopalakrishnan S and Rangarajan C 1972 *J. Geophys. Res.* **77** 1012
- Grotowski K, Kwiatkowski K and Jaworowski Z 1977 Report No. 984/PL of Institute of Nuclear Physics, Krakow, 1-11
- Jaworowski Z, Kownacka L, Grotowski K and Kwiatkowski K 1978 *Nucl. Technol.* **37** 159
- Jaworowski Z, Kownacka L and Bysiek M 1979 in *Natural radiation environment* Proc. Symp. III, Houston, Texas, p. 28-1
- Koide M and Goldberg E D 1981 *Earth Planet. Sci. Lett.* **54** 239
- Lambert G, Ardouin B, Sanak J, Lorus C and Pourchet M 1975 *Proc. Grenoble Symp.* p. 146, IAHS-AISH Publication No. 118
- Merlivat L, Jouzel J, Robert J and Lorus G 1975 *Proc. Grenoble Symp.* IAHS-AISH Publication No. 118, p. 138
- Nijampurkar V N, Bhandari N, Borole D V and Bhattacharya U 1981 PRL Report GLAC-81-03, Ahmedabad
- Nijampurkar V N, Bhandari N, Krishnan V and Vohra C P 1982 *J. Glaciol.* **28** 98
- Picciotto E, Crozaz G and de Breuck W 1964 *Nature (London)* **203** 398
- Rangarajan C, Gopalakrishnan S and Vohra K G 1970 *J. Geophys. Res.* **75** 1753
- Sadasivan S and Mishra U C 1982 Preprint
- World Armament and Disarmament, SIPRI Year Book, 1981, (London: Taylor and Francis), p. 382

Hypotheses for earthquake occurrences

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Abstract. Very little work has been done in generating alternatives to the Poisson process model. The work reported here deals with alternatives to the Poisson process model for the earthquakes and checks them using empirical data and the statistical hypothesis testing apparatus. The strategy used here for generating hypotheses is to compound the Poisson process. The parameter of the Poisson process is replaced by a random variable having prescribed density function. The density functions used are gamma, chi and extended (gamma/chi). The original distribution is then averaged out with respect to these density functions. For the compound Poisson processes the waiting time distributions for the future events are derived. As the parameters for the various statistical models for earthquake occurrences are not known, the problem is basically of composite hypothesis testing. One way of designing a test is to estimate these parameters and use them as true values. Moment-matching is used here to estimate the parameters. The results of hypothesis testing using data from Hindukush and North East India are presented.

Keywords. Compound Poisson processes; hypothesis testing; parameter estimation; waiting time distributions; Polya process; statistical models for earthquake occurrences.

1. Introduction

The debate whether the earthquake occurrences follow a Poisson process model is still inconclusive (Benioff 1951; Aki 1956; Shalanger 1960; Knopoff 1964; Lomnitz 1966; Ferraes 1967; Vere Jones 1970; Schlien and Toksoz 1970; Utsu 1972; Udias and Rice 1975) though a predominant view is that the Poisson process model is not particularly appropriate. One reason why the debate is still alive is that the Poisson process model has great conceptual appeal and those who rejected the Poisson process model have tried to restore it by removing the aftershocks and foreshocks from the sequence of earthquakes (Vere Jones 1970; Schlien and Toksoz 1970; Udias and Rice 1975). These attempts have lacked internal consistency because the definitions of the main event and the cluster of the main event and its aftershocks and foreshocks have been ad hoc. These have been in terms of the arbitrarily defined 'cluster length' which violates the assumption of independent events in a Poisson process model for the cluster centres, because two or more cluster centres are then forbidden within a cluster length. The second reason is that not much work is done in generating alternatives to the Poisson process model. The work of Vere Jones (1970) in which a contagious Poisson process model of the Neyman type has been suggested for the number of earthquakes per unit time is almost exceptional. No model can be effectively rejected unless more successful models are obtained. The third reason is that only some consequences of a Poisson

process model have earlier been studied. The present paper points out that different random variables derived from the stochastic process models may lead to different conclusions unless a really superior model is generated. Towards this end three different alternatives to the Poisson process model are generated here viz. Polya compound, χ -Poisson and compound $(\gamma-\chi)$ -Poisson. Under all the four hypotheses, distributions are obtained for the discrete random variable, number of earthquakes per unit time and density functions for continuous random variables such as waiting times for the next, second, third, etc. event.

The strategy used to generate alternatives to Poisson process model is compounding. Let

$$P(x = k | \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad \lambda > 0, \quad k = 0, 1, 2 \quad (1)$$

be the probability of k earthquakes per unit time under Poisson process model where λ is a parameter, called the rate of the process. Let λ be awarded a status of a random variable with density function $p(\lambda)$ over the range $[0, \infty)$. Then

$$\begin{aligned} P(x = k) &= \int_0^{\infty} P(x = k | \lambda) p(\lambda) d\lambda \\ &= E_{\lambda}[P(x = k | \lambda)], \end{aligned} \quad (2)$$

gives the probability of k earthquakes per unit time under a compound Poisson process where $E_{\lambda}[\cdot]$ is expectation with respect to the density function of λ (Fisz 1963; Johnson and Kotz 1969).

If the density function of λ is chosen to be γ , that is, if

$$p(\lambda) = \frac{a^v}{\Gamma(v)} \lambda^{v-1} \exp(-a\lambda) u(\lambda), \quad (3)$$

where $u(\lambda)$ is unit step function, and

$$\Gamma(v) = \int_0^{\infty} \lambda^{v-1} \exp(-\lambda) d\lambda \quad (4)$$

is the gamma function and where $a > 0$ and $v > 0$ are parameters, it has been shown that (Fisz 1963; Johnson and Kotz 1969) (2) becomes

$$P(x = k) = (-1)^k \binom{-v}{k} p^k q^v, \quad k = 0, 1, 2, \quad (5)$$

where

$$p = \frac{1}{1+a}, \quad q = 1 - p = \frac{a}{1+a}, \quad 0 < p < 1, \quad (6)$$

and

$$\binom{-v}{k} = \frac{(-v)(-v-1)\dots(-v-k+1)}{k!} \quad (7)$$

Equation (7) gives a negative binomial distribution. If v is a positive integer, it is sometimes called Pascal distribution (Johnson and Kotz 1969). When the postulates of

Poisson distribution, particularly independence, are suspect, the **negative binomial** distribution is a frequently suggested alternative (Johnson and Kotz 1969).

Compounding of Poisson distribution of (1) by rectangular, truncated normal and log-normal density functions has also been considered in literature (Johnson and Kotz 1969). Two new compound Poisson distributions are obtained in the next section.

2. Compound Poisson distributions

2.1 Compound Chi-Poisson distribution:

Let λ have a χ density function given by (Papoulis 1965)

$$p(\lambda) = \frac{2}{2^{n/2} \Gamma(n/2) \sigma^n} \lambda^{n-1} \exp(-\lambda^2/2\sigma^2) u(\lambda), \quad n > 1, \quad \sigma > 0. \quad (8)$$

Then (2) becomes

$$P(x = k) = \frac{2}{2^{n/2} \Gamma(n/2) \sigma^n k!} \int_0^\infty \lambda^{n+k-1} \exp[-(\lambda^2/2\sigma^2 + \lambda)] d\lambda \quad (9)$$

which is a compound Poisson distribution which will be named χ -Poisson. The problem is to evaluate an integral

$$C(m, a, b) = \int_0^\infty \lambda^m \exp[-(a\lambda^2 + b\lambda)] d\lambda. \quad (10)$$

We have (Spiegel 1968)

$$\begin{aligned} & \int_0^\infty \exp[-(a\lambda^2 + b\lambda + c)] d\lambda \\ &= \frac{1}{2} (\pi/a)^{1/2} \exp[(b^2 - 4ac)/4a] \operatorname{erfc}(b/2\sqrt{a}), \end{aligned} \quad (11)$$

where

$$\operatorname{erfc}(p) = \frac{2}{\sqrt{\pi}} \int_p^\infty \exp(-\lambda^2) d\lambda, \quad (12)$$

is the complementary error function. By putting $c = 0$ in (11) we get

$$\int_0^\infty \exp\{-(a\lambda^2 + b\lambda)\} d\lambda = \frac{1}{\sqrt{a}} \exp(b^2/4a) \int_{b/2\sqrt{a}}^\infty \exp(-\lambda^2) d\lambda. \quad (13)$$

Differentiating both sides of (13) with respect to b and using Leibnitz's theorem (Spiegel 1968) that

$$D^m(uv) = \sum_{r=0}^m \binom{m}{r} D^{m-r}u D^r v, \quad (1)$$

where D is a differential operator which is d/db in our case, we get

$$\begin{aligned} & \int_0^\infty \lambda^m \exp[-(a\lambda^2 + b\lambda)] d\lambda \\ &= \frac{(-1)^m}{\sqrt{a}} \left\{ \sum_{r=0}^m \binom{m}{r} \frac{d^r}{db^r} [\exp(b^2/4a)] \frac{d^{m-r}}{db^{m-r}} \left(\int_{b/2\sqrt{a}}^\infty \exp(-\lambda^2) d\lambda \right) \right\} \\ &= \frac{(-1)^m}{\sqrt{a}} \left\{ \sum_{r=0}^{m-1} \binom{m}{r} \frac{d^{m-r-1}}{db^{m-r-1}} \left[-\frac{1}{2\sqrt{a}} \exp(-b^2/4a) \right] \right. \\ & \quad \left. + \frac{d^r}{db^r} [\exp(b^2/4a)] + \left[\int_{b/2\sqrt{a}}^\infty \exp(-\lambda^2) d\lambda \right] \frac{d^m}{db^m} [\exp(b^2/4a)] \right\} \end{aligned} \quad (2)$$

Now, let

$$\phi(x) = \exp(-x^2/2), \quad \psi(x) = \exp(x^2/2), \quad (3)$$

and let

$$\phi^{(n)}(x) = (-1)^n \phi(x) H_n(x), \quad \psi^{(n)}(x) = \psi(x) \bar{H}_n(x), \quad (4)$$

where $H_n(x)$ and $\bar{H}_n(x)$ are polynomials and $\phi^{(n)}(x)$ and $\psi^{(n)}(x)$ are n th derivatives of $\phi(x)$ and $\psi(x)$ respectively. $H_n(x)$ are known as Hermite polynomials (Abramowitz and Stegun 1966) of degree n . They satisfy recurrence relation (Whalen 1971)

$$H_{n+1}(x) = xH_n(x) - nH_{n-1}(x), \quad H_0(x) = 1, \quad H_1(x) = x. \quad (5)$$

It can be easily shown that $\bar{H}_n(x)$ must satisfy the recurrence relation

$$\bar{H}_{n+1}(x) = x\bar{H}_n(x) + \bar{H}_n^{(1)}(x), \quad \bar{H}_0(x) = 1, \quad \bar{H}_1(x) = x. \quad (6)$$

Using (18) and (19) to tabulate the polynomials $H_n(x)$ and $\bar{H}_n(x)$, it is easily seen that the two sets of polynomials are closely related. In $H_n(x)$ and $\bar{H}_n(x)$ the absolute values of the coefficients of various powers of x are the same. However, whereas in $\bar{H}_n(x)$ the coefficients are positive, in $H_n(x)$ the coefficients of x^{n-2} , x^{n-6} , x^{n-10} , etc. are negative, where $n-2$, $n-6$, $n-10$, etc are non-negative integers. This relation between $H_n(x)$ and $\bar{H}_n(x)$ can be summarized by

$$\bar{H}_n(x) = (-j)^n H_n(jx). \quad (7)$$

Equation (20) could also be obtained more directly by using (16) and (17) after writing

$$\psi(x) = \exp[-(jx)^2]/2. \quad (8)$$

The polynomials $\bar{H}_n(x)$ can be named modified-Hermite polynomials because (20) is similar to

$$I_n(x) = (-j)^n J_n(jx), \quad (22)$$

for Bessel and modified Bessel functions $J_n(x)$ and $I_n(x)$, respectively (Bateman 1953). $H_n(x)$ and $\bar{H}_n(x)$ must now be regarded as known polynomials. Using (16) and (17), (15) becomes

$$\begin{aligned} C(m, a, b) &= \int_0^\infty \lambda^m \exp[-(a\lambda^2 + b\lambda)] d\lambda \\ &= \frac{1}{(2a)^{(m+1)/2}} \left\{ \sum_{r=0}^{m-1} \binom{m}{r} (-1)^r H_{m-r-1}[b/(2a)^{1/2}] \bar{H}_r[b/(2a)^{1/2}] \right. \\ &\quad \left. + (-1)^m \sqrt{\pi/2} \operatorname{erfc}[b/(2a)^{1/2}] \bar{H}_m[b/(2a)^{1/2}] \exp(b^2/4a) \right\} \quad (23) \end{aligned}$$

which now becomes a known function. Using (9), (10) and (23) we can write

$$\begin{aligned} P(x=k) &= \frac{2}{2^{n/2} \Gamma(n/2) k! \sigma^n} C(n+k-1, 1/2\sigma^2, 1) \\ &= \frac{2\sigma^k}{2^{n/2} \Gamma(n/2) k!} \left\{ \sum_{r=0}^{n+k-2} \binom{n+k-2}{r} (-1)^r H_{n+k-r-2}(\sigma) \bar{H}_r(\sigma) \right. \\ &\quad \left. + \sqrt{\pi/2} \operatorname{erfc}(\sigma/\sqrt{2}) (-1)^{n+k-1} \bar{H}_{n+k-1}(\sigma) \exp(\sigma^2/2) \right\}, \quad (24) \end{aligned}$$

which is the compound χ -Poisson distribution. Of course, the modified Hermite polynomials in (24) can be expressed in terms of Hermite polynomials with imaginary arguments using (20). A computer program to evaluate probabilities in (24) on DEC-20 has been written (Sharma 1982).

2.2 Compound (gamma/chi) — Poisson Distribution:

Let the density function of λ be

$$p(\lambda) = \frac{1}{C(n, a, b)} \lambda^n \exp[-(a\lambda^2 + b\lambda)] u(\lambda), \quad (25)$$

$a > 0, b > 0, n = 0, 1, 2, \dots$

where $C(n, a, b)$ is given in (23). This density function can be named extended (gamma/chi) because, if $a = 0$,

$$C(n, a, b) = C(n, 0, b) = \int_0^\infty x^n \exp(-bx) dx = \frac{\Gamma(n+1)}{b^{n+1}} \quad (26)$$

and $p(\lambda)$ of (25) reduces to the form

$$p(\lambda) = \frac{\lambda^n e^{-b\lambda} b^{n+1}}{\Gamma(n+1)} u(\lambda), \quad (27)$$

which can be seen to be a gamma density function from (3) and if $b = 0$

$$\begin{aligned} C(n, a, b) &= C(n, a, 0) = \int_0^{\infty} x^n \exp(-ax^2) dx \\ &= \frac{\Gamma\{(n+1)/2\}}{2a^{(n+1)/2}} \end{aligned} \quad (28)$$

and $p(\lambda)$ of (25) reduces to the form

$$p(\lambda) = \frac{2a^{(n+1)/2}}{\Gamma\{(n+1)/2\}} \lambda^n \exp(-a\lambda^2) u(\lambda), \quad n > 0, a > 0$$

which can be seen to be a χ density function from (8). Substituting (25) in (2) and using (10)

$$\begin{aligned} P(X = k) &= \int_0^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} \frac{1}{C(n, a, b)} \lambda^n \exp[-(a\lambda^2 + b\lambda)] d\lambda \\ &= \frac{C(n + k, a, b + 1)}{C(n, a, b)k!} \end{aligned} \quad (29)$$

where (23) can be used to get an explicit expression. This can be named compound (gamma/chi)-Poisson distribution. It subsumes gamma-Poisson and chi-Poisson distributions as special cases. A computer program to evaluate probabilities in (29) on DEC-20 has been written (Sharma 1982).

Probabilities under the Poisson, negative binomial, compound chi-Poisson and (gamma/chi)-Poisson distributions are shown in table 1 with a few chosen values of the parameters.

3. Compound Poisson process models

3.1 Poisson process model

Let X_t be the number of earthquakes during the time interval $[0, t)$ where $0 \leq t < \infty$. Then $\{X_t, 0 \leq t < \infty\}$ is a stochastic process, where for every t the random variable can take on integer values $k = 0, 1, 2, \dots$. Under certain ideal assumptions (Feller 1956; Fisz 1963) the earthquakes follow a Poisson process model given by

$$P\{X_t = k | \lambda\} = \frac{(\lambda t)^k}{k!} \exp(-\lambda t), \quad k = 0, 1, 2, \dots, \lambda > 0. \quad (30)$$

If the parameter λ is awarded a status of a random variable having density function $p(\lambda)$, we get

$$P(X_t = k) = \int_0^{\infty} \frac{(\lambda t)^k}{k!} \exp(-\lambda t) p(\lambda) d\lambda \quad (31)$$

as a compound Poisson process model (Fisz 1963).

Table 1. Probabilities for each distribution.

k	Poisson distribution $\lambda = 3.2808219$	Negative binomial distribution $a = 1.0118489$ $v = 3.3196956$	Compound chi-Poisson distribution $n = 2, \sigma = 2.65$	Compound (gamma:chi)- Poisson distribution $n = 3, a = 0.1,$ $b = 0.01$
0	0.0375973	0.1021264	0.1046892	0.0390846
1	0.1233501	0.1685158	0.1601306	0.0975718
2	0.2023448	0.1809124	0.1729216	0.1444864
3	0.2212858	0.1594551	0.1574826	0.1633476
4	0.1814998	0.1252216	0.1283219	0.1550099
5	0.1190937	0.0911185	0.0962408	0.1294238
6	0.0651208	0.0628011	0.0676109	0.0978149
7	0.0305214	0.0415600	0.0453229	0.0679416
8	0.0125169	0.0266475	0.0279501	0.0450210
9	0.0045629	0.0016659	0.0171359	
10	0.0014970	0.0102013	0.0107734	
11	0.0004465	0.0061399		
12	0.0001221	0.0036418		
13	0.0000308	0.0021332		

3.2 Polya process model

If λ has a gamma density function of (3), (31) becomes (Fisz 1963)

$$\begin{aligned}
 P(X_t = k) &= \int_0^{\infty} \frac{(\lambda t)^k}{k!} \exp(-\lambda t) \frac{a^v}{\Gamma v} \lambda^{v-1} \exp(-a\lambda) d\lambda \\
 &= \frac{v(v+1) \dots (v+k-1)}{k!} \left(\frac{t}{a+t} \right)^k \left(\frac{a}{a+t} \right)^v, \quad k = 1, 2, 3, \dots \\
 &= \left(\frac{a}{a+t} \right)^v, \quad k = 0
 \end{aligned} \tag{32}$$

using the definition of (4) and the properties of the γ functions.

3.3 Compound chi-Poisson process model

If λ has a chi density function of (8), (31) becomes

$$\begin{aligned}
 P(X_t = k) &= \int_0^{\infty} \frac{(\lambda t)^k}{k!} \exp(-\lambda t) \frac{2}{2^{n/2} \Gamma(n/2) \sigma^n} \lambda^{n-1} \exp(-\lambda^2/2\sigma^2) d\lambda \\
 &= \frac{2t^k}{2^{n/2} \Gamma(n/2) \sigma^n k!} \int_0^{\infty} \lambda^{n+k-1} \exp[-(\lambda^2/2\sigma^2 + \lambda t)] d\lambda
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2t^k}{2^{n/2} \Gamma(n/2) \sigma^n k!} C(n+k-1, (2\sigma^2)^{-1}, t) \\
&= \frac{2t^k \sigma^k}{2^{n/2} \Gamma(n/2) k!} \left\{ \sum_{r=0}^{n+k-2} \binom{n+k-1}{r} (-1)^r \right. \\
&\quad \left. H_{n+k-r-2}(\sigma t) \bar{H}_r(\sigma t) + (\pi/2)^{1/2} (-1)^{n+k-1} \right. \\
&\quad \left. \operatorname{erfc} \left(\frac{\sigma t}{\sqrt{2}} \right) \bar{H}_{n+k-1}(\sigma t) \exp(\sigma^2 t^2/2) \right\} \quad (33)
\end{aligned}$$

where (10) and (23) have been used. Equation (33) represents compound chi-Poisson process.

3.4 Compound (gamma/chi)-Poisson process model

If λ has an extended (gamma/chi) density function of (25), (31) becomes

$$\begin{aligned}
{}_1P(X_t = k) &= \int_0^\infty \frac{(\lambda t)^k}{k!} \exp(-\lambda t) \frac{\lambda^n \exp[-(a\lambda^2 + b\lambda)]}{C(n, a, b)} d\lambda \\
&= \frac{t^k C(n+k, a, b+t)}{k! C(n, a, b)} \quad (34)
\end{aligned}$$

where (10) has been used. An explicit expression can be obtained if substitutions are made for $C(n+k, a, b+t)$ and $C(n, a, b)$ from (23). Equation (33) represents compound (gamma/chi)-Poisson process.

4. Waiting time distributions for earthquakes

4.1 General theory

Let T_1 be the waiting time for the next earthquake, and let t_1 be a specific value which the random variable T_1 takes. Then

$$\begin{aligned}
\text{Prob}(T_1 < t_1) &= \text{Prob. (one or more earthquakes occur during time } t_1) \\
&= \sum_{k=1}^{\infty} P(X_{t_1} = k) = 1 - P(X_{t_1} = 0). \quad (35)
\end{aligned}$$

Let T_p be the waiting time for the p th earthquake, and let t_p be a specific value the random variable T_p takes.

Then

$$\begin{aligned}
\text{Prob}(T_p < t_p) &= \text{Prob}(p \text{ or more earthquakes occur during time } t_p) \\
&= \sum_{k=p}^{\infty} P(X_{t_p} = k) = 1 - \sum_{k=0}^{p-1} P(X_{t_p} = k). \quad (36)
\end{aligned}$$

Differentiating (35) with respect to t_1 , the density function for T_1 is obtained as (Massey 1971):

$$p_{T_1}(t_1) = - \frac{d}{dt_1} P(X_{t_1} = 0). \quad (37)$$

Similarly, differentiating (36) with respect to t_p , the density function for T_p is obtained as

$$p_{T_p}(t_p) = - \sum_{k=0}^{p-1} \frac{d}{dt_p} P(X_{t_p} = k), \quad p = 1, 2, \dots \quad (38)$$

of which (37) is a particular case.

4.2 Poisson process model

Substituting for $P(X_{t_1} = 0)$ in (37) from (30), we get (Massey 1971)

$$p_{T_1}(t_1) = \lambda \exp(-\lambda t_1) u(t_1), \quad (39)$$

which is a negative exponential density function. T_1 can also be considered the inter arrival time of the earthquakes. Thus, the interarrival times for the Poisson process are negative — exponentially distributed.

Substituting for $P(X_{t_p} = k)$ in (38) from (30), we get, after mundane adjustments

$$p_{T_p}(t_p) = \frac{\lambda^p t_p^{p-1}}{(p-1)!} \exp(-\lambda t_p) u(t_p). \quad (40)$$

From (3) it can be seen that the waiting time for the p th earthquake is γ -distributed if the earthquakes follow a Poisson process model.

4.3 Polya process model

Substituting for $P(X_{t_p} = k)$ in (38) from (32) and simplifying, we get

$$p_{T_p}(t_p) = \frac{v(v+1) \dots (v+p-1)a^v}{(a+t_p)^{v+p}} \frac{t_p^{p-1}}{(p-1)!} u(t_p), \quad (41)$$

which is the density function for waiting time for the p th earthquake under the Polya process model for earthquakes. This density function can also be obtained by compounding the gamma density function of (40) for T_p by assuming a gamma density function of (3) for λ (Sharma 1982). Hence, the density function of (41) can be named compound gamma-gamma. This also proves that (a) compounding a Poisson process by gamma-density function for λ to get the Polya process and then to obtain the waiting time density functions under Polya process model and (b) obtaining the waiting time density functions under Poisson process model and then compounding them by a gamma density function for λ , both give the same results. This is certainly reassuring and similar results would be expected for other compound processes.

4.4 Compound chi-Poisson process model

Substituting for $P(X_{t_p} = k)$ in (37) from (24), obtaining a relation (Sharma 1982)

$$H_n^{(1)}(x) = xH_n(x) - H_{n+1}(x), \quad (42)$$

for Hermite polynomials and using it together with

$$\bar{H}_n^{(1)}(x) = \bar{H}_{n+1}(x) - x\bar{H}_n(x), \quad (43)$$

which is obtained from (19) and simplifying, we get (Sharma 1982)

$$\begin{aligned} p_{T_1}(t_1) = & - \frac{2}{2^{n/2}\Gamma(n/2)} \sum_{r=0}^{n-2} \left\{ (-1)^r \binom{n-1}{r} \sigma H_{n-r-2}(\sigma t_1) \bar{H}_{r+1}(\sigma t_1) \right. \\ & - (-1)^r \binom{n-1}{r} \sigma H_{n-r-1}(\sigma t_1) \bar{H}_r(\sigma t_1) \\ & \left. + (-1)^n \sigma \bar{H}_{n-1}(\sigma t_1) + (-1)^{n-1} \sigma (\pi/2)^{1/2} \operatorname{erfc}(\sigma t_1/\sqrt{2}) \bar{H}_n(\sigma t_1) \exp(\sigma^2 t_1^2/2) \right\} \end{aligned} \quad (44)$$

As the same result can be obtained by compounding the negative exponential density function of (39) under the Poisson process model by assuming λ to be chi-distributed according to (8), the density function of (44) can be named compound chi-negative exponential.

Substituting for $P(x_{t_p} = k)$ in (38) from (24) we get

$$\begin{aligned} p_{T_p}(t_p) = & - \frac{2}{2^{n/2}\Gamma(n/2)} \sum_{k=1}^{p-1} \left\{ \frac{t_p^k}{k!} \frac{d}{dt_p} C(n+k-1, \frac{1}{2\sigma^2}, t_p) \right. \\ & \left. + \frac{t_p^{k-1}}{(k-1)!} C(n+k-1, \frac{1}{2\sigma^2}, t_p) \right\}. \end{aligned} \quad (45)$$

For reasons alluded to above this density function can be named compound chi-gamma density function. Using (23) an explicit expression for it can be obtained.

4.5 Compound (gamma/chi)-Poisson process model

Substituting for $P(X_{t_p} = k)$ in (37) from (34) and using (23), (42) and (43) to simplify, we get (Sharma 1982)

$$\begin{aligned} p_{T_1}(t_1) = & - \left[\sum_{r=0}^{n-1} \left\{ (-1)^r \binom{n}{r} H_{n-r-1}[b + t_1/(2a)^{1/2}] \bar{H}_{r+1}[b + t_1/(2a)^{1/2}] \right. \right. \\ & (2a)^{1/2} - (-1)^r \binom{n}{r} \frac{1}{(2a)^{1/2}} H_{n-r}[b + t_1/(2a)^{1/2}] \bar{H}_r[b + t_1/(2a)^{1/2}] \\ & \left. \left. + (-1)^{n+1} \frac{1}{(2a)^{1/2}} \bar{H}_n[b + t_1/(2a)^{1/2}] + \right. \right. \end{aligned}$$

$$\begin{aligned}
& + (-1)^n (\pi/2)^{1/2} \operatorname{erfc} (b + t_1/2\sqrt{a}) \frac{1}{(2a)^{1/2}} \bar{H}_{n+1} [b + t_1/(2a)^{1/2}] \\
& \exp(b + t_1)^2/4a \Big] \Big/ \\
& \left[\sum_{r=0}^{n-1} \binom{n}{r} (-1)^r H_{n-r-1} [b/(2a)^{1/2}] H_r [b/(2a)^{1/2}] + (-1)^n (\pi/2)^{1/2} \right. \\
& \left. \operatorname{erfc} (b/2\sqrt{a}) \bar{H}_n [b/(2a)^{1/2}] \exp(b^2/4a) \right]. \quad (46)
\end{aligned}$$

For reasons alluded to above this density function can be named compound (gamma/chi)-negative exponential density function.

Substituting for $P(X_{t_p} = k)$ in (38) from (34), we get

$$\begin{aligned}
p_{T_p}(t_p) = & - \frac{1}{C(n, a, b)} \sum_{k=1}^{p-1} \left\{ \frac{t_p^k}{k!} \frac{d}{dt_p} C(n+k, a, b+t_p) \right. \\
& \left. \frac{t_p^{k-1}}{(k-1)!} C(n+k, a, b+t_p) \right\} \quad (47)
\end{aligned}$$

which can be named compound (gamma/chi)-gamma density function for which explicit expression can be obtained by using (23).

5. Estimation of the parameters of various distributions

5.1 Poisson distribution

The moments of the Poisson distribution of (1) are (Johnson and Kotz 1969)

$$\begin{aligned}
E(x) &= \lambda, \quad E(x^2) = \lambda + \lambda^2, \\
E(x^3) &= \lambda + 3\lambda^2 + \lambda^3, \quad E(x^4) = \lambda + 7\lambda^2 + 6\lambda^3 + \lambda^4. \quad (48)
\end{aligned}$$

If expression for any population moment is equated with the corresponding sample moment, estimate $\hat{\lambda}$ for λ can be obtained. Thus

$$\hat{\lambda} = \frac{1}{N} \sum_{i=1}^N x_i, \quad (49)$$

could be the simplest estimate of λ where N is the number of samples. As the second and third central moments of the Poisson distribution are also equal to λ , other estimators for λ can be obtained.

5.2 Negative binomial distribution

The moments of the negative binomial distribution of (5) are (Fisz 1963):

$$\begin{aligned}
\text{mean} &= m_1 = vp/q, \\
\text{variance} &= \mu_2 = (v/qp) \left(1 + \frac{p}{q} \right). \quad (50)
\end{aligned}$$

$$a = [(\hat{\mu}_2/\hat{m}_1) - 1]^{-1}, \quad v = \hat{m}_1 a. \quad (51)$$

Therefore,

$$\hat{a} = [(\hat{\mu}_2/\hat{m}_1) - 1]^{-1}, \quad \hat{v} = \hat{m}_1 \hat{a}, \quad (52)$$

can be taken as the estimates of a and v , where $\hat{\mu}_2$ and $\hat{m}_1$ are sample variance and sample mean respectively. If v is assigned some value and only a is considered a parameter, we get

$$\hat{a} = v/\hat{m}_1. \quad (53)$$

5.3 Compound chi-Poisson and (gamma/chi)-Poisson distributions

For these distributions given by (24) and (29) respectively, expressions for the first two and three moments respectively can be written and equated with the corresponding sample moments. However, the resulting equations cannot be analytically solved. Therefore the parameters (a) n and σ (b) n , a and b respectively of the two distributions have to be evaluated by computing these distributions and their moments for various parameter values till the population moments match the sample moments.

Here also the value of n can be fixed and (a) σ and (b) a and b respectively can be obtained by matching the first and first two moments.

5.4 Gamma distribution

The m th moment for the gamma density function for the waiting time for the p th earthquake under Poisson process model for earthquake occurrences of (40) can be shown to be (Sharma 1982):

$$\begin{aligned} E(t_p^m) &= \frac{\lambda^p}{(p-1)!} \int_0^\infty t_p^{m+p-1} \exp(-\lambda t_p) dt_p \\ &= \frac{p(p+1) \dots (p+m-1)}{\lambda^m} \end{aligned} \quad (54)$$

Therefore, by putting $m = 1$, we can get

$$\hat{\lambda} = p/\hat{E}(t_p), \quad (55)$$

where $\hat{E}(t_p)$ is the sample mean of t_p . We can, of course, get estimates as

$$\hat{\lambda} = \left\{ \frac{p(p+1) \dots (p+m-1)}{\hat{E}(t_p^m)} \right\}^{1/m}, \quad (56)$$

where $\hat{E}(t_p^m)$ is the m th sample moment of t_p .

The m th moment for the compound gamma-gamma density function of (41) for the waiting time for the p th earthquake under Polya process model for earthquake occurrences can be shown to be (Sharma 1982):

$$E(t_p^m) = \frac{v(v+1) \dots (v+p-1)a^v}{(p-1)!} \sum_{j=0}^{n+p-1} \binom{m+p-1}{j} (-a)^{m+p-j-1} \int_0^{\infty} y^{j-v-p} dy. \quad (57)$$

In particular, by putting $p = 1, 2, 3$ and $m = 1, 2, 3$ in (57), we get

$$E(t_1) = \frac{a}{v-1}, \quad v > 1, \quad (58)$$

$$E(t_1^2) = \frac{2a^2}{(v-1)(v-2)}, \quad v > 2, \quad (59)$$

$$E(t_1^3) = \frac{6a^3}{(v-1)(v-2)(v-3)}, \quad v > 3, \quad (60)$$

$$E(t_2) = \frac{2a}{(v-1)}, \quad v > 1, \quad (61)$$

$$E(t_2^2) = \frac{6a^2}{(v-1)(v-2)}, \quad v > 2, \quad (62)$$

$$E(t_2^3) = \frac{24a^3}{(v-1)(v-2)(v-3)}, \quad v > 3, \quad (63)$$

$$E(t_3) = 3a/(v-1), \quad v > 1 \quad (64)$$

$$E(t_3^2) = \frac{12a^2}{(v-1)(v-2)}, \quad v > 2, \quad (65)$$

$$E(t_3^3) = \frac{60a^3}{(v-1)(v-2)(v-3)}, \quad v > 3. \quad (66)$$

It is clear from (57) that the m th moment for t_p exists only if $v > m$. That is, all the moments do not exist for the density functions of (41). Solving the pairs of equations (58) and (59), (61) and (62) and (64) and (65), respectively, we get

$$\hat{a} = \frac{\hat{E}(t_1) \hat{E}(t_1^2)}{\hat{E}(t_1^2) - 2\hat{E}^2(t_1)}, \quad \hat{v} = 2 \frac{\hat{E}(t_1^2) - \hat{E}^2(t_1)}{\hat{E}(t_1^2) - 2\hat{E}^2(t_1)} \quad (67)$$

$$\hat{a} = \frac{\hat{E}(t_2) \hat{E}(t_2^2)}{2\hat{E}(t_2^2) - 3\hat{E}^2(t_2)}, \quad \hat{v} = \frac{4\hat{E}(t_2^2) - 3\hat{E}^2(t_2)}{2\hat{E}(t_2^2) - 3\hat{E}^2(t_2)} \quad (68)$$

$$\text{and} \quad \hat{a} = \frac{\hat{E}(t_3) \hat{E}(t_3^2)}{3\hat{E}(t_3^2) - 4\hat{E}^2(t_3)}, \quad \hat{v} = 2 \frac{3\hat{E}(t_3^2) - 2\hat{E}^2(t_3)}{3\hat{E}(t_3^2) - 4\hat{E}^2(t_3)} \quad (69)$$

as estimates for parameters in the density functions $p_{T_1}(t_1)$, $p_{T_2}(t_2)$ and $p_{T_3}(t_3)$ respectively of (41). If the conditions on v in (58) to (66) are not satisfied, the estimates of a and v may not be valid. These estimates may or may not be consistent with (60), (63) and (66) respectively.

5.6 Compound chi-gamma and (gamma/chi)-gamma distributions

The moments of the compound chi-gamma and (gamma/chi)-gamma density functions of (45) and (47) for the waiting time for the p th earthquake under the compound chi-Poisson and (gamma-chi)-Poisson process models for earthquake occurrences can in principle be obtained, and equated with the corresponding sample moments to obtain the estimates of the parameters (a) n and σ and (b) n , a and b , respectively. However, the expression for $p_{T_p}(t_p)$ themselves are complicated and, therefore, this procedure is clearly impractical. The only hope is to tabulate the density functions and their moments on a computer for various values of the parameters till the sample moments match the computed population moments.

6. Hypothesis testing

6.1 The regions and data

The data of earthquakes in the Hindukush region are taken from the catalogue of epicentral locations of earthquakes published by the Indian Meteorological Department. For computing the number of earthquakes per unit time, 2395 earthquakes of magnitude greater than 3.5, for the period from January 1963 to December 1974, having focal depth less than 250 km in an area bounded by 62 to 76° E longitude and 30 to 39° N latitude are used. In tests for waiting time density functions, 1935 earthquakes were considered from a region restricted to 69 to 72° E longitude and 35 to 38° N latitude during the period from January 1970 to December 1976.

Another set of data of microearthquakes from the North-Eastern region of India is also used. These data were collected by the University of Roorkee and the Geological Survey of India under a joint project, for a period of 5.5 months from May 1979 to October 1979. There were seismic stations at Raliang (25.47° N, 92.43° E), Borjori (26.40° N, 92.94° E), Burnihat (26.06° N, 91.89° E) and Shillong (25.57° N, 91.88° E). The Shillong station was run by the Indian Meteorological Department. Magnitudes were between 2.2 to 5.1. Earthquake sources having distances more than about 300 km, i.e. those with S-P times greater than 40 seconds, were excluded. 235 events were used for the analysis.

6.2 Choice of a test

The likelihood ratio test was used. That is, if $P_1(x)$ and $P_0(x)$ were probabilities under the alternative and the null hypotheses, the alternative hypothesis was accepted if (Whalen 1971)

$$\frac{P_1(x)}{P_0(x)} \geq 1. \quad (70)$$

Similarly, for the continuous random variable t_p , the alternative hypothesis was accepted if (Whalen 1971)

$$\frac{p_1(t_p)}{p_0(t_p)} \geq 1 \quad (71)$$

where $p_1(t_p)$ and $p_0(t_p)$ are the density functions under the alternative and the null hypotheses.

Under multiple observations, the observations could be considered to be independent (though not always a correct assumption) and the alternative hypothesis was accepted if (Whalen 1971)

$$\prod_{i=1}^N \frac{P_1(x_i)}{P_0(x_i)} \geq 1 \quad \text{or} \quad \prod_{i=1}^N \frac{p_1(t_{pi})}{p_0(t_{pi})} \geq 1, \quad (72)$$

where x_i is the i th sample for number of earthquakes per unit time or t_{pi} is the i th sample for the waiting time for the p th earthquake, and N is the number of samples.

Alternatively, whenever the inequality (70) or (71) was satisfied for a particular sample, it was considered as one vote for an alternative hypothesis, and an alternative hypothesis was accepted if it collected a majority of votes. Thus, in the two decision rules, the integration of samples can be said to be before the decision and after the decision, respectively.

The tests discussed above assume that the parameters (λ , n , σ , a or b) are known. However, if the parameters are not known and have to be regarded as random variables, the hypotheses are called composite. One of the strategies in such a case is to obtain estimates of the unknown parameters and use these estimates as the operative values of the parameters to obtain the likelihood ratio (Whalen 1971). Normally, maximum likelihood estimates are used, but here we have used moment-matching estimates discussed in §5.

6.3 Tests for number of earthquakes

6.3a *Poisson versus negative binomial distribution:* For number of earthquakes per week occurring in Hindukush region as random variable, we have

$$\begin{aligned} \text{Sample mean} &= 3.281, \quad \text{second sample moment} = 17.287, \\ \text{third sample moment} &= 113.64, \quad \text{sample variance} = 6.523 \end{aligned} \quad (73)$$

Therefore, from (49) and (52)

$$\hat{\lambda} = 3.281, \quad \hat{a} = 1.012, \quad \hat{v} = 3.32. \quad (74)$$

Using (72), (1) (5) and (6), the negative binomial distribution is to be accepted if (Sharma 1982)

$$\begin{aligned}
& -N\hat{\lambda} + N\ln[(\hat{v} - 1)!] - N\hat{v}\ln\hat{q} + \\
& + \sum_{i=1}^{\hat{v}} k_i \ln(\hat{\lambda}/\hat{p}) - \sum_{i=1}^{\hat{v}} \ln[(\hat{v} + k_i - 1)!] \leq 0
\end{aligned} \tag{75}$$

The value of the left side in (75) comes out to be -1768.89 . Thus, the negative binomial distribution is to be accepted.

Taking the number of earthquakes per fortnight as a random variable, we get

$$\begin{aligned}
\text{Sample mean} &= 7.011; \text{ second sample moment} = 70.038; \\
\text{sample variance} &= 20.419
\end{aligned} \tag{76}$$

Therefore, from (49) and (52)

$$\hat{\lambda} = 7.011, \hat{a} = 0.527, \hat{v} = 3.710 \tag{77}$$

and the left side of (75) turns out to be -6513 suggesting again that the negative binomial distribution can be accepted.

Taking the number of micro-earthquakes per day in the NE India as a random variable, we get

$$\begin{aligned}
\text{Sample mean} &= 9.198; \text{ second sample moment} = 110.51; \\
\text{sample variance} &= 25.899
\end{aligned} \tag{78}$$

so that from (49) and (52) we get

$$\hat{\lambda} = 9.198, \hat{a} = 0.558, \hat{v} = 5.135, \tag{79}$$

and the left side of (75) turns out to be -11841.8 which once again suggests that the negative binomial distribution can be accepted.

Using the number of earthquakes per fortnight in the Hindukush region as a random variable and the voting scheme for decision-making, the votes in favour of the Poisson and negative binomial distributions are 153 and 187 respectively. Thus, the negative binomial distribution is favoured. Using the number of earthquakes per week in Hindukush region as a random variable, the votes in favour of the Poisson and negative binomial distributions are 448 and 282 respectively! In this case the Poisson distribution is favoured.

6.3b Poisson versus compound chi-Poisson distribution: Taking the number of earthquakes per week in the Hindukush region as a random variable, the parameters of the compound chi-Poisson distribution were estimated to be

$$\hat{n} = 2, \hat{\sigma} = 2.65, \tag{80}$$

With these parameters the population moments are

$$\text{First moment} = 3.316 \text{ and second moment} = 17.121. \tag{81}$$

These values are reasonably close to the sample moments of (73).

The votes in favour of the Poisson and compound chi-Poisson distributions were 404 and 326 respectively, thereby favouring the Poisson distribution.

6.3c *Poisson versus compound (gamma/chi)-Poisson distribution:* Taking the number of earthquakes per week in the Hindukush region as a random variable, the parameters of the compound (gamma/chi)-Poisson distribution were estimated as

$$\hat{n} = 3, \quad \hat{a} = 0.1, \quad \hat{h} = 0.01 \quad (82)$$

With these parameters the population moments are

$$\begin{aligned} \text{First moment} &= 3.566, \quad \text{second moment} = 17.593 \text{ and} \\ \text{third moment} &= 99.245 \end{aligned} \quad (83)$$

These values are reasonably close to the sample moments of (73).

The votes in favour of the Poisson and compound (gamma/chi)-Poisson distributions were 432 and 298 respectively, thereby favouring the Poisson distribution.

6.4 Tests for waiting time distributions

6.4a *Gamma-versus compound gamma-gamma distribution:* From (71), (40) and (41), the decision rule is to prefer compound gamma-gamma density function over the gamma density function if

$$\begin{aligned} g(t_p) = \hat{\lambda} t_p - (\hat{v} + p) \ln (\hat{a} + t_p) &\geq -\hat{v} \ln \hat{a} + p \ln \hat{\lambda} \\ - \sum_{j=0}^{p-1} \ln (\hat{v} + j) &= \Lambda_p \end{aligned} \quad (84)$$

where t_p is the waiting time for the p th earthquake, and the test is resolved by comparing a statistic $g(t_p)$ against the threshold Λ_p .

With inter-arrival time t_1 as the random variable and using (55) and (67), we get

$$\hat{\lambda} = 6.983 \times 10^{-6}, \quad \hat{a} = 1.411 \times 10^6 \text{ and } \hat{v} = 10.851 \quad (85)$$

With waiting time t_2 as the random variable and using (55) and (68), we get

$$\hat{\lambda} = 6.979 \times 10^{-6}, \quad \hat{a} = 1.996 \times 10^6 \text{ and } \hat{v} = 14.930 \quad (86)$$

With waiting time t_3 as the random variable and using (55) and (69), we get

$$\hat{\lambda} = 6.978 \times 10^{-6}, \quad \hat{a} = 2.555 \times 10^6 \text{ and } \hat{v} = 18.830. \quad (87)$$

Using (85) to (87) for $p = 1, 2, 3$ and (84), the thresholds Λ_p turn out to be

$$\Lambda_1 = -167.896, \quad \Lambda_2 = -245.808 \text{ and } \Lambda_3 = -322.392. \quad (88)$$

The data used are from the Hindukush region and the time is measured in seconds.

The votes in favour of gamma and compound gamma-gamma density functions were (a) 864 and 670, (b) 906 and 627, (c) 870 and 662 respectively for waiting times t_1 , t_2 and t_3 , thus the Poisson process model is preferred to Polya process model.

6.4b *Gamma versus compound chi-gamma and (gamma/chi)-gamma distributions*

The expressions for compound chi-gamma and (gamma/chi)-gamma density functions are complicated and these density functions have not been computed yet. Therefore, they have not been tested against the gamma distribution yet.

7. Discussion

7.1 *Analysis of results*

(a) While comparing various stochastic process models for earthquake occurrences the decision of hypothesis test may depend on the definition of the random variable chosen for the test. If the test is based on the number of earthquakes in a fortnight the negative binomial distribution is favoured. The Poisson distribution is accepted if the unit of time is a week (for the data from Hindukush region using voting scheme).

(b) The decision to accept or to reject a particular hypothesis may depend on how the test is set up. When the random variable was the number of earthquakes per week, the Poisson distribution was accepted on the basis of a voting scheme whereas the negative binomial distribution was preferred when various samples were treated as independent samples.

(c) When stochastic process models for earthquake occurrences are being tested, different results may be obtained on the basis of different implications of the stochastic process models. If the number of earthquakes per unit time is used for the hypothesis test, the Polya process model may be accepted, but on the basis of waiting times for the first, second, third, etc. earthquakes Poisson process model may be preferred.

(d) There are situations wherein the Poisson process model is not the best and the Polya process model seems superior. But, also in situations where the Poisson process model is preferred the difference in the number of votes gathered by the two hypotheses under comparison is not very large. That is, the alternatives to the Poisson process model proposed here are not very poor. In fact, if instead of deciding the issue by simple majority and taking a hard decision, if the soft decision were entertained, that is, if a particular hypothesis was to be accepted only if it got $(50 + \Delta)\%$ votes and if the decision was to be deferred if the votes gathered by the winning hypothesis were less than $(50 + \Delta)\%$, it is conceivable that the decisions in this paper would have been deferred until larger data sets are available for $\Delta = 12\%$.

7.2 *Limitations of the analysis*

(a) It is known that the mean, variance and the third central moment are all equal to λ for the Poisson distribution. It can be seen that this equality does not hold for the data from (73), (76) and (78). Similarly, values of $\hat{a}$ and $\hat{v}$ estimated from the different waiting time statistics are significantly different under Polya process model, as can be seen from (85)–(87). This might be interpreted to mean that neither the Poisson nor the Polya process model is particularly good for the data considered here. Nevertheless it can also be used as an argument that the parameters of a stochastic process model should not be estimated by matching just the requisite number of moments for a

chosen random variable. But an attempt may be made to match more than the requisite number of moments for a moderate number of random variables simultaneously under a suitable matching criterion. Alternatively, estimators such as maximum likelihood or maximum entropy estimators may be sought. The results of the hypothesis test may depend on how the parameters are estimated.

(b) After the parameters are estimated by a method, the estimate of the density function or the probability is not obtained by substituting the estimate instead of parameter. That is, if $\hat{\lambda}$ is obtained from (49), $\hat{\lambda}$ is a maximum likelihood estimator of λ (Johnson and Kotz 1969), but then $\hat{\lambda}^k \exp(-\hat{\lambda})/k!$ is not a maximum likelihood estimator of $P(x = k)$. In fact, the minimum variance unbiased estimator of $P(x = k)$ is shown to be (Barton 1961 and Glasser 1962)

$$\hat{P}(x = k) = \binom{N\hat{\lambda}}{k} \left(\frac{N-1}{N} \right)^{N\hat{\lambda}} \frac{1}{(N-1)^k} \quad (89)$$

A similar situation would arise for other distributions. As an example, let $P(x = k; n, a, b)$ be a distribution of (29) where parameters n, a, b are explicitly shown. Once the estimates $\hat{n}, \hat{a}, \hat{b}$ are obtained, for likelihood ratio test we have used

$$\hat{P}(x = k; n, a, b) = P(x = k; \hat{n}, \hat{a}, \hat{b}) \quad (90)$$

as an estimate of $P(x = k; n, a, b)$. That this is erroneous can be easily shown by expanding $P(x = k; n, a, b)$ as a function of n, a, b as Taylor's series around $\hat{n}, \hat{a}, \hat{b}$. Then

$$\begin{aligned} P(x = k; n, a, b) &= P(x = k; \hat{n}, \hat{a}, \hat{b}) + \\ &+ (n - \hat{n}) \frac{\partial P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n} + (a - \hat{a}) \frac{\partial P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a} \\ &+ (b - \hat{b}) \frac{\partial P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial b} + \frac{(n - \hat{n})^2}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n^2} \\ &+ \frac{(a - \hat{a})^2}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a^2} + \frac{(b - \hat{b})^2}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial b^2} \\ &+ (n - \hat{n})(a - \hat{a}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n \partial a} \\ &+ (n - \hat{n})(b - \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n \partial b} \\ &+ (a - \hat{a})(b - \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a \partial b} \end{aligned} \quad (91)$$

wherein as $n \neq \hat{n}$, $a \neq \hat{a}$, $b \neq \hat{b}$, the terms other than the first on the right side, do not vanish. Taking expectation with respect to the joint density function of $\hat{n}, \hat{a}, \hat{b}$, (91) becomes

$$\begin{aligned}
 P(x = k; n, a, b) &= E[P(x = k; \hat{n}, \hat{a}, \hat{b})] \\
 &+ \frac{\text{var}(\hat{n})}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n^2} + \frac{\text{var}(\hat{a})}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a^2} \\
 &+ \frac{\text{var}(\hat{b})}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial b^2} + \text{covar}(\hat{n}, \hat{a}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n \partial a} \\
 &+ \text{covar}(\hat{n}, \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n \partial b} + \text{covar}(\hat{a}, \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a \partial b}
 \end{aligned} \tag{92}$$

assuming, that the series in (92) converges, that higher order terms can be neglected, that $\hat{n}, \hat{a}, \hat{b}$ are unbiased estimators of n, a, b and that the derivatives of $P(x = k; \hat{n}, \hat{a}, \hat{b})$ are independent of $(a - \hat{a})(a - \hat{a})(b - \hat{b})$, etc. Thus, in general

$$E[P(x = k; \hat{n}, \hat{a}, \hat{b})] \neq P(x = k; n, a, b) \tag{93}$$

that is $P(x = k; \hat{n}, \hat{a}, \hat{b})$ is not an unbiased estimator of $P(x = k; n, a, b)$ in whatever way $\hat{n}, \hat{a}, \hat{b}$ are obtained. To estimate an error, low order moments of the estimators of parameters must be estimated. There are many methods of estimating parameters. For example, four methods of estimating parameters of the negative binomial distribution are described (Johnson and Kotz 1969). Such an analysis should be done for all the methods. As this exercise has not been conducted in this paper, the results need not be reliable.

(c) Compounding has been used here to generate alternative hypotheses for earthquake occurrences in a heuristic way. An attempt has not been made to attach physical meaning to the parameters of the compounding distribution or to the compounding distribution as a whole. Thus the work reported here needs further investigation.

(d) The alternative hypotheses generated here relate only to earthquake occurrences and do not take into account the location of the epicentres, the magnitudes of the earthquakes, etc. A more comprehensive and utilitarian statistical model for earthquakes must include these aspects.

(e) The computed probabilities for the compound chi-Poisson and (gamma/chi)-Poisson distributions violated the necessary constraint that the probabilities must always be positive, for large values of k . Thus, the formulae for these distributions are not numerically appropriate. Numerically appropriate formulae for them should be obtained. These may be in terms of the recurrence relations between $P(x = k)$, $P(x = k - 1)$, ..., $P(x = k - m)$. The choice of time period of one week was dictated by this numerical difficulty.

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References

- Abramowitz M and Stegun I A 1966 *Handbook of mathematical functions*, Nat. Bur. of Std. Appl. Math. Ser 55 (Washington D.C. U.S. Government Printing Office) pp. 510
- Aki K 1956 *Zisin* 8 205
- Barton D E 1961 *Biometrika* 48 227
- Bateman H 1953 *Higher transcendental functions II* (New York: McGraw-Hill) pp. 5
- Benioff H 1951 *Bull. Geol. Soc. Am.* 62 331
- Feller W 1965 *An introduction to probability theory and its applications* (New York: John Wiley) Vol. 1, pp. 400
- Ferrares S G 1967 *Mexico Geophys. Int.* 7 43
- Fisz M 1963 *Probability theory and mathematical statistics* (New York: John Wiley) pp. 164, 276, 298
- Glasser G J 1962 *Technometrics* 4 409
- Johnson N L and Kotz S 1969 *Discrete distributions* (Boston, Houghton: Mifflin Company) pp. 44, 90, 94, 122, 131, 183
- Knopoff L 1964 *Bull. Seismol. Soc. Am.* 5 1871
- Lomnitz C 1966 *Rev. Geophys.* 4 377
- Massey L D 1971 *Probability and statistics* (New York: McGraw-Hill) pp. 114, 115
- Papoulis A 1965 *Probability, random variables and stochastic processes* (New York: McGraw-Hill) pp. 250
- Schlien S and Toksoz M M 1970 *Bull. Seismol. Soc. Am.* 60 1765
- Shalanger (Ben Menahum) A 1960 *Gerlands Beitr. Geophys.* 69 68
- Sharma N K 1982 *Generation and testing of hypotheses for earthquake occurrences*, M.Tech. Thesis, University of Roorkee, Roorkee (Unpublished) pp. 17, 37, 41, 43, 54, 60, 65, 68, 70, 72
- Spiegel M R 1968 *Mathematical handbook of formulas and tables, Schaum's outline series* (New York: McGraw-Hill) pp. 98
- Udias A and Rice J 1975 *Bull. Seismol. Soc. Am.* 65 809
- Utsu T 1972 *J. Fac. Sci. Hokkaido University Geophys. Ser. VII* 4 1
- Vere-Jones T 1970 *J. R. Statist. Soc.* 832 1
- Whalen A D 1971 *Detection of signals in noise* (New York: Academic Press) pp. 126, 246, 250, 252, 254

Electron microscopy of olivine with perfect cleavage

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Abstract. Electron microscopy (SEM and TEM) of unusual olivine (Fo = 88) crystals, with perfect (010) cleavages, from Chalk Hills, Salem, Tamil Nadu has been carried out. SEM studies reveal the effect of compressive stress. Microstructures by TEM showed the abundance of curved dislocations with jogs, kinks and dipoles, indicative of the dominant climb, characteristic of high temperature deformation. The stacking fault fringes observed in olivine are due to mechanical weakening caused by nonstructural chemical constituents. The evidences for this come from fluid-microinclusions observed and higher amount of K, Na, Rb and Sr in the olivine. Hydrolytic weakening through dislocation glide motion, assisted by H₂O containing incompatible elements, may break the metal-oxygen bonds. This leads to perfect parting of crystallographic planes under deformational stress, during solid emplacement of the dunite. The easily cleavable planes are those with the largest interplanar spacings.

Keywords. Olivine mineral; electron microscopy; deformation; perfect cleavage.

1. Introduction

While studying the ultramafites in Chalk Hills, Salem, Tamil Nadu (Lat. 11° 40' and 11° 45'; Long. 78° 05' and 78° 11'), outcrops of dunites with lustrous sheen, resembling mica have been noticed. They are exposed amidst the unmetamorphosed massive peridotites and are named 'schiller dunites' as a field term. These dunites contain tabular and prismatic grains of olivine with perfect cleavage. Normally olivine has imperfect cleavages, particularly among the magnesium-rich end-members. So far there is only one report on the occurrence of olivine with perfect cleavages (Hawkes 1946). No detailed mineralogical studies have been reported even on this.

Petrological and mineralogical studies have indicated that a major part of the ultramafites from Chalk Hills are residual peridotites left behind by a partial melting event in the subcrustal continental mantle (Murthy 1982). This ultramafic residue found its way to the crust through transcrustal faults. The evidences for the solid emplacement come from petrographic observations *viz* recrystallisation, extreme fracturing and crude alignment of grains. Since the schiller dunites exist amidst these residual peridotites, it is possible that the development of perfect cleavages may arise through special conditions of deformation, accompanied by recrystallisation.

Another point of general interest is that olivine is the major mineral of the upper mantle and that the mechanical behaviour of this mineral largely determines the plastic flow of asthenosphere (Carter 1976). Hence deformation mechanisms in olivine crystals are of great geophysical interest. Recent researches have shown that there are more than one type of peridotites (Coleman 1977); those crystallised from a magma (cumulates) and those which are residual part of a partial melting process in the mantle. The

latter, when emplaced in the crust, displays textures similar to metamorphic rocks. Extensive deformation during solid emplacement leads to high dislocation density and substructural changes. Although considerable information on the xenolith peridotites from alkaline basalts and kimberlites has been obtained through electron microscopy, little is known about the peridotites tectonically emplaced in the continental crust. Hence it is thought worthwhile to investigate the mechanism of generation of perfect cleavages in olivine using transmission electron microscope (TEM), coupled with SEM, x-ray powder diffraction, infrared spectrum, chemical and petrographic studies. Electron microscopy has revealed the importance of hydrolytic weakening under stress possibly leading to the development of perfect cleavages in olivine.

2. Experimental

Olivine from schiller dunite is designated as olivine 1 and that from the associated massive dunite as olivine 2. For transmission electron microscopic studies, the samples were finely ground in acetone medium and two drops of the solution were deposited on a holey carbon grid. An electron microscope (Philips FM 301) with double tilt holder was used for the present study. A stereoscan (Cambridge S150) was used for the scanning electron microscopic studies of the polished and etched samples of the mineral. Diffractograms were recorded of the powdered mineral in Philips PW1140 diffractometer using CuK_α radiation. Infrared spectra were recorded in KBr using Perkin-Elmer 590 spectrometer. Chemical analysis were carried out by wet methods using a combination of complexometry, colorimetry and flame photometry.

3. Results and discussion

Chemical composition of the olivine from schiller dunite (olivine 1) and that of an olivine from the associated massive dunite (olivine 2) is given in table 1, along with the number of cations per formula unit. Both the samples have high forsterite, Mg_2SiO_4 , content ($\text{Fo} = 88$ and 92 respectively). The chemical differences observed are comparatively higher amounts of alkalis and water in olivine 1, while olivine 2 has higher Cr and Ni contents. Preliminary mass spectral analysis with spark source has indicated higher K (600 ppm), Rb (5 ppm) and Sr (32 ppm) contents in olivine 1, which are unusually high for olivines from residual peridotites.

Both the olivines are colourless under the petrographic microscope. Olivine 1 has perfect (010) cleavage and moderate (100) parting. The schiller effect is due to the reflection of light from the perfectly cleaved (010) planes. In rock slices, the cleavage in olivine is so good that it can be mistaken for orthopyroxene (figure 1). Optical microscopy also shows that there are relict olivine crystals with imperfect cleavages within the schiller dunite, indicating that the cleavages are secondarily produced through certain solid-state processes. The absence of perfect cleavage in the relict olivine crystals is not a directional feature, as verified by the studies on rock slices at orthogonal directions of the same specimen. Petrographic studies have shown the absence of secondary phases formed by the alteration of olivine.

Scanning electron micrographs reveal that the grain boundaries in schiller dunite are distorted (figure 2a) far more than those of massive dunite from Chalk Hills (figure 2b). The type of folding of the grain boundary corresponds to a compressive stress along

Table 1. Chemical composition of olivine.

	Olivine 1	Olivine 2
SiO ₂	40.03	40.82
TiO ₂	0.01	0.00
Al ₂ O ₃	0.59	0.25
Fe ₂ O ₃	0.62	0.20
FeO	10.85	7.72
MnO	0.12	0.09
MgO	46.57	50.07
CaO	0.40	0.35
Na ₂ O	0.24	0.08
K ₂ O	0.11	0.02
P ₂ O ₅	0.03	0.14
H ₂ O ⁺	0.37	0.02
H ₂ O ⁻	0.08	0.01
CO ₂	0.06	0.00
Ni (ppm)	1120	2600
Cr (ppm)	635	2840
* Number of ions on the basis of 4 (O) atoms.		
Si	0.996	0.007
Ti	0.002	0.000
Al	0.017	0.007
Fe (+3)	0.012	0.004
Fe (+2)	0.226	0.158
Mn	0.003	0.002
Mg	1.727	1.822
Ca	0.011	0.009

* Number of ions on the basis of 4 (O) atoms.

the horizontal axis, in the specimen, as seen in the micrograph. The cleavage planes are aligned perpendicular to the compressional stress. Petrographically these planes are identified to be (0k0) type. They generate striations on etching (figure 2c). The original grain boundary is barely visible to the left side of figure 2c, the striations are running across this boundary indicating that cleavage is not primary feature. That the dunites were in the high pressure field during their formation is confirmed by the presence of microcrystals of spinel in the massive dunites (figure 2d). It has been demonstrated experimentally that spinel-peridotites have deeper origin in the mantle, when compared to plagioclase-periodotites (Wyllie 1971).

X-ray powder diffraction data indicate that olivine 1 has nearly the same *d*-values as those of olivine 2. In the Mg₂SiO₄-Fe₂SiO₄ solid solution series, the unit cell dimensions increase with increasing replacement of Mg²⁺ by Fe²⁺. The cell dimensions determined in the present case (table 2) are in accordance with this generalisation. Besides, the *d*₁₁₀ spacings correspond to the compositional range given in table 1 (Yoder and Sahama 1957). The conspicuous difference in the x-ray pattern of olivine 1 is the high relative intensity of (020) reflection. *I*₀₂₀ / *I*₀₂₁ ratio is 2.7 when compared to

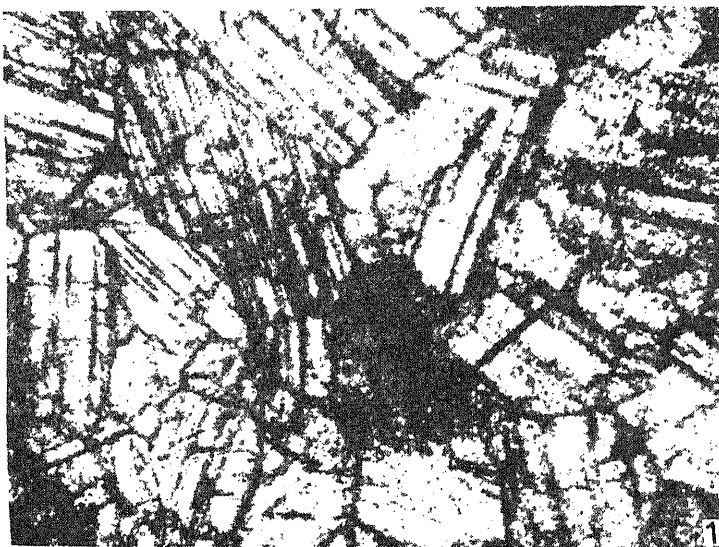


Figure 1. Photomicrograph of schiller dunite under plane polarised light ($\times 65$)

1.05 for olivine 2 (figure 3). This can be explained on the basis of the preferred orientation of crystallites in the x-ray powder mount. Crystals exhibiting pronounced cleavage such as mica, kaolinite or chlorite orient preferentially even when reduced to lower particle size by grinding. The particles, in these cases, align such that the plane of the platelet is parallel to the axis of the specimen. The crystal plane parallel to the platelet, thus, has the greatest chance of reflecting causing increase in intensity of reflection from cleavage plane (Klug and Alexander 1974; Azároff and Buerger 1958) (figure 3). In order to show that the cause of increased intensity of (020) reflection is due to preferred packing effect, the powdered specimen of olivine 1 is mixed with finely divided Lindemann glass (Klug and Alexander 1974). The relative intensity, I_{020}/I_{021} , is lowered with the extent of dispersion, thus showing the effect of preferred orientation. Further, the orientational effect has been tested by comparing the ratio of the integrated intensities of (020) and (021) reflections measured in symmetrical reflection geometry to their ratio measured in symmetrical transmission geometry, following the method of Weiss (1966). It is seen that this ratio does not obey the Weiss-equation, further confirming the presence of preferred orientation.

There are no other detectable crystallographic or structural changes for the olivine 1. This is further supported by the infrared absorption spectrum of this specimen, when compared with that of olivine 2 (figure 4). The absorption bands correspond to those arising from independent SiO_4 tetrahedra.

From the above observations, it can be concluded that the generation of perfect cleavage has more to do with the microstructural changes than to the variations in molecular structure. In this context, TEM studies have been very useful, particularly in comparison with the substructure of olivine 2. Microstructural changes occur due to deformations during the solid emplacement of dunites. Experimentally and naturally-deformed silicate minerals indicate that deformation is achieved by slip, mechanical

Table 2. X-ray powder diffraction data.

<i>hkl</i>	Olivine 1		Olivine 2	
	<i>d</i> (Å)	Relative Intensity	<i>d</i> (Å)	Relative Intensity
020	5.126	100	5.126	100
021	3.090	37	3.909	95
101	3.739	8	3.739	15
111	3.507	10	3.493	25
002	3.008	10	2.998	29
130	2.780	37	2.773	100
131	2.522	35	2.522	90
112	2.468	38	2.462	71
041	2.357	10	2.356	27
210	2.322	11	—	—
122	2.275	19	2.270	34
140	2.261	21	2.254	46
132	2.036	4	2.036	15
150	1.881	6	1.877	15
151	—	—	1.794	7
222	1.755	29	1.7554	42
241	—	—	1.675	17
061	1.674	8	1.642	34
133	1.645	15	1.622	22
004	1.501	21	1.499	23
331	—	—	1.3961	27
171	1.3645	16	1.353	12
322	1.355	7	1.351	12

Unit cell parameters:

<i>a</i>	4.763 (Å)	4.756 (Å)
<i>b</i>	10.28 (Å)	10.27 (Å)
<i>c</i>	6.011 (Å)	5.991 (Å)

twinning or shear (martensic) transformations, controlled by dislocation motion. The predominant plastic flow mechanism in olivine is through the formation of deformation lamellae parallel to the slip planes (Green and Radcliffe 1972a; Carter and Ave'Lallemant 1970). Slip mechanisms vary with temperature and less sensitively with strain rates. Green and Radcliffe (1972b) observed that with increasing stress, small grains are formed by the edge dislocations climbing to form (100) boundaries and screw dislocations cross-slipping to form (001) boundaries, producing rectangular subgrains elongated parallel to (010). TEM studies of olivine 2 reveal the subgrains with (100) kink boundaries (figure 5a). Careful observations show 'picket fence' structure at the kink boundaries, which are the dense tangled zones of dislocations. The edge components of dislocations loops of $\{0K1\}$ $[100]$ climb into such kink boundaries, leaving the screw components of these loops extending across the band. The screws are pinned to their edge components in the boundaries and are seen to bow out.



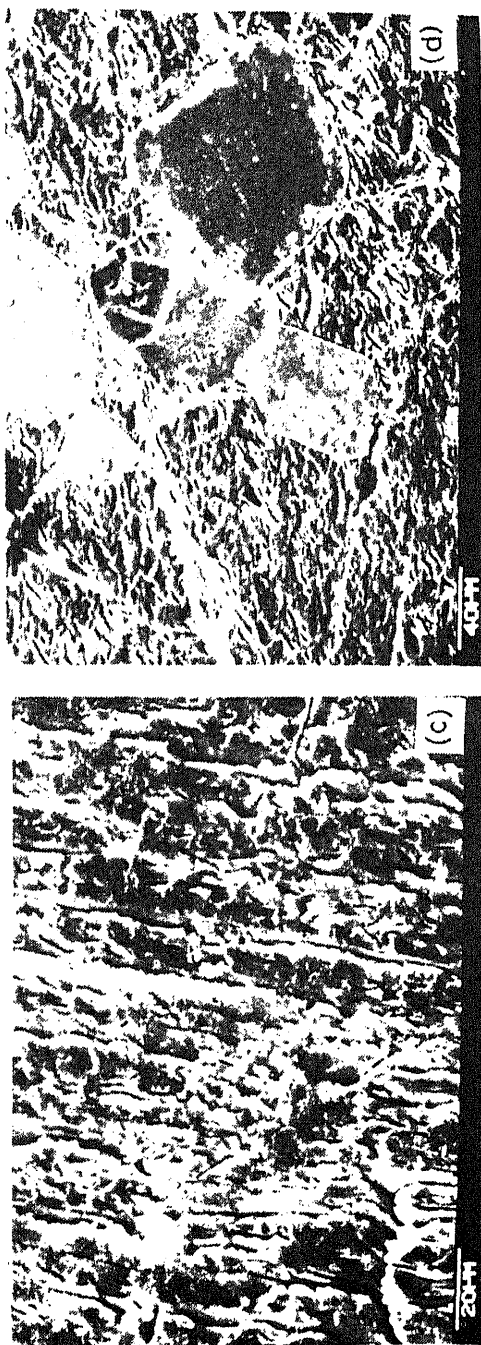


Figure 2. SEM micrographs of dunites. (a) Schiller dunite with buckled grain boundaries. (b) Massive dunite with nearly straight grain boundaries. (c) The parting planes of olivine from schiller dunite. (d) Microcrystals of spinel in massive dunite.

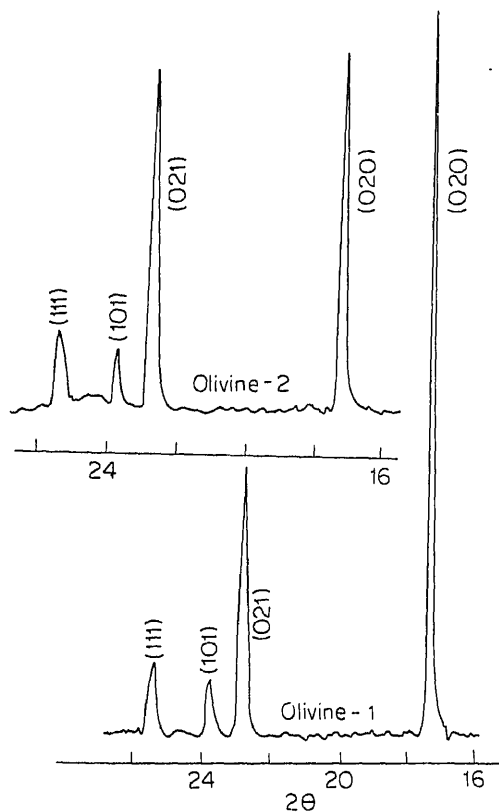


Figure 3. The relative high intensity of (020) x-ray reflection in olivine with perfect cleavage. Note the difference in intensity for olivine 2. Both the diffractograms were recorded at identical conditions (2000 counts).

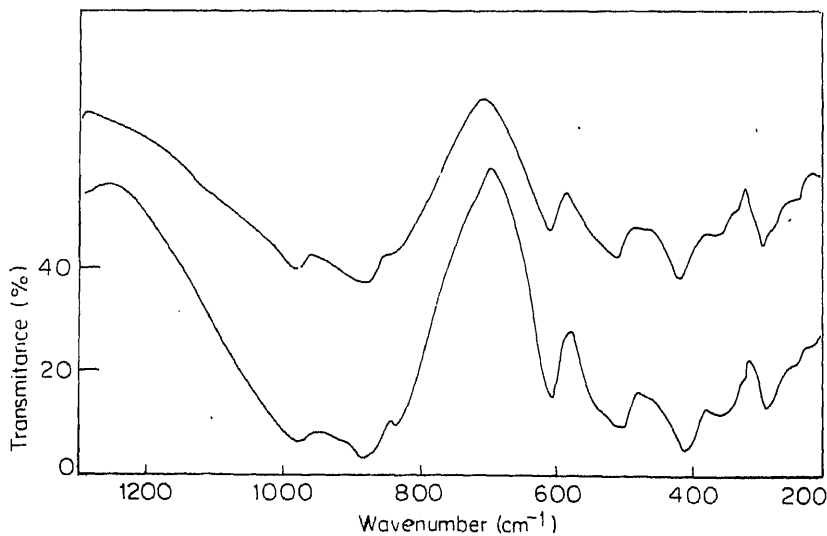


Figure 4. Infrared absorption spectra of olivine 1 with perfect cleavage and that of olivine 2.

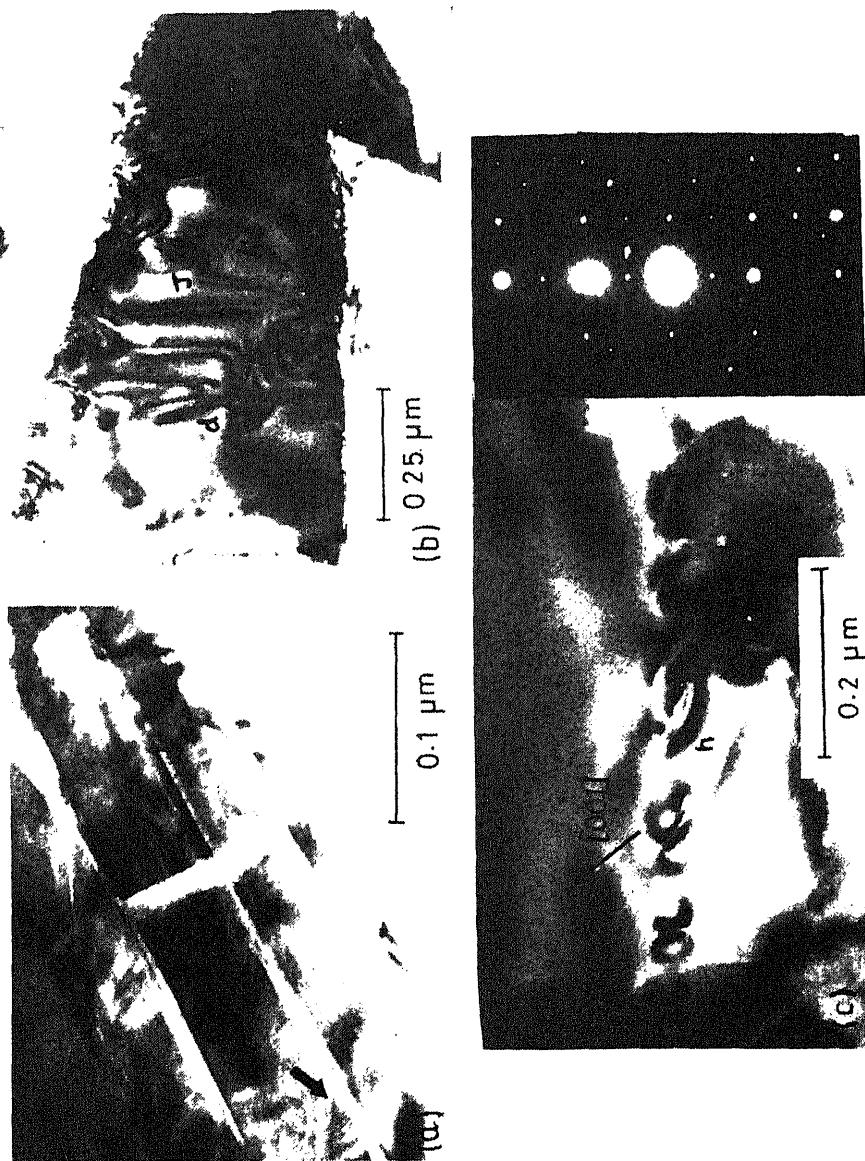
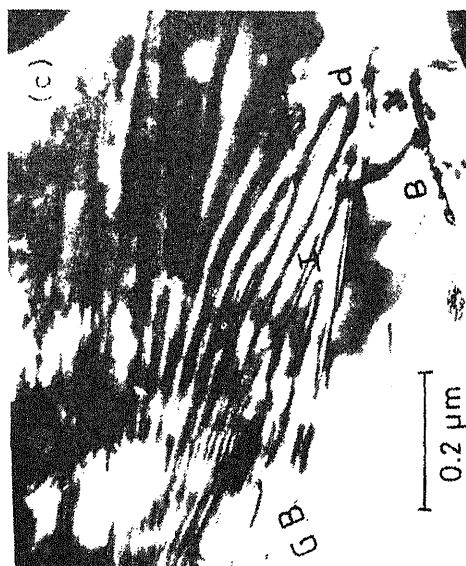
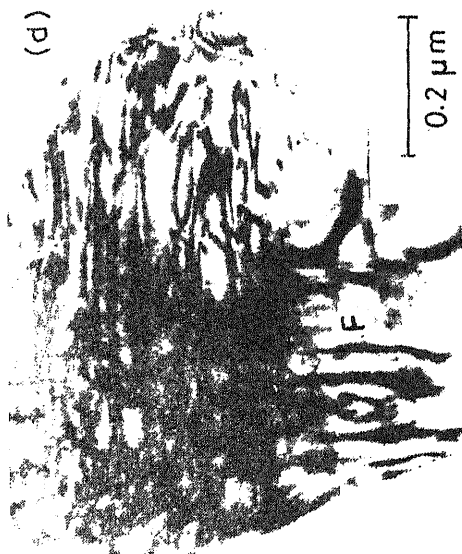
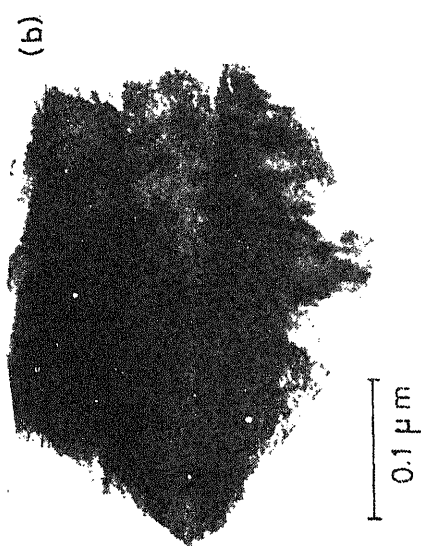


Figure 5. TEM micrographs of olivine 2. (a) Subgrains with (100) kink boundaries. Arrow indicates the bowing of screw dislocations. (b) Curved dislocations with jogs (*J*) kink (*K*) and dipoles (*d*). Note the picket fence structure (*p*) at the grain boundaries. (c) Helical dislocations (*h*) in highly recovered region and the corresponding selected area diffraction.



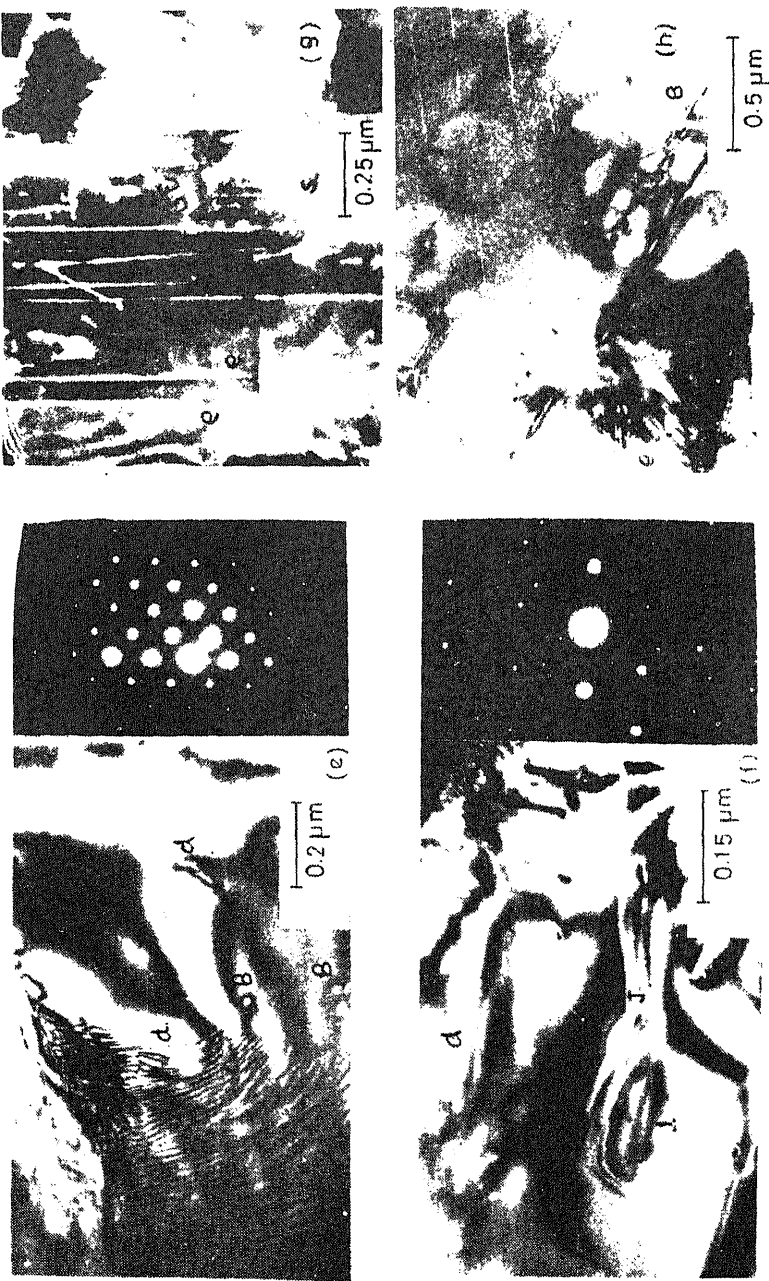


Figure 6. TEM micrographs of olivine 1. (a) Ill-developed kink-boundaries. (b) Nearly obliterated kink-boundaries with the dislocations anchored to the walls. (c) Curved dislocations (*J*) with kinks and dipoles (*d*). Note the region with low dislocation density (lower portion) after the low angle grain boundary (GB). The bubbles (*B*) are aligned to form a stringer. (d) Tangled dislocations, some of which are nearly linear. The stacking fault (*F*) is seen to the right side. (e) Recovered portion being separated from a region with curved dislocation by small angle grain boundary. Note dipoles (*d*) and clustered bubbles (*B*) with the corresponding selected area diffraction. (f) Highly recovered regions with low dislocation density being separated by a low angle grain boundary. Note the loops (*l*), dipoles (*d*) and jogs (*J*) with the selected area diffraction. (g) Well delineated edge dislocations (*e*) which align themselves to form small angle grain boundaries (GB). Infrequently present are the screw dislocations (SS). (h) Bubbles (*B*) individual as well as aligned into stringers and edge dislocations (*e*) in a recovered region.

Another important TEM observation with olivine 2 is the absence of straight dislocations particularly those parallel to low index directions. On the other hand the dislocations are highly curved, which is an indication of the dominance of climb. Jogs, kinks and dipoles are well evident in figure 5b, demonstrating the occurrence of extensive cross-slip of screw dislocations and the onset of climb. The loops and helical dislocations are noticed in olivine 2 (figure 5c), further supporting that climb is extensive. Compared to the results of experimentally deformed olivine, these observations indicate that olivine 2 is deformed above 1000° C where the influence of structural (Peierls) forces is less important, while climb mechanism dominates.

For olivine 1, the kink boundaries are ill-developed or nearly obliterated (figure 6a). Notably absent are the bowed screw dislocations across the subgrain boundaries. There is dominance of edge components. Jogs on these dislocations, that are anchored to (001) or (010) walls (figure 6b), are indicative of cross-slip. High density of nonlinear dislocations along with low angle grain boundaries are common in many crystals (figures 6c,d). Regions which are considerably recovered are also seen (figures 6e,f). The dislocation loops and dipoles may be limited by small angle grain boundary (figure 6e). On the whole, there is complete absence of straight dislocations in olivine 1. Dipoles, loops and networks are dominant, indicating that cross-slip is extensive and that climb mechanism predominates. In this respect, the substructure of olivine 1 has common features with that of olivine 2. However, well-isolated as well as collective edge dislocations, in the latter leading to small angle grain boundary, (figure 6g) is characteristic only of olivine 1. There are a few screws athwart the edge dislocations (figure 6g) indicating that the velocity of propagation for edge dislocations, are considerably different from that of screw dislocations. Another feature is the stacking fault fringes observed in olivine 1 (figure 6e). They are along the (100) plane. In olivine crystal structure, the oxygens lie in sheets parallel to this plane in approximately hexagonal close packing. Normally, stacking fault energy is high in olivine structure (Christie and Ardell 1976). However, it can be lowered in the presence of nonstructural chemical components, the presence of which may be inferred from the occurrence of bubbles (figures 6f,h). Green and Radcliffe (1975), and Green (1976), have found, by TEM studies, that minute bubbles exist in olivine which are nothing but fluid inclusions similar to noble gas bubbles in irradiated metals and oxides. Green and Radcliffe (1975) have shown that such bubbles contain H₂O and CO₂ rich volatile phases with higher amount of incompatible elements (K, Rb, Ba, Sr, U, Th and lighter rare earths). These bubbles will thus account for the place of residence of the incompatible elements in olivine 1.

Small amounts of volatile phases have profound mechanical weakening effect during deformation of olivine (Post 1973). Tests on hypersthene revealed mechanical weakening by water (Griggs 1967, 1974) in accordance with the hydrolytic model proposed by Griggs and Blacic (1965). Dunite samples deformed in hydrous confining media are weaker than vacuum-dried samples (Blacic 1972), deformed in anhydrous conditions. Large strains resulting from hydrolytic weakening requires diffusion of H₂O, H⁺, or OH⁻. In the orthosilicates, dislocation glide motion may be achieved by the breakages of metal-oxygen bonds (Christie and Ardell 1976), which may be further assisted by the incompatible elements possibly present as dissolved species along with volatile constituents. Breakage of metal-oxygen bonds which is of lower bond energy than of Si-O bonds may lead to parting and, further, to the development of cleavage. When the stress normal to a crystallographic plane exceeds a critical value, the plane

(010) planes for olivine, the crystals tend to cleave along these planes. Thus, the critical stress assisted by the modified interatomic forces which in turn are due to the presence of nonstructural chemical constituents, have caused the development of perfect cleavages in olivine 1.

4. Conclusions

The residual ultramafites to Chalk Hills are deformed at comparatively higher temperatures. Both massive and the schiller dunites have the same deformational impress. However, the latter has higher amount of nonstructural chemical constituents which can hydrolytically weaken the olivine crystals generating larger strains. Dislocation glide motion may thus be achieved by the breakage of metal-oxygen bonds leading to the development of perfect cleavage on the crystallographic planes with largest interplanar spacings, under compressive stress. The rare development of perfect cleavages may require the unique combination of higher incompatible elements and volatile concentrations as well as higher temperature and stress conditions. There are no quantitative data for the latter parameters. The higher abundance levels of incompatible elements may be due to the improper separation of the partial melt from the residue in isolated small pockets.

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References

- Azároff L V and Buerger M J 1958 *The powder method in X-ray crystallography* (New York: McGraw-Hill) p. 258
- Blacic J D 1972 *Geophys. Monogr.* **16** 109
- Carter N L 1976 *Rev. Geophys. Space Phys.* **14** 301
- Carter N L and Ave'Lallemant H G 1970 *Geol. Soc. Am. Bull.* **81** 2181
- Christie J M and Ardell A J 1976 *Electron Microscopy in mineralogy* (ed.) H R Wenk (Berlin: Springer-Verlag) p. 374
- Coleman R G 1977 *Ophiolites* (Berlin: Springer-Verlag) p. 15
- Green H W 1976 *Electron microscopy in mineralogy* (ed.) H R Wenk (Berlin: Springer-Verlag) p. 443
- Green H W and Radcliffe S V 1972a *Earth Planet. Sci. Lett.* **15** 239
- Green H W and Radcliffe S V 1972b *Geol. Soc. Am. Bull.* **83** 847
- Green H W and Radcliffe S V 1975 *Geol. Soc. Am. Bull.* **86** 846
- Griggs D T 1967 *Geophys. J.* **14** 19
- Griggs D T 1974 *J. Geophys. Res.* **79** 1653
- Griggs D T and Blacic J D 1965 *Science* **147** 292
- Hawkes H E 1946 *Am. Mineral.* **31** 276
- Klug H P and Alexander L E 1974 *X-ray diffraction procedures* (New York: Wiley Interscience) p. 368
- Murthy S R N 1982 *Petrogenesis of the ultramafic rocks from high grade terrains*, Ph.D. thesis, Indian Institute of Science, Bangalore p. 174
- Post R L 1973 *The flow laws of Mt. Burnett Dunite*, Ph.D. thesis, Univ. of California, Los Angeles, p. 272
- Weiss R J 1966 *X-ray determination of electron distributions* (New York: John Wiley) p. 96
- Wyllie P J 1971 *The dynamic earth* (New York: McGraw-Hill) p. 70
- Yoder H S and Sahama T G 1957 *Am. Mineral.* **42** 475

Granite-molybdenite association in Kerala in relation to taphrogenic metallogeny

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Abstract. Two molybdenite-bearing granites from the Kerala region, namely, the Chengannoor and the Ambalavayal granites are discussed here in terms of their salient geochemical characters. The geochemistry of the host rocks indicates a peraluminous, calc-alkaline to alkali-calcic nature and the element levels show good fit with the proposed 'finger-prints' for granite-molybdenite systems. Fluid inclusion studies suggest that Mo was probably partitioned in vapour phase, which subsequently combined with available sulphur. Regional distribution of molybdenite strongly favours the view of a molybdenum province in the southern part of the Indian shield. Associations of rare metal mineralization with the rift-controlled acid magmatic phase in the Kerala region are suggestive of a late Precambrian — Early Palaeozoic taphrogenic metallogeny.

Keywords. Granite; molybdenite; geochemistry; fluid inclusions; continental margin; taphrogenic metallogeny.

1. Introduction

The Peninsular India comprises largely Precambrian rocks including granites, granitic gneisses, migmatites and charnockites. The Kerala region forms a part of the southern peninsular shield and of the western continental margin of India. Occurrences of various younger acid intrusives in the Precambrian crystallines of the region with probable relation to rift-tectonics are noted (Santosh and Nair 1983; Nair and Santosh 1983). Recent studies attribute a late Precambrian-early Palaeozoic age to the acid magmatic phase (Nair and Vidyadharan 1982; Odom 1982; Soman *et al* 1983). Even though molybdenite findings are reported from Kerala (GSI News 1981) and from the adjacent terranes of Tamil Nadu, Andhra Pradesh and Sri Lanka (Meenakshisundaram *et al* 1976; Subramonian *et al* 1979), detailed studies on the nature of mineralization and its correlation with the petrochemical characters of the host rocks are lacking. The present work summarises the salient geochemical characters of the host rocks and physicochemical aspects of associated molybdenite mineralization based on studies on two molybdenite-bearing granites of Kerala, namely, the Chengannoor granite of Alleppey District and the Ambalavayal granite of Wynad District.

2. Geological setting

Both the Chengannoor and the Ambalavayal granites are emplaced within Precambrian high grade metamorphics (figures 1a, b). The Chengannoor granite, a roughly E-W elongated body, is intrusive into charnockite-cordierite gneiss sequence, whereas the

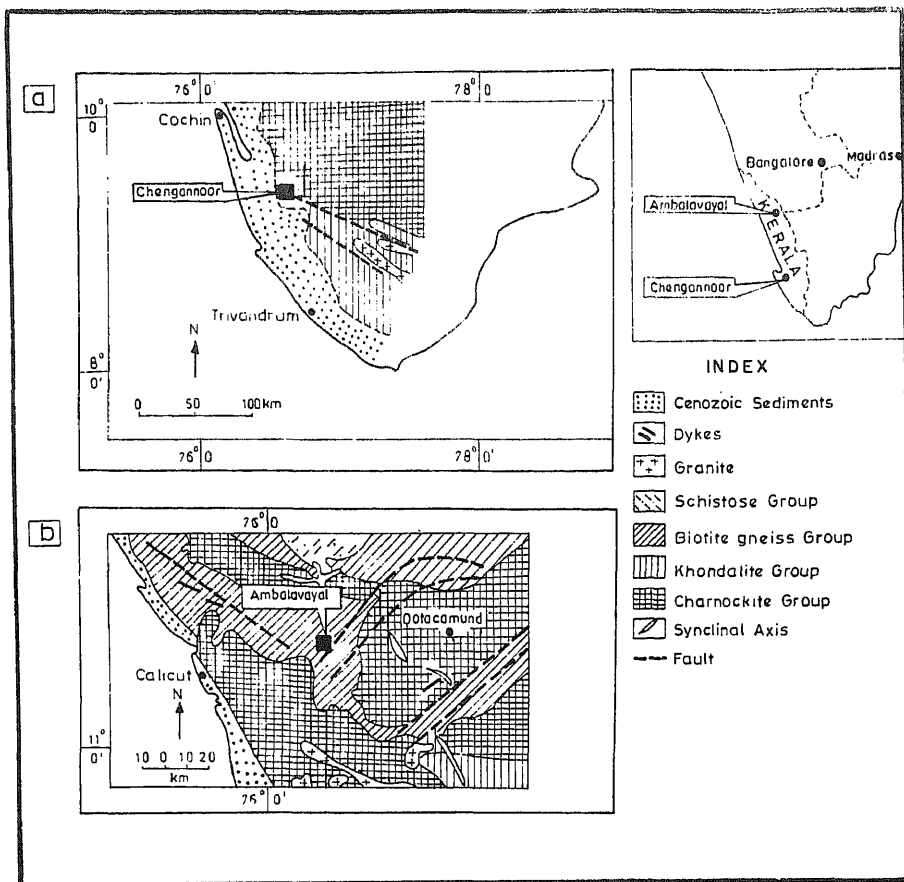
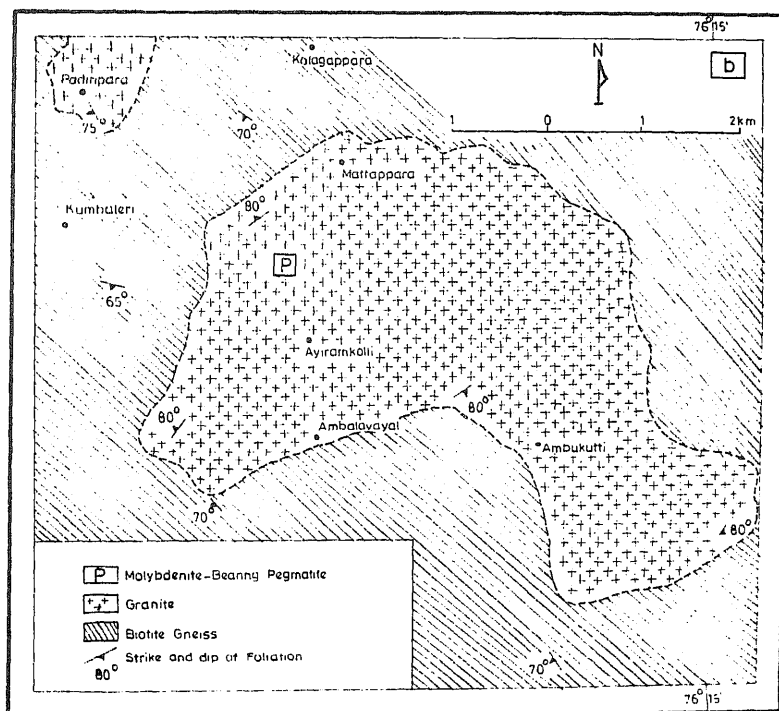
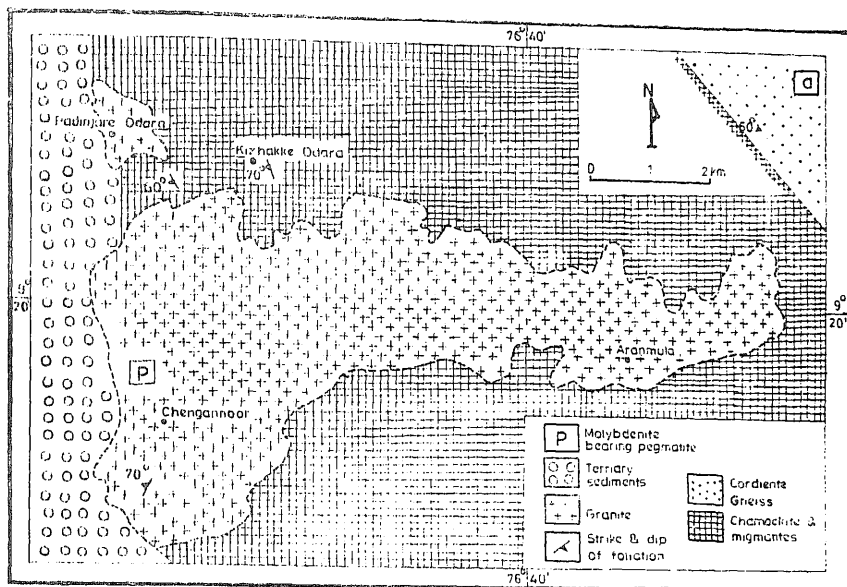


Figure 1. Regional geological map (after Rao 1976) (a) of southern Kerala showing location of Chengannoor granite; (b) of northern Kerala showing location of Ambalavayal granite.

country rock for the Ambalavayal granite is biotite-gneiss (figures 2a, b). Association of gravity 'low' with the Ambalavayal granite, as observed from the Bouguer anomaly map of Manantoddy-Gudalur area implies a subsurface extension of the body (Queresby *et al* 1969).

The Chengannoor granite occurs along the Achenkovil lineament, a major fault-lineament of southern Kerala which extends in the NW-SE direction. The Ambalavayal granite occurs along the extension of the NW-SE trending Bavali lineament, a deep-seated fault which is known to have controlled the emplacement of acid and basic magmatic rocks (Nair and Santosh 1983). It also falls along the extension of the Kabani fault, an ENE-WSW trending taphrogenic lineament which marks the boundary between the Archaean granulitic mobile belt and the Dharwar craton (Katz 1978).

Pegmatites and aplites of varying dimensions (generally less than 2 m in width) traverse both the granites. In Ambalavayal, the pegmatites contain large crystals of pink feldspar (measuring upto 20 cm), slightly altered hornblende (measuring upto 15 cm) and quartz. A local biotite-bearing variety is also seen at Ambalavayal.



3. Petrography

The Chengannoor granite is a pink potassic granite having mesoperthitic K-feldspar, plagioclase of oligoclase-andesine composition and quartz as principal minerals with hornblende as the mafic constituent. The geology and petrochemistry of the granite, attributing a post-kinematic evolution, are discussed in detail elsewhere (Santosh and Nair 1983).

Pink feldspar and interlocking quartz form the principal minerals in the Ambalavayal granite with hornblende as the conspicuous mafic constituent, the lineation of which imparts a general gneissosity to the granite. The feldspar assemblage of the granite is of subsolvus nature with subequal amounts of rarely perthitic K-feldspar and sodic plagioclase, which, along with quartz, are set in a hypidiomorphic granular texture (figure 3). Locally, the K-feldspar grows into megacrysts exhibiting faint microcline cross-hatching. In addition to the strongly pleochroic greenish hornblende, sphene, apatite and zircon also occur in minor amounts. Calcite, epidote and muscovite are seen as alteration products. The opaque is generally represented by magnetite, which shows hematite replacement along the grain boundaries. Molybdenite occurs as minor stringers and flakes along grain boundaries. Other sulphide phases present are pyrite and chalcopyrite, the latter being generally found as inclusions in the former. Based on preliminary petrochemical assessments, Nair *et al* (1982) deduced a late to post-kinematic emplacement history for the Ambalavayal granite.

4. Molybdenite mineralization

In Chengannoor, molybdenite occurs as stringers in pegmatites and as local disseminations in the charnockitic country rocks. Molybdenite, the major sulphide phase of the granite and pegmatites of Ambalavayal, is found as disseminations in the granite or as flaky aggregates in the pegmatites, associated with other silicate minerals. The major part of mineralization here is localised along the contact of the granite with the country rock and in the endocontact region the aplitic granite shows ubiquitous disseminations of molybdenite. Flaky aggregates measuring upto 20 cm were recovered from the pegmatites of the nearby Ayiramkolli quarry.

Flakes of molybdenite exhibit a bluish schiller. Under incident light, it shows high reflectivity, strong bireflectance and ink-blue anisotropism. It is seen as leafy aggregates adhering to the silicate minerals and more commonly shows preferential distribution along grain boundaries of quartz and feldspar (figure 4). It also occurs as thin stringers along cleavage traces and microfractures of feldspar and hornblende.

5. Geochemical characters

The host rock for molybdenite mineralization in Ambalavayal is adamellite (quartz monzonite) as depicted by $\text{Na}_2\text{O}-\text{K}_2\text{O}$ relationship (figure 5). Comprehensive chemical data show aerial transition to granodiorite also. The Chengannoor granite similarly shows an adamellite-granodiorite transition. Some salient geochemical data of both the granites as applicable to granitic host rocks for molybdenite are given in table 1.

Both the granites are of peraluminous nature, typical of all molybdenite-bearing

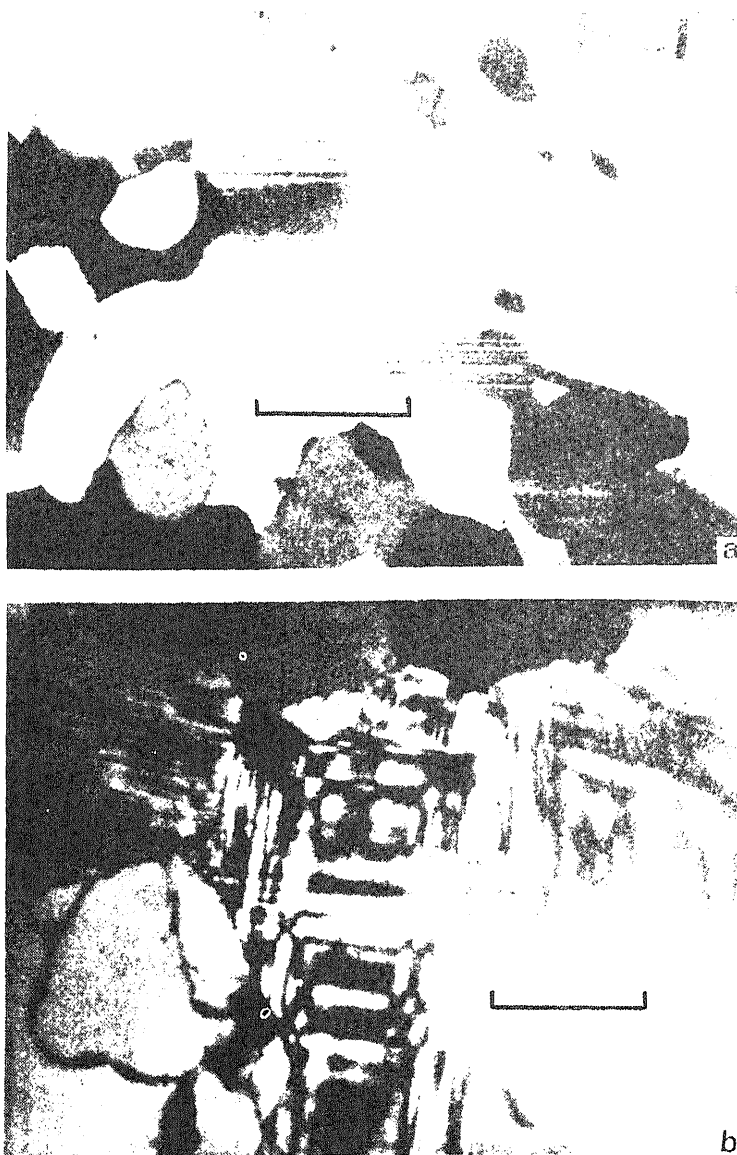


Figure 3. Photomicrographs (transmitted light) of Ambalavayal granite showing the sub-solvus feldspar assemblage (a) albite with lamellar twinning; (b) microcline megacryst. Bar scales represent 3 mm.

granites. The A/CNK values of Ambalavayal granite range from 1.09 to 1.30 with a mean at 1.18 whereas the Chengannoor granite shows an average of 1.39. The SiO_2 (av. 74.51) and Na_2O (av. 3.48) of Chengannoor granite are comparable with those of typical granite host rocks as given by Mutschler *et al* (1981) and comparable with the values of the molybdenum-bearing granite of Wankatana River, Sierra Leone (Mamao 1962). The lower K_2O values (av. 3.64) of Ambalavayal and SiO_2 (av. 69.39) of

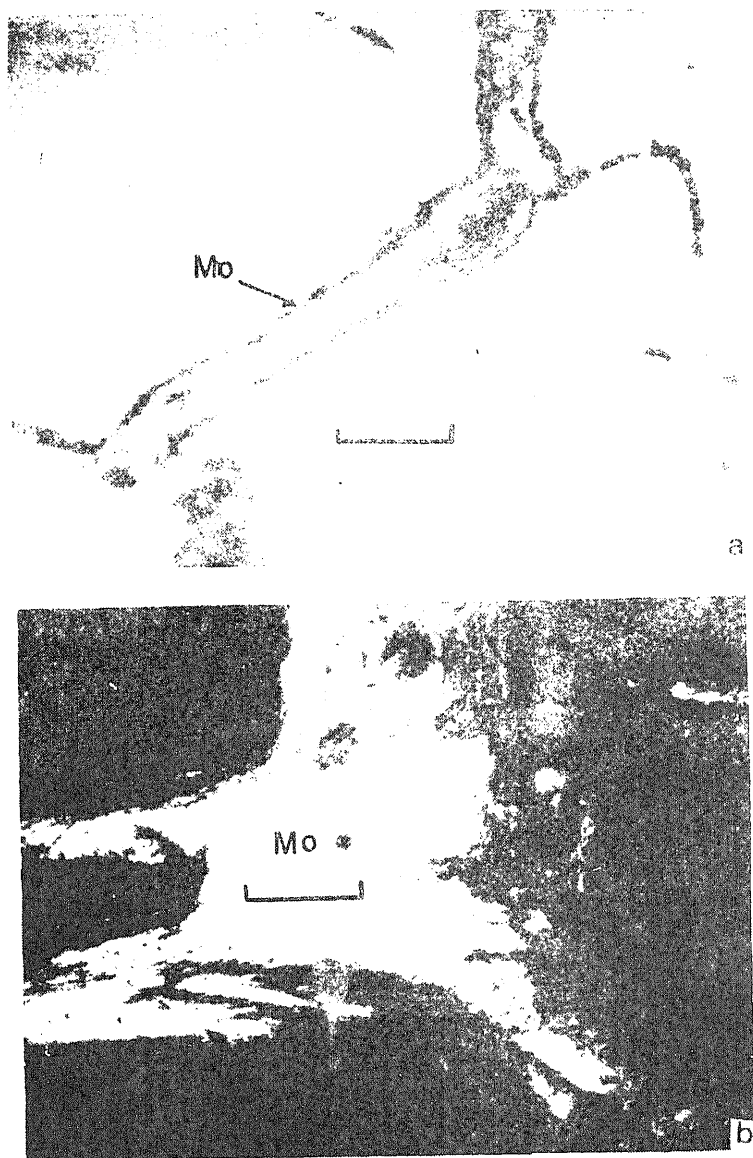


Figure 4. (a) and (b) Photomicrographs (incident light) of molybdenite flakes occurring along grain boundaries of silicates in Ambalavayal granite. Bar scales represent 1.5 mm.

Chengannoor show deviation, but similar cases are recognised elsewhere also (Mutschler *et al* 1981, table 7). Normative $Q - Ab - Or$ plots of both the rock types fall within the field of magmatic granites as defined by Tuttle and Bowen (1958). They also lie within the compositional spectrum of unaltered source rocks for molybdenite (figure 6). The mean $Q + Ab + Or$ for Ambalavayal (86.08) and Chengannoor (84.18) granites show good fit with the estimated mean of 80%. When normalised to 100%, the mean values of Q (44.34), Ab (30.59) and Or (25.07) of the Ambalavayal granite and

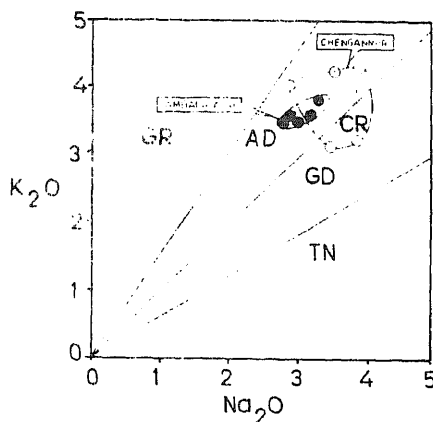


Figure 5. Na-O-K-O relations (after Harpum 1963). Fields of host rocks from Chengannoor and Ambalavayal are shown.

Q (34.28), Ab (29.51) and Or (36.21) of the Chengannoor granite are consistent with the finger-prints, $Q \geq 20\%$, $Ab \geq 20\%$ and $Or \geq 20\%$ for granite-molybdenite systems.

A classification scheme for the host rocks as proposed by Westra and Keith (1981) is adopted here. Thus, K_2O values at 57.5% SiO_2 ($K_{57.5}$) for Ambalavayal range from 2.65 to 3.02 with a mean at 2.81, indicating the source rock to be a high K calc-alkaline type. Mean $K_{57.5}$ of Chengannoor (3.49) shows it to be alkali-calcic. Plots in mol%

Table 1. Salient geochemical features of Ambalavayal and Chengannoor granites.

	Al-1	Al-2	Al-3	Al-4	Al-5	Mean	CR	GS	WR
SiO_2	76.96	71.95	74.00	73.94	75.72	74.51	69.39	75.73	74.50
TiO_2	0.12	0.18	0.18	0.19	0.12	0.16	0.38	0.11	
Al_2O_3	12.24	15.30	13.26	14.28	13.28	13.67	15.65	12.62	13.60
Fe_2O_3	0.78	1.04	1.30	0.84	0.76	0.94	2.68	0.78	1.40
FeO	1.40	1.94	1.99	2.48	1.54	1.87	1.05	0.49	0.30
MgO	0.48	0.48	0.48	0.16	0.48	0.42	1.62	0.49	
CaO	1.35	1.12	1.57	1.57	1.57	1.44	0.53	0.69	1.00
Na_2O	2.81	3.26	3.23	2.91	2.83	3.01	3.48	3.25	3.90
K_2O	3.76	3.78	3.64	3.55	3.49	3.64	4.19	5.29	5.00
$Q + Ab + Or$	88.21	84.44	85.92	86.05	85.76	86.08	84.18		
$K_{57.5}$	2.81	3.02	2.83	2.76	2.65	2.81	3.49		
$Al_2O_3/Na_2O + K_2O +$ $+ CaO$ (mol. %)	1.10	1.30	1.09	1.22	1.17	1.18	1.39		
Mo (ppm)	19	16	20	40	6	20.2	9		

Al-1 to Al-5 — Analyses of Ambalavayal granite (after Nair *et al* 1982). CR — Mean of 8 analyses of Chengannoor granite (after Santosh and Nair 1983). GS — Mean of Granite-Molybdenite systems (after Mutschler *et al* 1981, table 2). WR — Analysis of Wankatana River granite (from Marmo 1962).

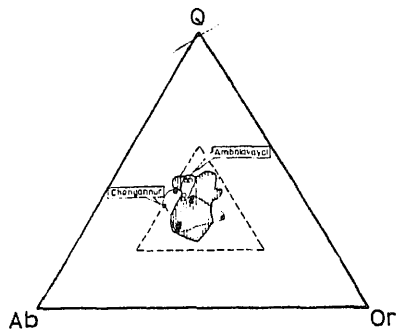


Figure 6. Q–Ab–Or plots of the host rocks. Inner triangle represents the field of magmatic granites (Tuttle and Bowen 1958) and stipled area represents the field of unaltered source rocks for molybdenum (after Mutschler *et al* 1981).

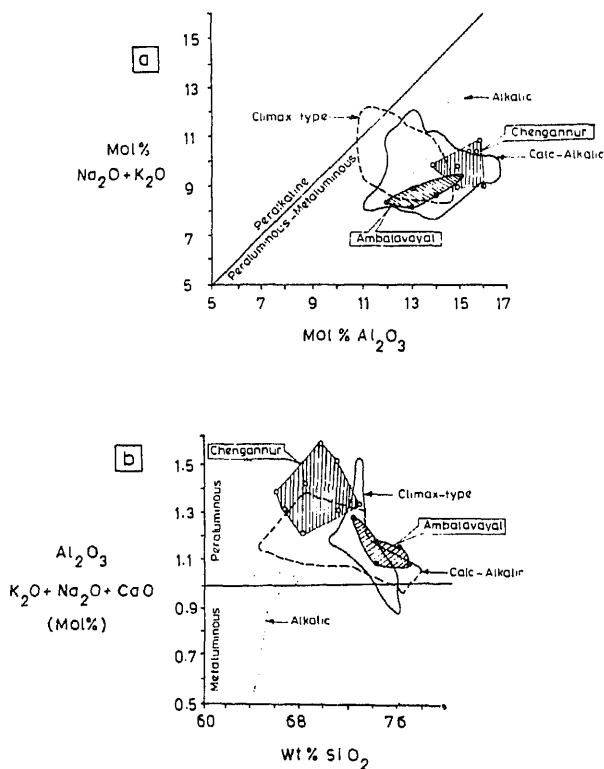


Figure 7. Host rock classification diagrams (after Westra and Keith 1981) (a) mol. % $\text{Na}_2\text{O} + \text{K}_2\text{O}$ vs Al_2O_3 ; (b) mol. % $\text{Al}_2\text{O}_3 / \text{CNK}$ vs wt. % SiO_2 . Fields of Chengannur and Ambalavayal granites are shown.

$\text{Na}_2\text{O} + \text{K}_2\text{O}$ vs Al_2O_3 (figure 7a) and mol% A/CNK vs wt% SiO_2 (figure 7b) further substantiate the calc-alkalic affinity of the Ambalavayal granite and the transition-to-alkali-calcic of Chengannoor.

Whole-rock Mo levels of Ambalavayal granite show an average of 20.2 ppm and that of Chengannoor show 9 ppm, being much higher than the 0.4–0.7 ppm range recognised in granitic host-rocks associated with molybdenum deposits (Pokalov 1977).

6. Fluid inclusion studies

A number of doubly-polished thin plates of quartz associated with molybdenite from pegmatites of Ambalavayal granite were prepared for fluid inclusion studies. Preliminary studies reveal the presence of primary, pseudosecondary and secondary inclusions with variable phase-ratios (figure 8), corresponding to a multistage growth regime of the mineral (Roedder 1979). Predominantly gaseous and liquid-gaseous inclusions with high vapour-liquid ratios are the most frequent types among them. In some plates, inclusions with variable gas-ratios are found to co-exist syngenetically along the same micro-fracture. Inclusions showing a CO_2 rich phase at room temperature, indicative of entrapment from a volatile-rich medium, are also present. Although inclusions with solid phases of NaCl (upto 20% by volume) are locally present, they are generally infrequent.

Studies on fluid inclusions in quartz from the Thiruvalla pegmatite, related to the Chengannoor granite also reveal the dominance of gas-liquid and liquid-gaseous inclusions. Heating data suggest a dominant crystallization phase of quartz between 320–400° C (Santosh 1981).

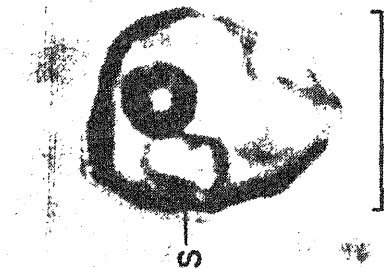
7. Physico-chemical parameters of mineralization

Based on petrochemistry, late-magmatic potassic alterations in Chengannoor and Ambalavayal granites are recognised (Santosh and Nair 1983; Nair *et al* 1982). Close spatial relationship between molybdenite mineralization and late magmatic potassium enrichment is characteristic of most of the molybdenite deposits of the world (*cf* Mutschler *et al* 1981; Pokalov 1977). The occurrence of disseminated molybdenite in K-feldspar of aplites and pegmatites in the Ambalavayal granite is consistent with similar observations (Wallace *et al* 1978) illustrating the close relation between K and Mo enrichment during final magma crystallization. In agreement with the experimental deduction of Isuk and Carman (1981), it may be assumed that late magmatic alkali-silicate melts were the carriers of Mo in these granites.

In alkali-calcic magmas, initial level of Mo is consistently low. Its concentration is achieved by magma differentiation, fractional crystallization, liquid-state thermogravitational diffusion and magma convection (Westra and Keith 1981). Mo concentration in the melt during fractional crystallization is governed by its speciation. It may occur as Mo^{4+} and can substitute for Ti^{3+} , Fe^{3+} and Al^{3+} in early crystallizing minerals like magnetite, sphene and ilmenite. In granodioritic, quartz monzonitic and granitic melts, molybdenum is present as a molybdate anion complex and is concentrated in the residual melt. Suitable conditions of precipitation occur mainly in the pegmatitic-pneumatolytic stage where a lowering of pressure and/or temperature leads to resurgent 'boiling' triggering Mo deposition (Krauskopf 1964).



a



b



Figure 8. Photomicrographs of fluid inclusions in quartz associated with molybdenite from pegmatites of Ambalavayal granite. (a) Predominantly gaseous inclusion. (b) Gas-liquid inclusion with solid phase of NaCl. (c) Primary gas-liquid inclusions. (d) Pseudo-secondary gas-liquid inclusions. The bar scales in (a) and (b) represent 50 millimicrons and those in (c) and (d) represent 200 millimicrons.

A phase of vapour saturation in the magma with subsequent release of low density pneumatolytic fluids and high density hydrothermal liquids is indicated by the abundance of gaseous CO₂-rich and gas-liquid inclusions in the quartz studied. Experimental data of Isuk and Carman (1981) also indicate a preferential partitioning of Mo into the vapour phase. Syngenetic inclusions with various gas-liquid ratios imply that the vapour saturation subsequently resulted in resurgent 'boiling' of the media (Roedder 1971).

The salinity of the transporting medium was only moderate, as aqueous-liquid mixed inclusions with solid phase of NaCl are generally rare. Recent investigations (Isuk and Carman 1981) show evidences of molybdenum-silicate and/or molybdenum hydroxylated silicate complexing, avoiding the necessity to invoke the mediation of chloride complexes, usually recognised for ore transport in rare-metal pegmatites (Jaireth *et al* 1982). It is probable that in the case of these granites, Mo was carried as silicate complexes which eventually exsolved during vapour generation (Whitney 1975) releasing Mo into potential ore-fluids. The occurrence of molybdenite generally associated with silicate mineral phases in the granite and pegmatites substantiate this view.

Experimental data (Isuk and Carman 1981) show that Mo-silicate complexes become less stable at temperatures higher than 650° C. Simultaneous with vapour-separation, there is considerable drop in temperature, precipitating Mo, which subsequently combined with the available sulphur, as indicated by the 320–400° C range in Chengannoor. Pokalov (1977) gives a temperature range of 300–370° C recorded from gas-liquid inclusions for the mineralizing media of fine-flaky molybdenite associated with quartz. Thompson (1982) also observes that calc-alkaline and alkali-calcic molybdenum deposits are invariably formed at temperatures below 400° C.

8. Taphrogenic metallogeny

Peraluminous nature and calc-alkaline to alkali-calcic affinity are typical of some of the granitic intrusives of Kerala. Petrochemical features including the higher levels of transition elements favour their origin from magmas evolved at deep-crustal or mantle-depths. Preferential dispositions of the intrusives along regional taphrogenic lineaments are conclusive of magmatism related to distensional tectonics associated with rifting of the continental margin (Nair and Santosh 1983; Nair *et al* 1983). Mantle-origin of the magma for those rocks genetically related to molybdenum deposits and rift-systems are noted elsewhere also (*cf* Westra and Keith 1982; Walser and Einarsson 1982).

Late Precambrian ages are known for some of the granites of Kerala (*eg* Nair and Vidyadharan 1982). Recent radiometric data attribute early Palaeozoic ages to the Chengannoor (550 m.y., Soman *et al* 1983) and the Ambalavayal (505 ± 20 m.y., Odom 1982) granites. The widely recognised pegmatitic phase in Kerala which carry rare metal and gem-stone mineralization (Paulose 1957; Soman *et al* 1982) also yield early Palaeozoic ages, being genetically related to the granitic phase. These pegmatites also show preferential localisation and orientation along NW-SE and E-W trending fractures. In mobile belts, tensional faulting is known to trigger taphrogenic mineralization (Gableman 1971). The taphrogenic faults supply juvenile mineralization through the release of metals and volatile fluids from the lower crust or mantle. Thus

the metallogeny associated with the granitic and pegmatitic phases in the Kerala region suggests an episode of late Precambrian-early paleozoic taphrogenic metallogeny.

Molybdenite occurrences in diverse rock types are reported from the adjacent terranes of Tamil Nadu, Andhra Pradesh and Sri Lanka (Meenakshisundaram *et al* 1976; Subramonian *et al* 1979). The regional distribution of molybdenite is in favour of the concept of a molybdenum province in the southern part of the Indian shield. Spatial relationship of individual molybdenite occurrences and their parent rocks with seismically active deep faults, margins of shear belts or major fault-lineaments are brought out in figure 9, which further suggest a regional metallogenic episode related to the taphrogenesis of the continental margin.

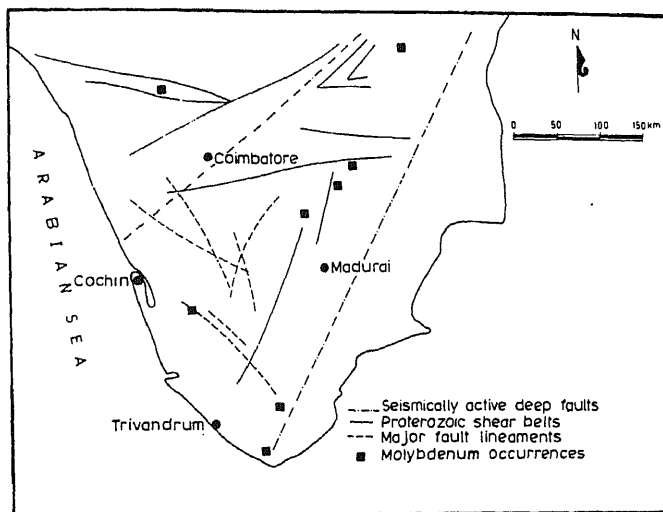


Figure 9. Part of Peninsular India showing the locations of reported molybdenite occurrences, Proterozoic shear belts (after Drury and Holt 1980), seismically active deep faults (after Grady 1971) and major fault lineaments.

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References

- Drury S A and Holt R W 1980 *Tectonophysics* **65**T
- Gableman J W 1971 *Metallotectonic evidence for continental drift* Rep. Divn. of Production and Materials Management (Washington, U.S.A.) 41
- Geological Survey of India News 1981 **2** 12
- Grady J C 1971 *J. Geol. Soc. India* **12** 56
- Harpur J R 1963 *Rec. Geol. Surv. Tanganyika* **10** 80

- Isuk E E and Carman J H 1981 *Econ. Geol.* **76** 2222
- Jairath S, Santosh M and Mohan P C 1982 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **91** 143
- Katz M B 1978 *J. Geol. Soc. India* **5** 185
- Krauskopf K B 1964 *Econ. Geol.* **59** 22
- Marmo V 1962 *Overseas Geol. Miner. Res.* **8** 416
- Meenakshisundaram S, Subramonian K S and Vemban N A 1976 *Geol. Surv. India Misc. Pub.* **23** II 580
- Mutschler F E, Wright E G, Luddington S and Abbot J T 1981 *Econ. Geol.* **76** 874
- Nair N G K and Santosh M 1983 *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* (in press)
- Nair N G K, Santosh M, Thampi P K and Balasubramonian G 1982 *Q. J. Geol. Min. Met. Soc. India* **54** 28
- Nair N G K, Santosh M and Thampi P K 1983 *N. Jb. Miner. Abh.* (in press)
- Nair M M and Vidyadharan K T 1982 *J. Geol. Soc. India* **23** 46
- Odom L A 1982 *Final Report* (unpublished), U.N. Case No. 81-10084, p. 10
- Paulose C V 1957 *Econ. Geol.* **52** 702
- Pokalov V T 1977 in Smirnov V I (ed.) *Ore deposits of the USSR* (London: Pitman) Vol. 3, 125
- Queresby M N, Krishnabrahmam N and Aravamadhu P S 1969 *J. Indian Geos. Assoc.* **2** 77
- Rao P S 1976 *Geol. Surv. India Misc. Pub.* **34** III 51
- Roedder E 1971 *Econ. Geol.* **66** 98
- Roedder E 1979 in *Geochemistry of hydrothermal ore deposits* (ed.) H L Barmes (New York: John Wiley) 684
- Santosh M 1981 *Petrology and fluid inclusion studies of Thiruvalla and Odara pegmatites of Kerala and Bandarchua and Khuronia pegmatites of Bihar*, M.Sc. thesis (unpublished) University of Roorkee, Roorkee, India 59
- Santosh M and Nair N G K 1983 *J. Geol. Soc. India* **24** 291
- Soman K, Golubyev V N and Santosh M 1983 *Indian J. Earth Sci.* **10** (in press)
- Soman K, Nair N G K, Golubyev V N and Arkelyan M N 1982 *J. Geol. Soc. India* **23** 458
- Subramonian K S, Meenakshisundaram S, Sarkar G P and Murthy M V N 1979 *Geol. Surv. India Misc. Pub.* **34** II 193
- Thompson T B 1982 *Econ. Geol.* **77** 709
- Tuttle O F and Bowen N L 1958 *Geol. Soc. Am. Mem.* **74** 153
- Wallace S R, Mackenzie W B, Blair R G and Muncaster N K 1978 *Econ. Geol.* **73** 325
- Walser G and Einarsson O 1982 *Geol. Rdsch.* **71** 213
- Westra G and Keith S B 1981 *Econ. Geol.* **76** 844
- Whitney J A 1975 *Econ. Geol.* **70** 346

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